Model-Checking Event Structures, Part 2

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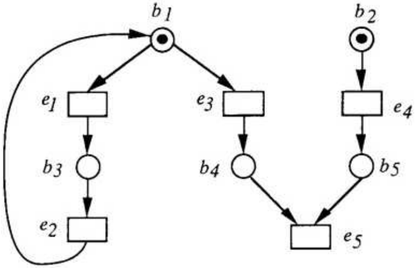
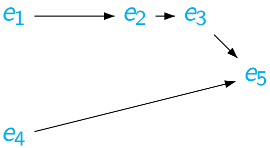
Concurrent systems

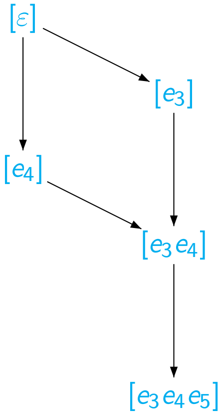


- Convenient to view each execution as a labelled partial order



Mazurkiewicz traces

- Actions are enriched with independence relation specifying which pairs are independent
  - Symmetric, irreflexive
  - Typically derived from structure of underlying system
    - Actions performed by disjoint sets of components
- In a linearization, adjacent independent actions can be swapped to yield an equivalent linearization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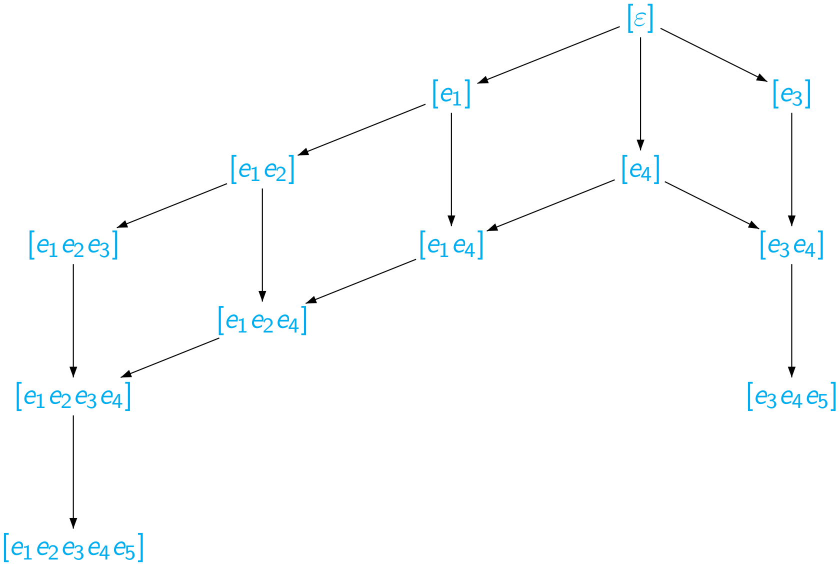

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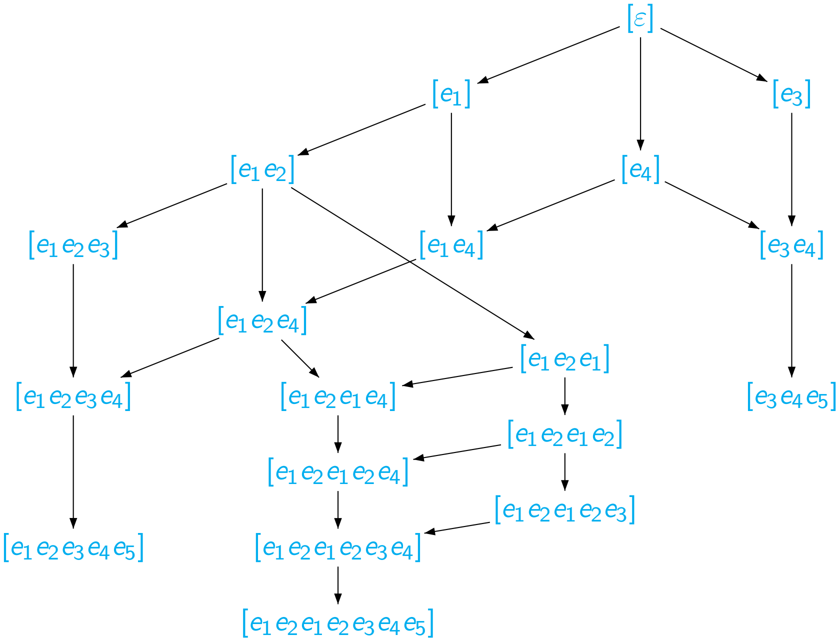
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- $t \# t'$ if t and t' are not compatible

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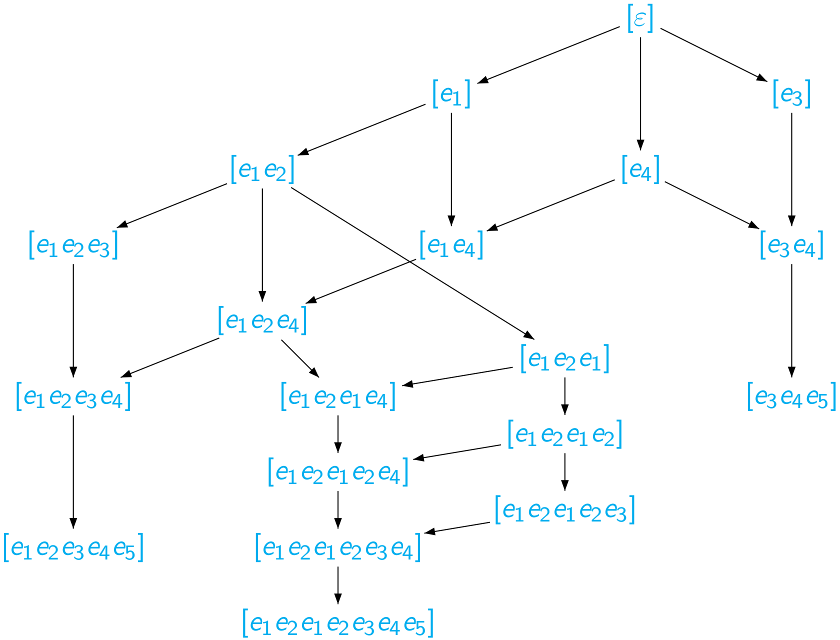
- $t \# t'$ if t and t' are not compatible
- Identify events with prime traces
  - Prime trace: Only one maximal element
  - “Earliest” occurrence of an action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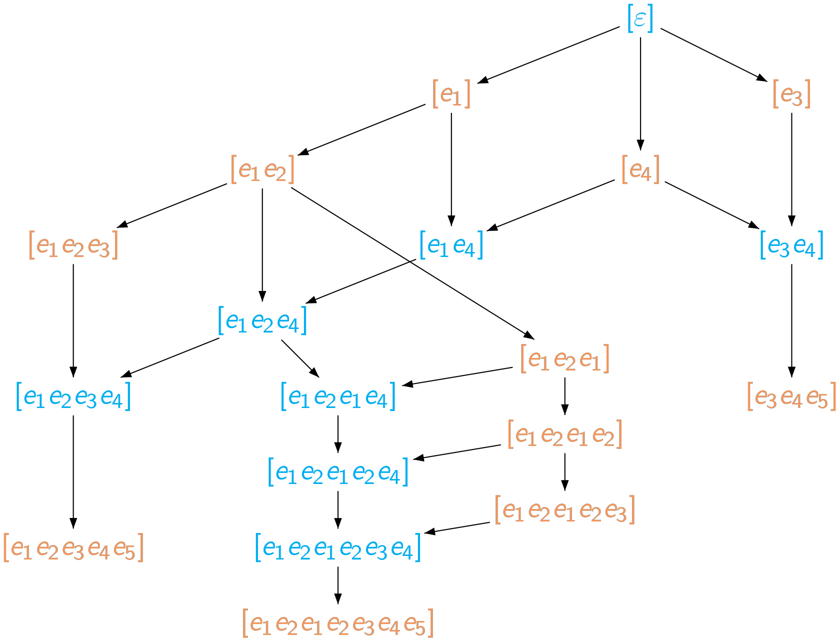

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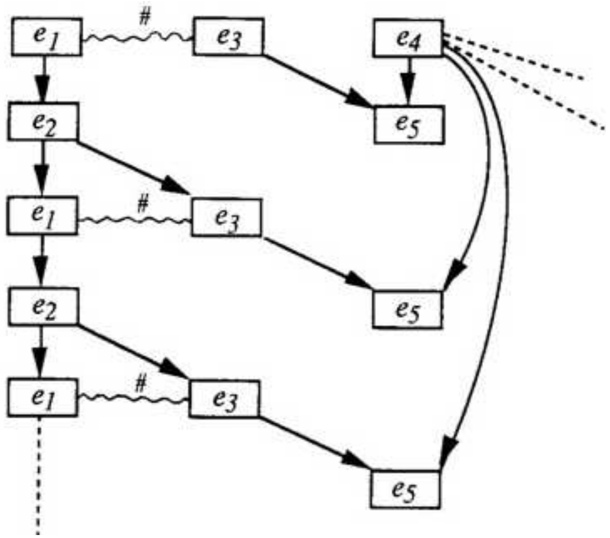
Event Structures ...



(Labelled) Event Structures

Formally, an event structure is of the form $ES = (E, \leq, \#, \lambda)$

- E is the set of event occurrences
- $\leq$ is the causality relation (a partial order)
- $\#$ is a binary conflict relation
  - Irreflexive, symmetric
- Conflict is inherited via causality
  - $e\#f$ and $f \leq f'$ implies $e\#f'$
- $\lambda : E \rightarrow \Sigma$ labels each event occurrence with an action
- Two events are concurrent if they are not related by $\leq$ or $\#$ — $e \; co \; f$

Trace event structures

Let (Σ, I) be a trace alphabet

$ES = (E, \leq, \#, \lambda)$ is a trace event structure if

- $e \#_\mu f \Rightarrow \lambda(e) \neq \lambda(f)$
  - Determinacy!
- If $e \lessdot f$ or $e \#_\mu f$, $(\lambda(e), \lambda(f)) \notin I$
- If $(\lambda(e), \lambda(f)) \notin I$ then $e \leq f$ or $f \leq e$ or $e \# f$.

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Fact

Any event structure constructed from the traces of a deterministic concurrent system is a trace event structure

Event structures as relational structures

Instead of temporal logics, consider

- First-Order Logic (FOL)
- (Variations of) Monadic Second Order logics (MSO)

FOL and MSO are logics over relational structures — a set with a collection of relations defined over the set

Labelled event structures give rise naturally to relational structures

- $ES = (E, \leq, \#, \lambda)$ labelled by $\Sigma = \{a_1, a_2, \ldots, a_n\}$
- Corresponding relational structure is $(E, \leq, \#, \ell_{a_1}, \ell_{a_2}, \ldots, \ell_{a_n})$
  - Each ℓ_{a_i} is a unary predicate such that $\ell_{a_i}(e)$ is true iff $\lambda(e) = a_i$

FOL and MSO

Relational structure $(E, \leq, \#, \ell_{a_1}, \ell_{a_2}, \dots, \ell_{a_n})$

- $\{x, y, \dots\}$: variables representing individual events
- $\{X, Y, \dots\}$: variables representing sets of events

FOL

$$x = y \mid x \leq y \mid x \# y \mid \ell_a(x) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists x.\varphi(x)$$

MSOL

$$x = y \mid x \leq y \mid x \# y \mid \ell_a(x) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists x.\varphi(x) \mid \exists X.\varphi(x)$$

The model-checking problem

- We are given a regular trace language L
  - Set of traces whose linearizations is a regular language
- From the prime traces, those with a single maximal event, we can extract an event structure ES_L
- Given a formula φ in FOL/MSO, does $ES_L \models \varphi$?

MSO over trace event structures is undecidable

[Walukiewicz]

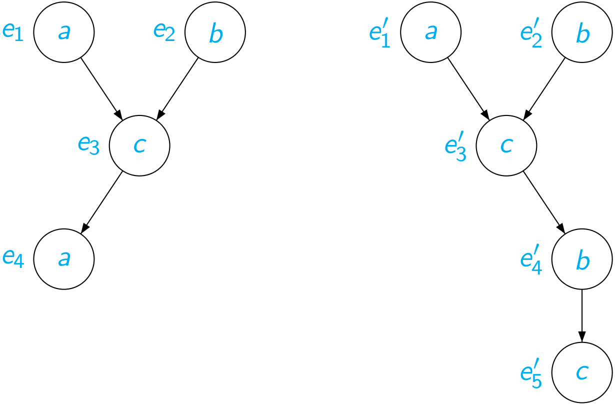
- Alphabet $\{a, b, c\}$ with $I = \{(a, b), (b, a)\}$
- Consider trace language generated by words of the form $a^* b^* c)$
- Each prime trace/event $[a^j b^k c]$ encodes a grid point (j, k)
- Set variables describe an assignment of colours to these events
- MSO can describe that this colouring/tiling of the grid is valid
- To get around this, restrict MSO to Monadic Trace Logic (MTL)
  - Quantify over conflict-free subsets of E

FOL over trace event structures is decidable

- Let $\varphi(x_1, x_2, \ldots, x_k)$ be an FOL formula
- φ defines a k-ary relation over events
  $R_\varphi = \{(e_1, e_2, \ldots, e_k) \mid ES \models \varphi(e_1, e_2, \ldots, e_k)\}$
- Recall that each event is actually a prime trace, so R_φ is a relation over traces in L
- Combine each tuple $(t_1, t_2, \ldots, t_k) \in R_\varphi$ into a single braided trace (over a new alphabet)
- Model-checking R_φ is equivalent to checking that the set of braided traces corresponding to R_φ is non-empty
- For each formula φ, the braided traces corresponding to R_φ form a regular trace language

Braiding traces

Overlap traces as far as possible, recording for each overlapped event, which components participate in that event



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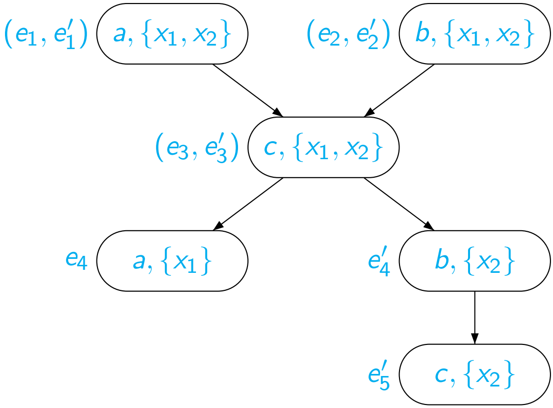

Braiding traces . . .

- Braided traces over new alphabet Σ_B with symbols (a, Y) where
  - $a \in \Sigma$ is a letter from the original alphabet
  - $Y \subseteq \{x_1, x_2, \ldots, x_k\}$
- $((a, X), (b, Y)) \in I_B$ if $(a, b) \in I$ or $X \cap Y = \emptyset$

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Observation

- If $(a, X) \leq (b, Y)$ in a braided trace, then $Y \subseteq X$
  - The second component monotonically decreases along each chain of dependent letters
- This property can be checked by a finite-state automaton

Braiding traces . . .

Theorem

For each FOL formula $\varphi(x_1, x_2, \ldots, x_k)$, the corresponding braided trace language is regular

Proof

By induction on the structure of φ

Braiding traces . . .

φ is $x = y$

Braiding traces . . .

φ is $x = y$

- $(t1, t2) \in R_\varphi$ iff $t_1 = t_2$
- Braided trace is isomorphic to t_1 (and t_2)
- Each action is labelled $\{x_1, x_2\}$

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- Braided trace is isomorphic to t_1 (and t_2)
- Each action is labelled $\{x_1, x_2\}$
- Check that projection onto Σ is a prime trace in L
  - Note: If L is a regular trace language, the prime traces of L also form a regular trace language
- Check that second component of each label is $\{x_1, x_2\}$

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φ is $x \leq y$

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Braiding traces . . .

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φ is $x\#y$

- $(t1, t2) \in R_\varphi$ iff t_1 and t_2 diverge
- At least one action each labelled only $\{x_1\}$ and $\{x_2\}$
- Braided trace restricted to
  - actions labelled $\{x_1, x_2\}$ or $\{x_1\}$ is isomorphic to t_1
  - actions labelled $\{x_1, x_2\}$ or $\{x_2\}$ is isomorphic to t_2

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- Check that projections $\{x_1, \ldots\}$ and $\{x_2, \ldots\}$ are both prime traces in L
- Check that there is at least one event each with second component of label $\{x_1\}$ and $\{x_2\}$
- Check that second component decreases monotonically along each chain of dependent letters

Braiding traces . . .

φ is $\exists y.\psi(y, x_1, \ldots, x_k)$

- By induction hypothesis, braided trace language for R_ψ is regular
- Define a natural projection operator to eliminate y from a set of braided traces
  - Project onto $(x_1, \ldots, x_k) \Rightarrow$ drop y from each event's label
  - Erase any event whose initial label was $\{y\}$ (and hence now has an empty label)
- If B is a regular language of braided traces over variables $\bar{x}$, its projection onto any subset of $\bar{x}$ is also regular
- Braided trace language for φ is obtained by projecting the language for ψ onto $(x_1, x_2, \ldots, x_k)$

Braiding traces ...

φ is $\neg\psi$: Easy

φ is $\psi_1 \wedge \psi_2$

- $\psi_1(x_1, \ldots, x_k)$ and $\psi_2(y_1, \ldots, y_m)$ so braided traces for φ are over $(x_1, \ldots, x_k, y_1, \ldots, y_m)$
- In general, some variables overlap between ψ_1, ψ_2
$$\varphi(\bar{x}, \bar{y}, \bar{z}) = \psi_1(\bar{x}, \bar{z}) \wedge \psi_2(\bar{y}, \bar{z})$$
- Define an "expansion" operator:
  - B, a set of braided traces over $\bar{u} = (u_1, u_2, \ldots, u_k)$
  - $\bar{v} = (v_1, v_2, \ldots, v_m)$, a new set of variables
  - $B \uparrow \bar{v}$: all braided traces over $(\bar{u}, \bar{v}) = (u_1, \ldots, u_k, v_1, \ldots, v_k)$ whose projection onto $\bar{u}$ lies in B.
- Then, the language for φ is $(B_{\psi_1} \uparrow \bar{y}) \cap (B_{\psi_2} \uparrow \bar{x})$

MTL

- MTL is MSO with set quantifiers restricted to conflict-free subsets of E
- In FOL proof, each individual variable x is assigned an event e, which can be regarded as a prime trace
- Can we represent conflict-free subsets of E as traces?

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- If $X \subset E$ is conflict-free, so is $\downarrow X$
- Thus, $\downarrow X$ is a trace (not necessarily prime)
- Not all events in $\downarrow X$ are part of the subset
  - Add a tag from $\{\perp, \top\}$ to indicate which events in $\downarrow X$ belong to X and which do not

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- Can again assign a set of braided traces with each formula φ
- Show by induction on φ that this set is regular

In perspective

FOL over traces can express all natural temporal modalities

- $ES, e \models A_{\leq}\varphi$ if at every f such that $e \leq f$, $ES, f \models \varphi$
- $ES, e \models E_{\#}\varphi$ if there exists f such that $e \# f$ and $ES, f \models \varphi$
- $ES, e \models A_{co}\varphi$ if at every f such that $e \; co \; f$, $ES, f \models \varphi$
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What more remains to be done?

In perspective . . .

- System is presented as a regular trace language
- Implicitly, we assume a deterministic machine recognizing the language
- Model-checking is typically applied to a given system model
  - May be nondeterministic
  - Distinction between labelled and unlabelled systems in models like Petri nets
- What is the status of branching-time model-checking for labelled concurrent systems?

In perspective . . .

- In sequential systems, model-checking is intimately connected to automata theory
  - Tree automata
  - Alternating automata (on strings and trees)
- In concurrent systems, the theory of “string” automata is reasonably well-understood
  - Asynchronous automata, Zielonka’s theorem
- How do we define alternating automata on traces?