Time Dependent Phenomena in String Theory

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hep-th/0610053, hep-th/0602107, Sumit Das, Jeremy Michelson, KN, Sandip Trivedi; hep-th/0412337, David Morrison, KN;
hep-th/0406039, David Morrison, KN, Ronen Plesser.]

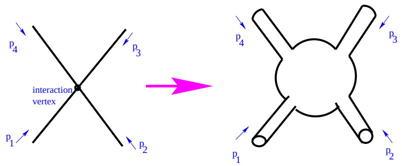
- Overview of string theory ...
- Cosmologies and their holographic duals.
- Phases of unstable 3-complex dimensional geometries.

A broad overview

- Four fundamental forces in Nature: gravity, electromagnetism, weak and strong nuclear forces.
- Besides gravity, remarkable progress in our understanding via quantum mechanics $\rightarrow$ Standard Model of elementary particles.
- Theoretical framework is quantum field theory.
- Einstein's general relativity predicts that gravity become strong in $e.g.$ black holes and the very early universe.
- Would like a quantum theory of gravity too.

Barebones of string theory

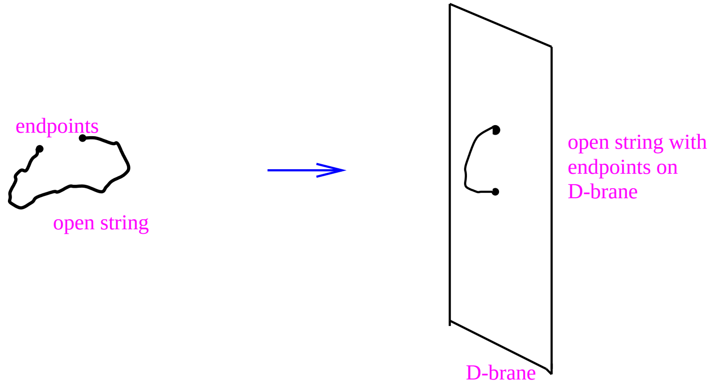
- Promising theory of quantum gravity: consider 1-dimensional extended objects.



- Lorentz invariant smearing of interactions: short distance loop divergences controlled.
- Low energy physics: General Relativity, Yang-Mills, …
- Perturbatively, strings must propagate in at most 10 dimensions for quantum consistency.
- Rich consistent mathematical structure.
- No experiment to date: energies naively too high.

String spectra, D-branes, . . .

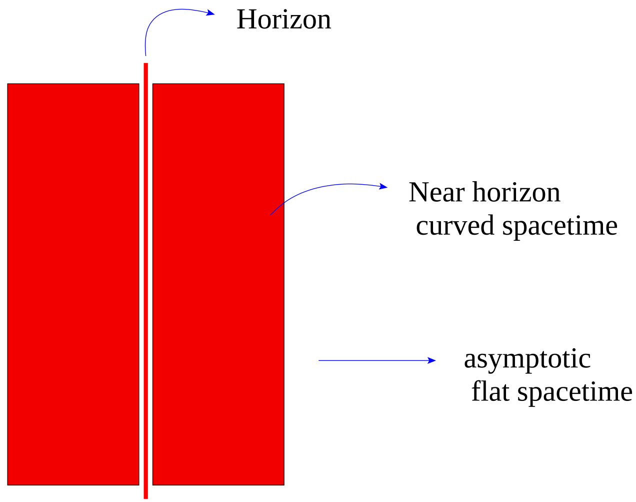
- Strings come in two types: open (segments) and closed (loops).
- Massless (bosonic) fluctuations: open $\longrightarrow$ vector (gauge) fields; closed $\longrightarrow$ gravitons, scalars, tensors.
- There exist extended dynamical hypersurfaces, D-branes $\longrightarrow$ boundaries where open strings can end.



String spectra, D-branes, . . . cont'd.

- D-branes come in various different dimensions. They are charged objects.
- Low lying fluctuations of D-branes are governed by open string fluctuations: thus low energy theory on D-branes is gauge theory.
- Gauge theory on stack of N D-branes in flat space is nonabelian $U(N)$ maximally supersymmetric Yang-Mills theory, plus stringy corrections.

Gravitational descriptions of D-branes



- D-branes are heavy objects. A stack of D-branes can be described in terms of a black-hole-like spacetime geometry, with an extended horizon and near-horizon curved spacetime, plus stringy corrections.

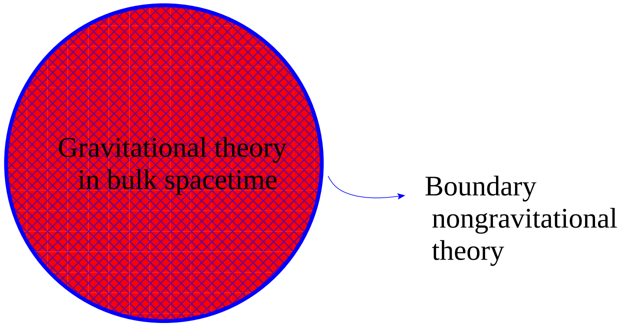
Gauge-theory/gravity duality

- Thus a stack of D-branes has two *independent* descriptions: nonabelian gauge theory (open) and string theory on black-hole-like spacetime (closed), plus stringy corrections.
- There exist scaling limits that suppress the stringy corrections, decoupling the open string sector from the closed string sector.
- Then the *gauge theory* $\equiv$ *gravitational theory*, via equality of the partition functions

$$Z_{gauge} = Z_{gravity} \ ,$$

with a specific prescription on their evaluation. When one theory is strongly coupled, the other is weakly coupled.

Gauge/gravity duality and holography



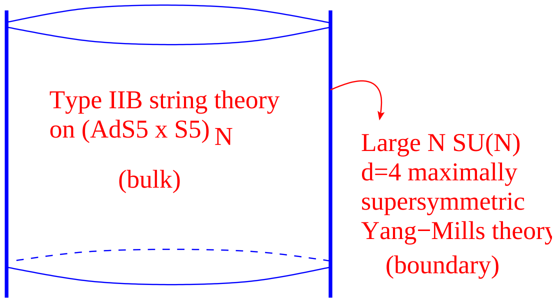
- In many cases, “holography” (’t Hooft, Susskind) is manifest: a gravitational theory (string theory) in a bulk spacetime is dual to a theory without gravity “living” on the boundary of the spacetime, $i.e.$ in one less dimension.
- Various consistency checks and generalizations of the canonical example (Maldacena, 1997), $i.e.$ Type IIB string theory on $AdS_5 \times S^5$ dual to $\mathcal{N}{=}4$ $d = 4$ Super Yang-Mills theory.

Some key string theory questions

- Role and nature of time in string theory ? Can one address what happens at the beginning or the end of time, $i.e.$ Big Bang or Crunch, in string theory models ?
- Vacuum structure of string theory ? Lots of vacua ($i.e.$ solutions to the equations of) in string theory: broken spacetime supersymmetry means metastable and unstable ones too. Good approximations to our Universe ?
- Smooth quantum (stringy) completion of classical spacetime geometry ?

General Relativity breaks down at singularities: so want "stringy" description.

AdS/CFT duality Maldacena; Gubser,Klebanov,Polyakov; Witten,...



- Type IIB string theory on $(AdS_5 \times S^5)_N$ (bulk) $\equiv$ large N $\mathcal{N}=4$ d=4 $SU(N)$ super Yang-Mills theory (boundary).
- Isometry group of $AdS_5 \times S^5$ is $SO(2,4) \times SO(6) \rightarrow$ conformal symmetry, R-symmetry of $\mathcal{N}=4$ SYM.
- 32 supercharges (max.) in bulk matches the supersymmetry and superconformal symmetry of $\mathcal{N}=4$ SYM.
- Various detailed consistency checks (correlators, etc); extensive generalizations.

AdS/CFT duality cont'd.

- $(AdS_5 \times S^5)_N$ (bulk): nontrivial metric (Poincare coords)

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}$$

and 5-form field strength. Dilaton (scalar) $\Phi = const$.

- $\mathcal{N}$=4 d=4 SYM (bndry): field content $A_\mu, \phi^i, i = 1\ldots 6$, +fermions. Spectrum organized in terms of superconformal chiral primary operators $O(x)$ and descendants.

- The precise correspondence (ϵ is a regulator):

$$\begin{aligned} \langle e^{\int d^4x O(x)\phi_0(x)}\rangle_{CFT} &= Z_{string}[\phi(\vec{x},z)|_{z=\epsilon} = \epsilon^{4-\Delta}\phi_0(\vec{x})] \\ &= e^{-S_{sugra}[\phi_0(\vec{x})]+\ldots} . \end{aligned}$$

AdS/CFT: modes and deformations

- String mode of mass $m^2 \leftrightarrow$ CFT operator of scaling dimension $\Delta = 2 + \sqrt{4 + m^2}$.
- Non-normalizable modes: $\phi(\vec{x}, z) \rightarrow \epsilon^{4-\Delta}\phi_0(\vec{x})$, frozen at boundary. Boundary values of these modes act as sources for CFT operators. Turning on such modes in the bulk corresponds to turning on sources for these operators in the dual gauge theory.
- Normalizable modes: $\phi(\vec{x}, z) \rightarrow \epsilon^{\Delta}\phi_0(\vec{x})$, fluctuating, subleading at the boundary. Turning these on in the bulk corresponds to changing the state of the gauge theory.
- These can be used to study deformations of $AdS_5 \times S^5$ and their duals.

Time-dependent deformations: cosmologies

w/ Das, Michelson, Trivedi

- Non-normalizable deformations of the metric and dilaton:

$$
\begin{aligned}
ds^2 &= \frac{1}{z^2}(\tilde{g}_{\mu\nu} dx^\mu dx^\nu + dz^2) + ds^2_{S^5} \ , \\
\Phi &= \Phi(x^\mu) \ , \qquad \text{also nontrivial } 5-\text{form} \ .
\end{aligned}
$$

- This is a solution in string theory if

$$
\begin{aligned}
\tilde{R}_{\mu\nu} &= \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi \ , \\
\frac{1}{\sqrt{\tilde{g}}}\,\partial_\mu(\sqrt{\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\Phi) &= 0 \ ,
\end{aligned}
$$

$i.e.$ if it is a solution to a 4-dim Einstein-dilaton system.

Time-dependent cosmologies

- Consider

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \sum_{i=1}^{3} t^{(2p_i)} dx^i dx^i .$$

One obtains solutions generalizing Kasner-like cosmologies

$$\Phi = \alpha \log t \, , \qquad \sum_i p_i = 1 \, , \qquad \frac{\alpha^2}{2} = 1 - \sum_i p_i^2 .$$

Can be generalized to other $\tilde{g}_{\mu\nu}$.

- These contain spacelike cosmological singularities.

Null-time cosmologies

- Consider (x^+ is a lightlike coordinate)

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = e^{f(x^+)}(-2dx^+ dx^- + dx^i dx^i) \ , \ \ \Phi = \Phi(x^+).$$

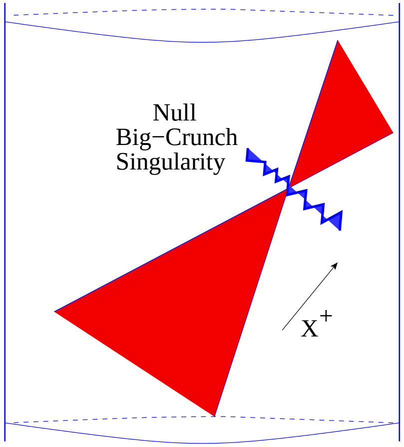
  These are solutions if $\quad (f' = \frac{\partial f}{\partial x^+})$

$$\frac{1}{2}(\partial_+ \Phi)^2 = \tilde{R}_{++} = \frac{1}{2}(f')^2 - f''.$$

  Infinite family of solutions parametrized by dilaton $\Phi(x^+)$.

- These preserve some fraction of lightcone supersymmetry.

Nature of null Big-Bang/Crunch singularity



- These contain null Big-Bang (Crunch) cosmological singularities when the transverse space shrinks as $e^f \to 0$, at say $x^+ = 0$. Then the curvature along infalling null geodesics ξ^μ diverges

$$\tilde{R}_{ab}\xi^a\xi^b = \tilde{R}_{++}e^{-2f} \to \infty \,, \qquad e^f \to 0.$$

Diverging compressional tidal forces along infalling null geodesic congruence.

Gauge theory duals to cosmologies

Conjecture: Type IIB string theory on these backgrounds is dual to $\mathcal{N}=4$ $d=4$ SYM on a base space $\tilde{g}_{\mu\nu}$ with a time dependent gauge coupling $g_{YM}^2 = e^{\Phi}$.

- natural extension of AdS/CFT.
- checks include the dictionary above to map the modes and dual operators, the limiting duality for small perturbations to $AdS_5 \times S^5$, D-brane probe analysis etc.
- What is the gauge theory description of the Big-Bang or Big-Crunch cosmological singularities in the bulk ? Is the gauge theory dual nonsingular ?

The dual gauge theory

For the null cosmologies, the gauge theory dual lives on base space $\tilde{g}_{\mu\nu} = e^{f(x^+)}\eta_{\mu\nu}$ conformal to flat space, and has a null-time-dependent gauge coupling $g_{YM}^2 = e^{\Phi(x^+)}$ which vanishes as $x^+ \to 0$.

$[Typical\ example:\ \ e^{f(x^+)} = \tanh^2 x^+\ ,\ \ e^{\Phi} \sim |\tanh\frac{x^+}{2}|^{\sqrt{8}}\ .]$

These null solutions are special for two reasons:

- The trace anomaly $T_\mu{}^\mu$ for these theories vanishes: therefore the gauge theory is insensitive to conformal factor $e^{f(x^+)}$ and lives on 4D flat space.
  Vanishing trace anomaly $\Rightarrow$ under Weyl rescaling of the metric, partition function unchanged. Conformally dressed operators $e^{f(x^+)\Delta/2}\mathcal{O}(x)$ have well-behaved correlations.

The nonsingular gauge theory

- It is possible to find new (nonlocal) variables $\tilde{A}_\mu = e^{-\Phi(x^+)/2} A_\mu$ which are weakly coupled near the singularity, and encode a completely non-singular description of the system.
  In terms of the $\tilde{A}_\mu$ variables, the gauge field part of the action becomes (F =field strength)

  $$\int e^{-\Phi} \mathrm{tr} F^2 \to \int \left[\mathrm{tr} \tilde{F}^2 + \mathcal{O}(e^{\Phi/2})(\text{interaction terms}) \right]$$

  Since coupling $g_{YM}^2 = e^\Phi \to 0$ near the singularity at $x^+ = 0$, the $\tilde{A}_\mu$ theory is $free\ gauge\ theory$ there.
- $e^{\Phi(x^+)}$ is lightlike $\Rightarrow$ no particle production.

$Result$: The gauge theory is $nonsingular$.

Perspective: desingularizing cosmological singularities

Does physics break down at a Big-Bang/Crunch ?
Here we have null Big-Bang/Crunch cosmological singularities in the interior of the Anti deSitter spacetime, where a priori physics could break down.

However the dual gauge theory shows that there exists $some$ description in the “space of dynamical variables” which gives a controlled understanding of the time-dependent singularity.

What happens from the point of view of a bulk observer ?
Stringy time dependent physics.

Unstable backgrounds in string theory

- Supersymmetry is a powerful handle, controlling quantum corrections to physical observables.
- In the null cosmologies above, some fraction of (lightcone) susy is preserved. However in general, time dependence means spacetime susy is broken.
- Roughly speaking, susy means that spacetime is stable under small fluctuations: if susy broken, spacetime typically contains instabilities, and small fluctuations cause runaway behaviour. String spectra in such backgrounds often contain tachyons (spacetime fields with $m^2 < 0$).
- Important to understand the dynamics of these unstable backgrounds and their phase structure.

String worldsheet RG flows

- The worldsheet theory is a 2-dim. field theory with action

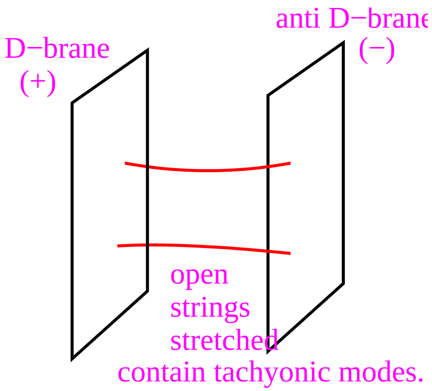
$$S \sim \int d^2\sigma \, (g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu \eta^{ab} + \ldots)$$

This is a conformal fixed point (under renormalization) if $R_{\mu\nu}(X) = 0 + \ldots$, $i.e.$ if Einstein's equation holds.

- There exist renormalization group flows from fixed points representing unstable spacetimes to new, more stable endpoints. If there exists a CFT description, these RG flows are induced by tachyons in the string spectrum.
- If $worldsheet$ susy unbroken, RG flows can be tracked.
- RG flow matches qualitative properties of on-shell time evolution, $e.g.$ flow directions and fixed points.

D-brane annihilation: open string tachyons

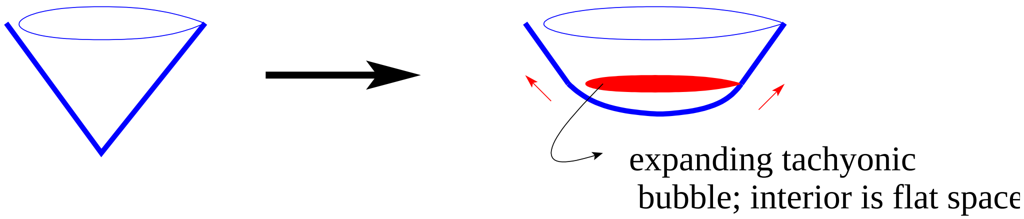
Sen,...



- Tachyons appear in open string spectrum around coincident Dp-brane–anti-Dp-brane pair ($p+1$-dim subspace in a fixed background spacetime).
- In the limit $g_s \to 0$ (zero string coupling), gravitational backreaction can be ignored: conceptually this is field theory without gravity.
- Tachyon condenses on worldvolume as branes decay.

Unstable geometries: localized closed string tachyons

- Closed string tachyons complicated: gravity involved. In general, hard to understand "decay of spacetime".
- Want to isolate instability to localized (noncompact) regions in an otherwise stable spacetime background.



- Consider spacetimes with conical singularities of the form $\mathbb{R}^{7,1} \times \mathbb{C}/\mathbb{Z}_N$ or $\mathbb{R}^{5,1} \times \mathbb{C}^2/\mathbb{Z}_N$ (Adams, Polchinski, Silverstein; and others): tachyon condensation corresponds to blowing up cycles (compact subspaces) initially collapsed at the singularity. Singularity fully resolved eventually.

3-dimensional ($\mathbb{C}^3/\mathbb{Z}_N$) orbifolds ?

Morrison,KN,Plesser; Morrison,KN

- Full spacetime: $\mathbb{R}^{3,1}[x^{0123}] \times \mathcal{M}[\{x^{4,5}, x^{6,7}, x^{8,9}\} \equiv z^{1,2,3}]$.
  Orbifold action: $\mathbb{C}^3/\mathbb{Z}_N(k_1, k_2, k_3),\ z^i \to e^{2\pi i k_i/N} z^i$.
  (susy if $\mathbb{Z}_N \in SU(3),\ i.e.\ \sum_i k_i = 0(mod\, 2N)$.)
- Singularities in complex dimension 3 are particularly interesting: rich connections of tachyon physics with resolution theory in algebraic geometry. Toric singularities are special, described by combinatorial data.
- There are terminal singularities: no physical blowup modes (tachyons or moduli).
- No canonical resolution: resolutions of distinct topology related by flip and flop transitions.

$\mathbb{C}^3/\mathbb{Z}_N$ orbifolds: CFT and geometry

Unlike point particles which would see a singular space, string modes can "wind" around singularity: these localized "twisted sector" excitations (in Conformal Field Theory) map precisely to blowup modes governing the resolution of the singularity.
Analyzing all metric blowup modes (all twisted states) shows:

- Singularity always resolved in (Type II string) theories with spacetime fermions, $i.e.$ no terminal singularities.
- In (Type 0) string theories with no spacetime fermions but containing a delocalized tachyon, there is in fact a terminal singularity $\mathbb{C}^3/\mathbb{Z}_2(1,1,1)$, which arises as a generic endpoint – this singularity has no tachyons and no moduli.

Gauged linear sigma models

Witten; Morrison,Plesser;...

Elegant way to realize the various phases of spacetime geometry. 2-dim $U(1)^r$ gauge theory with $(2,2)$ worldsheet susy and massless chiral superfields $\Psi_i \to e^{iQ_i^a\lambda}\Psi_i$, with charge matrix Q_i^a. No superpotential. There are couplings to $t_a = ir_a + \frac{\theta_a}{2\pi}$ (Fayet-Iliopoulos parameters, θ-angles). The potential energy is

$$U = \sum_a \frac{(D_a)^2}{2e_a^2} + 2\sum_{a,b} \bar{\sigma}_a\sigma_b \sum_i Q_i^a Q_i^b |\Psi_i|^2.$$

Vacuum structure ($U = 0$) exhibits several "phases" as r_a vary

$$D_a \sim \sum_i Q_i^a |\Psi_i|^2 - r_a = 0.$$

Nonzero $r_a \Rightarrow$ some fields are Higgsed: light fields (moduli of GLSM) describe spacetime geometry of string propagation.

RG flows in the GLSM

The FI parameters have a quantum (1-loop) renormalization

$$r_a(\mu) = (\sum_i Q_i^a) \log \frac{\mu}{\Lambda} \,,$$

Λ being the energy scale at which the r_a are defined to vanish. Under RG flow in the GLSM, the 2-dim theory evolves from free gauge theory in the ultraviolet ($\mu \gg e_a$), through various phases as the energy scale μ decreases through $e_a \gg \mu \gg \Lambda$ ($r_a < 0$) to $\mu \ll \Lambda$ ($r_a > 0$). This is equivalent to Ricci flow $\frac{d}{dt} g_{\mu\nu} = -R_{\mu\nu}$.

[For 2-sphere, $|\phi_1|^2 + |\phi_2|^2 = r//U(1), \ (\phi_1, \phi_2) \to (e^{i\theta}\phi_1, e^{i\theta}\phi_2)$, we have $r = r_0 + 2\log\frac{\mu}{\Lambda} \quad \equiv \quad R^2 = R_0^2 - t$, $i.e.$ sphere shrinks: slow at early (RG) times, as $R \sim R_0 - \frac{t}{2R_0} + \ldots$.]

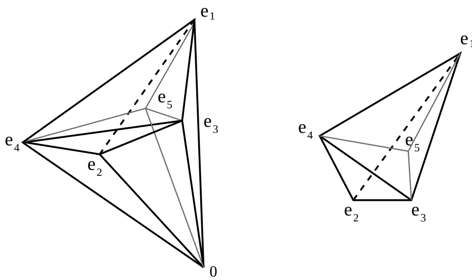
Unstable conifolds KN

Interesting phenomena associated to new toric singularities of the "conifold" kind, labelled by a charge vector

$Q = (\ n_1 \quad n_2 \quad -n_3 \quad -n_4\ ), \quad \sum Q_i \neq 0.$

- Nonsusy quotient of the susy conifold $z_1 z_4 - z_2 z_3 = 0, z_i \in \mathbb{C}^4$. Can be described as nontrivial fibrations over 2-spheres in two topologically distinct ways ("small resolutions").
- The geometry before quotienting can be deformed (singularity replaced by 3-sphere): however the quotienting obstructs this deformation.
- These geometries $\mathbb{R}^{3,1} \times \mathcal{C}$ admit spacetime fermions if $\sum_i Q_i = even$. Their phase structure is cascade-like, containing lower-order conifold-like singularities.

Geometry of unstable conifolds



- Above is the conifold “toric fan” defined by integral lattice vectors $e_{1,2,3,4}$ satisfying $\sum_i Q_i e_i = 0$.
- This combinatorial data describes the geometry as

$$n_1|\phi_1|^2 + n_2|\phi_2|^2 - n_3|\phi_3|^2 - n_4|\phi_4|^2 = r \; //U(1),$$

  the $U(1)$ acting as $\phi_i \to e^{iQ_i\beta}\phi_i$ on the GLSM fields ϕ_i.
- $r < 0$ and $r > 0$ describe fibrations over topologically distinct 2-spheres $\mathbb{P}^1_-$ and $\mathbb{P}^1_+$, with residual $\mathbb{C}^3/\mathbb{Z}_k$ orbifold singularities on their loci.

Phases of unstable conifolds

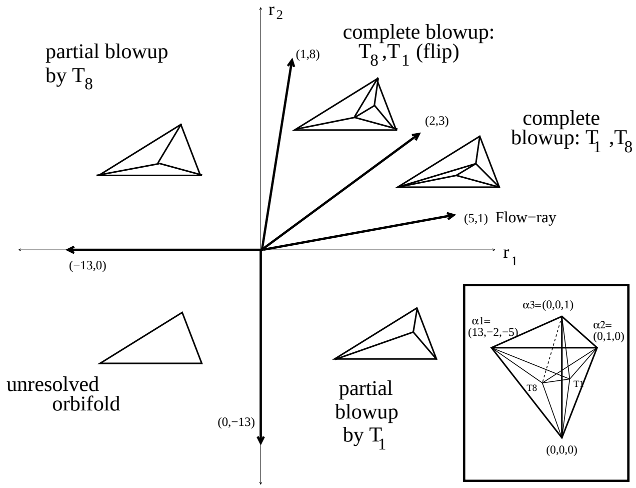
GLSM RG flow shows that $r = (\sum_i Q_i) \log \frac{\mu}{\Lambda}$.
$i.e.$ r flows from $r \ll 0$ $(\mu \gg \Lambda)$ to $r \gg 0$ $(\mu \ll \Lambda)$.
$\mathbb{P}^1_-$ unstable to shrinking, while $\mathbb{P}^1_+$ grows $[q = -\sum_i Q_i > 0]$.

Let $R_0^2 = \log \frac{\mu_0}{\Lambda}$, $\mu_0 \gg \Lambda$, and recast 1-loop RG flow of r. Then:

- Early ($t \sim 0$) times: $R_- = q^{1/2}\sqrt{R_0^2 - t} \sim R_0 - \frac{t}{R_0}$.
  Shrinking of $\mathbb{P}^1_-$ accelerates towards the singular region.
- Late ($t \gg R_0^2$) times: $R_+ = q^{1/2}\sqrt{t - t_0} \sim \sqrt{t} - \frac{t_0}{\sqrt{t}}$. $\mathbb{P}^1_+$ first rapidly grows, then decelerates.
- Singular region near $r = 0$: transient intermediate state. Quantum (worldsheet instanton) corrections large here.
- Residual orbifold singularities also resolve themselves by tachyon condensation: full phase structure rich.

Phases of an Orbifold: $\mathbb{C}^3/\mathbb{Z}_{13}(1,2,5)$, two tachyons T_1, T_8

Morrison, KN



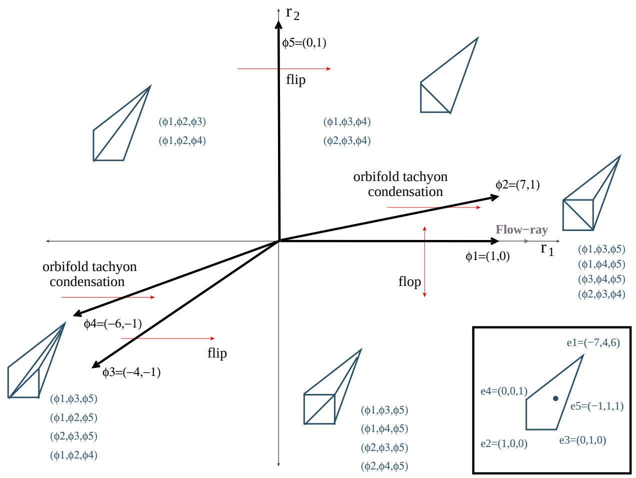
- Fields $\Psi_i = \phi_1, \phi_2, \phi_3, T_1, T_8$, charge matrix

$$Q_i^a = \begin{pmatrix} 1 & 2 & 5 & -13 & 0 \\ 8 & 3 & 1 & 0 & -13 \end{pmatrix}.$$

- Large r_1, r_2 : stringy corrections (worldsheet instantons) ignored.

Phases of an unstable conifold:

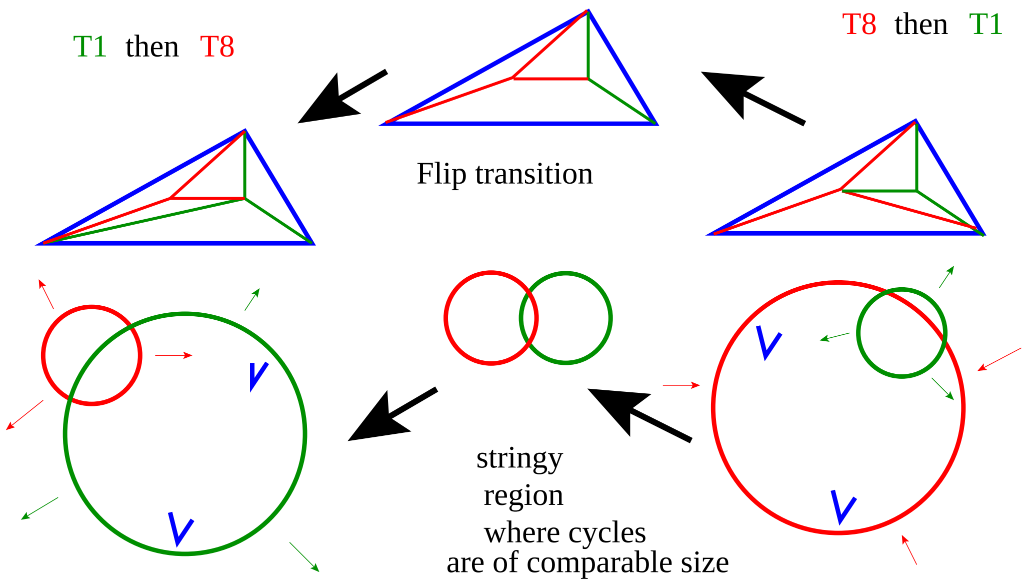
$Q = (\ 1\ \ 7\ \ -4\ \ -6\ )$ **KN**



- Fields $\Psi_{1,\ldots,5}$, charge matrix $Q_i^a = \begin{pmatrix} 1 & 7 & -4 & -6 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{pmatrix}$.
- Red arrows along Flow-ray $(1,0)$ are RG flowlines. Each phase is a distinct resolution of singularity ($i.e.$ triangulation of fan).
- Occasionally, the geometry decays to lower-order conifold-like singularity, here $Q = (\ 1\ \ 1\ \ -1\ \ -1\ )$ $[\sum_i Q_i = 0 \Rightarrow susy]$.

Topology change: flip transitions

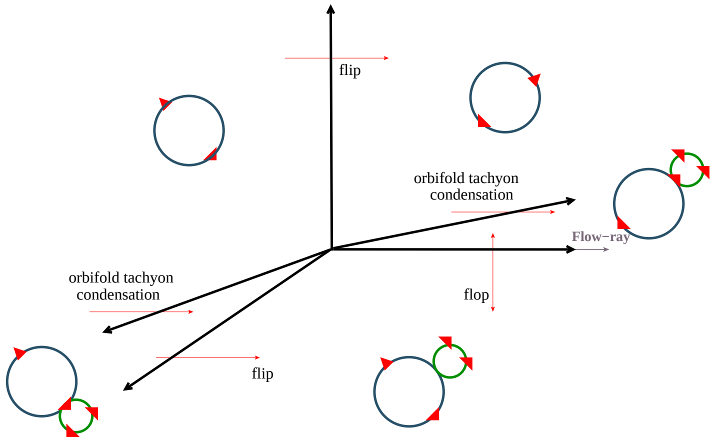
Orbifolds: generically there are multiple tachyons: if a more dominant tachyon condenses during the condensation of some tachyon, there is a blowdown of one cycle and a topologically distinct cycle blows up, mediating mild dynamical topology change.



Inherent time dependence here: geometry dynamically evolves towards less singular resolution.

Topology change: flip transitions

Conifolds: The blowdown of one 2-sphere $\mathbb{P}^1_-$ and the blowup of a topologically distinct 2-sphere $\mathbb{P}^1_+$ is a $flip\ transition$: it mediates mild dynamical topology change, since $\mathbb{P}^1_\pm$ have distinct intersection numbers with various cycles of the full (compactified) geometry.



Inherent time dependence here: geometry dynamically evolves towards least singular resolution.

Open questions

- String compactifications involving the above singularities, spacetime point of view . . .
  D-brane physics in the dynamics of unstable geometries.

- Controlled descriptions of more general time dependent phenomena in string theory ? Understanding of spacelike Big-Bang singularities and realistic cosmologies ? Beginning/end of time in our Universe ?