Relaxing conditions yields undecidability

Timed automata are decidable – one-slope variables
"initialized" by default, det. jumps

2 slope variables $\dot{x} = k_1$ or $\dot{x} = k_2$
but not initialized

"Simple" rectangular automaton

- exactly one x is not a clock
- initially all variables are 0, all resets to 0
- all rectangles are compact (bounded & closed)

Thm: Reachability is undecidable for simple automata with one two-slope variable.

$\exists y$ s.t. $\dot{y} \in \{k_1, k_2\}$ in each state

2 counter machines

Instructions

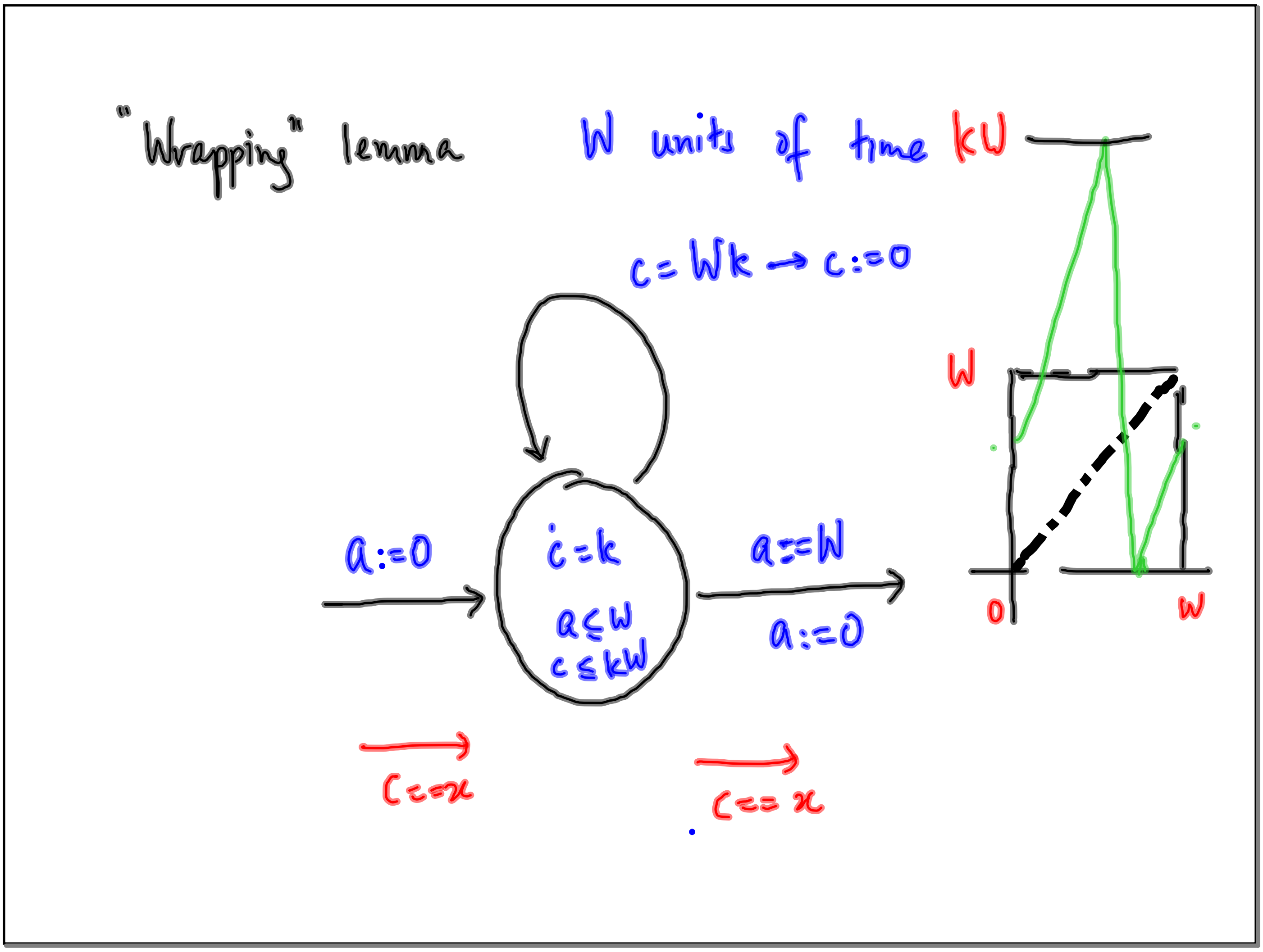
1:

2: ← c_1++, c_1--

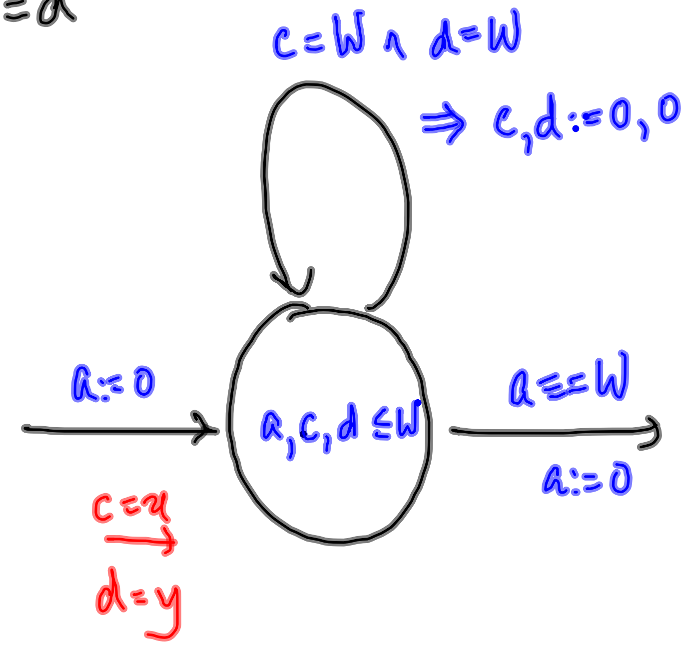
c_2++, c_2--

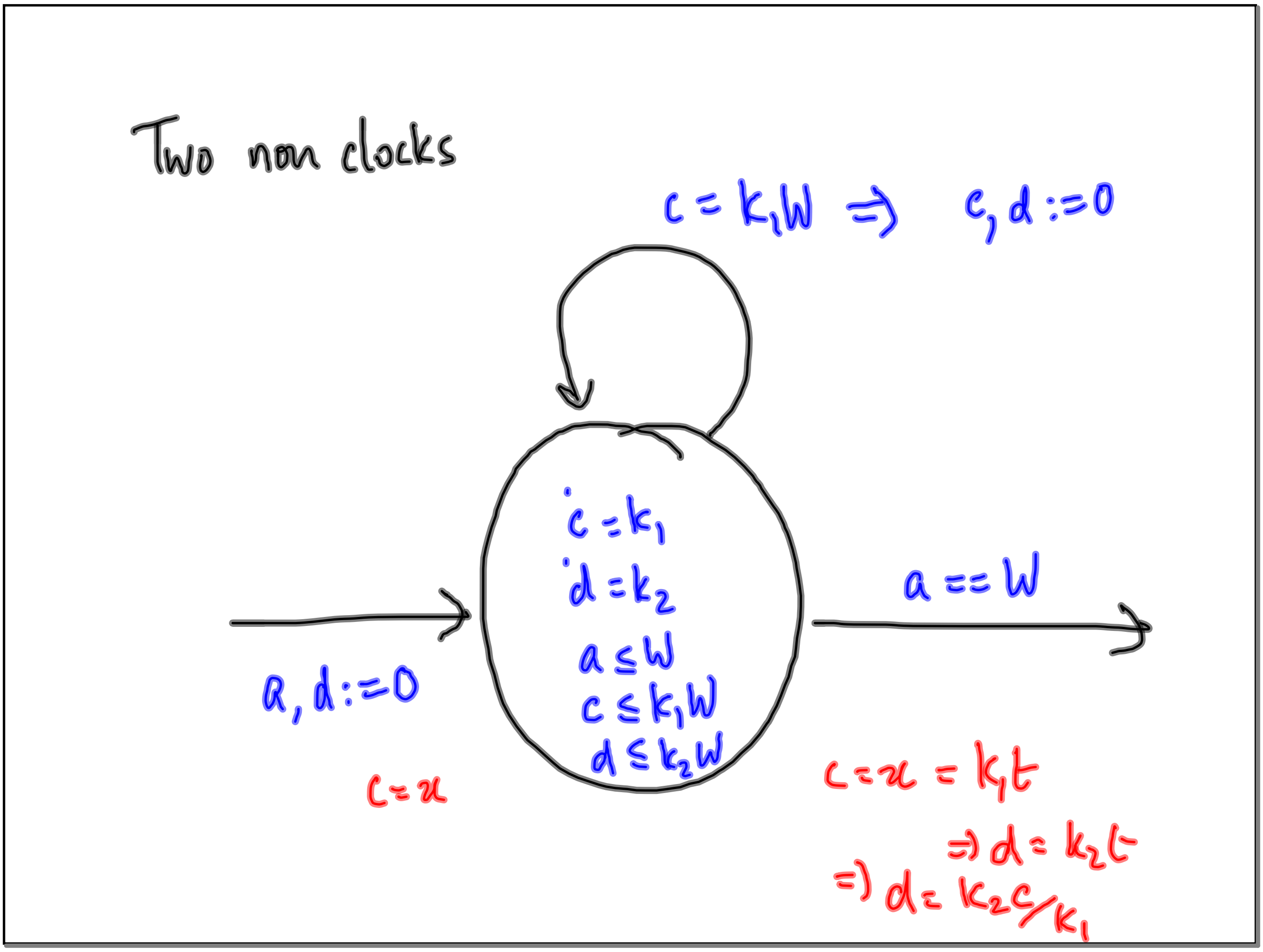
⋮ if c_i==0 goto ℓ

n: ← halt


"Wrapping" lemma
W units of time kW
c = Wk → c := 0
a := 0
ċ = k
a ≤ W
c ≤ kW
a == W
a := 0
c := x
c == x
W
0
W

Testing c=d




Two non clocks
$c = k_1 W \Rightarrow c, d := 0$
$\dot{c} = k_1$
$\dot{d} = k_2$
$a \leq W$
$c \leq k_1 W$
$d \leq k_2 W$
$a == W$
$a, d := 0$
$c = x$
$c = x = k_1 t$
$\Rightarrow d = k_2 t$
$\Rightarrow d = k_2 c / k_1$

Simulation of 2 counter machine

Counters C — clock c

D — clock d

3 more clocks a, b, b'

One non-clock z — slopes. k_1, k_2

$k_1 > k_2 > 0$

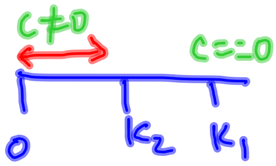
Value of $C = u$ is encoded $c = k_1 \cdot \left(\frac{k_2}{k_1}\right)^u$

$D = u$ is encoded $d = k_1 \cdot \left(\frac{k_2}{k_1}\right)^u$

$C = u$ $\quad c = k_1 \cdot \left(\frac{k_2}{k_1}\right)^u$

$C = 0$ $\quad c = k_1$

$C \geq 1$ $\quad c \in (0, k_2]$

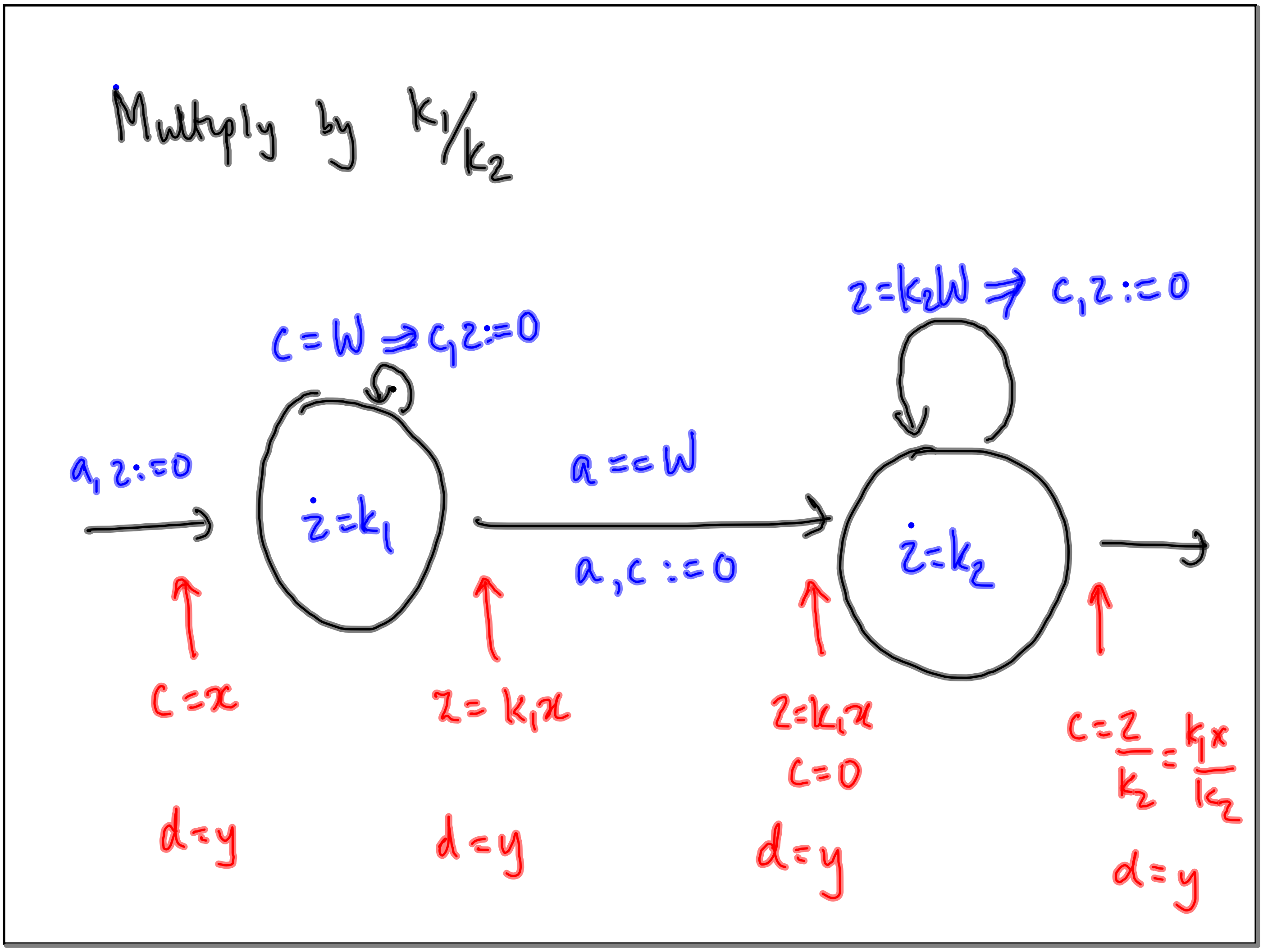


if $C == 0$ $\quad c \in [k_1, k_1]$

$C != 0$ $\quad c \in (0, k_2]$

decrement / increment — multiply by $\frac{k_2}{k_1}$

divide by $\frac{k_2}{k_1}$ = multiply by $\frac{k_1}{k_2}$


Multiply by k1/k2
c = W ⇒ c, z := 0
z = k2W ⇒ c, z := 0
a, z := 0
ż = k1
a == W
a, c := 0
ż = k2
c = x
d = y
z = k1x
d = y
z = k1x
c = 0
d = y
c = z/k2 = k1x/k2
d = y

$k_1 > 0 > k_2$

$k_1 > 0$, $k_2 = 0$

$\vdots$

Different encodings of C, D