

Classical Mechanics II: Midsemester examination

Total: 35 marks

Time: 2 hrs.

For all questions below, you may find it useful to use Newton's laws as a guide/check.

(1) Consider a particle moving subject to a time-dependent Lagrangian $L = \frac{1}{2}Ct^A\dot{x}^2$, with C, A constant parameters.

(a) Find the equation of motion and solve for the particle trajectory. [3 mks]

(b) Consider a redefinition to the variable $y = t^{A/2}x$. Find the new Lagrangian in terms of the variable y . Find the new equation of motion and solve this. [5 mks]

(c) Are there any conserved quantities (in both variables)? [2 mks]

(2) Consider a particle of mass m attached to a spring of constant k and orbiting in a plane around a fixed point.

(a) What is the Lagrangian for this system? [2mks.]

(b) Find the equations of motion, using possible conserved quantities. [2 mks.]

(c) When does the particle have no radial motion, for fixed angular momentum? [3 mks]

(3) Consider a particle of mass m moving in a (z -independent) cylindrical potential $U(r) = -\frac{\alpha}{r}$, where $r = \sqrt{x^2 + y^2}$, with x, y, z being cartesian coordinates. The particle has constant z -momentum p_z .

(a) Find the equilibrium radial distance from the effective potential as a function of the angular momentum l about the z -axis. [4 mks.]

(b) Find the time-dependence of the cartesian coordinates x, y, z for this equilibrium trajectory. Assume some convenient initial conditions. What is the shape of the trajectory qualitatively? [5 mks].

(4) Consider a particle of mass m moving in a 1-dimensional potential $V = kx^2 + \lambda x^4$, with k, λ being arbitrary real parameters, $\lambda > 0$.

(a) For $k > 0$, find the equilibrium position of the particle. Expand the potential about this position and find the frequency of small oscillations about it. [3 mks]

(a) For $k < 0$, find the equilibrium position of the particle and the frequency of small oscillations about it. [6 mks]