

Quantum Mechanics III: Endsemester examination

Total: 50 marks. Time: 4 hrs.

(1) Consider the BCS ground state wavefunction $|\Psi_{BCS}\rangle = \prod_k (u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger) |0\rangle$.

(i) Calculate $\langle \Psi_{BCS} | \Psi_{BCS} \rangle$. [3 mks]

(ii) Calculate $\langle \Psi_{BCS} | c_{-k\downarrow} c_{k\uparrow} | \Psi_{BCS} \rangle$, and express it in terms of the gap and the 1-particle energy. [4 mks]

(iii) Calculate $\langle \Psi_{BCS} | b_{k\uparrow} | \Psi_{BCS} \rangle$. [3 mks]

(2) Show that the London equation, along with the Maxwell equations, implies that the gauge field (vector potential) becomes massive, satisfying an equation of the form (in Coulomb gauge) $\nabla^2 \vec{A} = \frac{1}{\Lambda^2} \vec{A}$, with Λ being the penetration depth. [5 mks]

(3) Consider the electron-phonon Hamiltonian ($\epsilon_k, \omega_k, M_{kk'}$ are general)

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \sum_k \omega_k c_k^\dagger c_k + \sum_{kk'} (M_{kk'} c_{k'-k}^\dagger a_k + M_{kk'}^* c_{k'-k}^\dagger a_k^\dagger a_{k'}) .$$

(i) Write down some elementary Feynman diagrams. [3 mks]

(ii) Consider the corrections in 2nd order perturbation theory (possibly diagrammatically) to the self-energy of the electron and phonon and simplify them to momentum integrals (you do not need to evaluate them). [6 mks]

(4) Consider the Bogoliubov de Gennes equations for an interface between a normal metal and a superconductor with gap Δ . Solve these equations for the effectively 1-dim case where an electron is incident on the interface from the normal side with sufficiently high energy to induce injection of a Bogoliubov quasiparticle with energy $E_k = \Delta$ (at threshold, energy equal to the gap) into the superconductor. Assuming $\Delta \ll \epsilon_F$, find the approximate wavefunctions on the normal and superconducting sides, using continuity. [13 mks]

(5) Consider the Hamiltonian for an electron system with a 2-body interaction

$$H = \sum_p \frac{p^2}{2m} a_p^\dagger a_p + \sum_{p_1, p_2, q} \frac{v}{q^n} a_{p_1+q}^\dagger a_{p_2-q}^\dagger a_{p_2} a_{p_1} ,$$

where v is some appropriate dimensionful constant, and n is some real number.

(i) Write down some elementary Feynman diagrams and consider the corrections in 2nd order perturbation theory (possibly diagrammatically) to the (i) ground state energy of the electron gas, (ii) self-energy of the electron. Simplify them to momentum integrals (you do not need to evaluate them). [8 mks]

(ii) Estimate the possible divergences in the infrared and ultraviolet limits that appear in the integrals. Is there any n for which there is no apparent IR divergence? [5 mks]