

# Searching and Sorting

## Setup

- Use `time` library to time executions

```
In [1]: import time
```

## Naive search by scanning the list

```
In [2]: def naivesearch(v,l):  
        for x in l:  
            if v == x:  
                return(True)  
        return(False)
```

## Binary search

```
In [3]: def binarysearch(v,l):  
        if l == []:  
            return(False)  
  
        m = len(l)//2  
  
        if v == l[m]:  
            return(True)  
  
        if v < l[m]:  
            return(binarysearch(v,l[:m]))  
        else:  
            return(binarysearch(v,l[m+1:]))
```

## Checking correctness on input [0,2,...,50]

```
In [4]: l = list(range(0,51,2))  
  
        for i in range(51):  
            print((i,naivesearch(i,l)),end=",")  
            print()  
  
        for i in range(51):  
            print((i,binarysearch(i,l)),end=",")  
            print()
```

```
(0, True),(1, False),(2, True),(3, False),(4, True),(5, False),(6, True),(7, False),(8, True),(9, False),(10, True),(11, False),(12, True),(13, False),(14, True),(15, False),(16, True),(17, False),(18, True),(19, False),(20, True),(21, False),(22, True),(23, False),(24, True),(25, False),(26, True),(27, False),(28, True),(29, False),(30, True),(31, False),(32, True),(33, False),(34, True),(35, False),(36, True),(37, False),(38, True),(39, False),(40, True),(41, False),(42, True),(43, False),(44, True),(45, False),(46, True),(47, False),(48, True),(49, False),(50, True),  
(0, True),(1, False),(2, True),(3, False),(4, True),(5, False),(6, True),(7, False),(8, True),(9, False),(10, True),(11, False),(12, True),(13, False),(14, True),(15, False),(16, True),(17, False),(18, True),(19, False),(20, True),(21, False),(22, True),(23, False),(24, True),(25, False),(26, True),(27, False),(28, True),(29, False),(30, True),(31, False),(32, True),(33, False),(34, True),(35, False),(36, True),(37, False),(38, True),(39, False),(40, True),(41, False),(42, True),(43, False),(44, True),(45, False),(46, True),(47, False),(48, True),(49, False),(50, True),
```

## Performance comparison across $10^4$ worst case searches in a list of size $10^5$

- Looking for odd numbers in a list of even numbers

```
In [5]: l = list(range(0,200000,2))  
  
        starttime = time.perf_counter()  
        for i in range(3001,23000,2):  
            v = naivesearch(i,l)  
        elapsed = time.perf_counter() - starttime  
        print()  
        print("Naive search", elapsed)  
  
        starttime = time.perf_counter()  
        for i in range(3001,23000,2):  
            v = binarysearch(i,l)  
        elapsed = time.perf_counter() - starttime  
        print()  
        print("Binary search", elapsed)
```

Naive search 21.177326752993395

Binary search 2.5567972129938425

## Selection sort

```
In [6]: def SelectionSort(L):
n = len(L)
if n < 1:
    return(L)
for i in range(n):
    # Assume L[:i] is sorted
    mpos = i
    # mpos is position of minimum in L[i:]
    for j in range(i+1,n):
        if L[j] < L[mpos]:
            mpos = j
    # L[mpos] is the smallest value in L[i:]
    (L[i],L[mpos]) = (L[mpos],L[i])
    # Now L[:i+1] is sorted
return(L)
```

## Selection sort performance is more or less the same for all inputs

```
In [7]: import random
random.seed(2023)
inputlists = {}
inputlists["random"] = [random.randrange(100000) for i in range(5000)]
inputlists["ascending"] = [i for i in range(5000)]
inputlists["descending"] = [i for i in range(4999,-1,-1)]
for k in inputlists.keys():
    tmlist = inputlists[k][:]
    starttime = time.perf_counter()
    SelectionSort(tmlist)
    elapsed = time.perf_counter() - starttime
    print(k,elapsed)
```

```
random 0.6429454580065794
ascending 0.6363217170001008
descending 0.6335939559940016
```

## Insertion sort, iterative

```
In [8]: def InsertionSort(L):
n = len(L)
if n < 1:
    return(L)
for i in range(n):
    # Assume L[:i] is sorted
    # Move L[i] to correct position in L[:i]
    j = i
    while(j > 0 and L[j] < L[j-1]):
        (L[j],L[j-1]) = (L[j-1],L[j])
        j = j-1
    # Now L[:i+1] is sorted
return(L)
```

## Insertion sort performance

- On already sorted input, performance is very good
- On reverse sorted input, performance is worse than selection sort

```
In [9]: import random
random.seed(2023)
inputlists = {}
inputlists["random"] = [random.randrange(100000) for i in range(5000)]
inputlists["ascending"] = [i for i in range(5000)]
inputlists["descending"] = [i for i in range(4999,-1,-1)]
for k in inputlists.keys():
    tmlist = inputlists[k][:]
    starttime = time.perf_counter()
    InsertionSort(tmlist)
    elapsed = time.perf_counter() - starttime
    print(k,elapsed)
```

```
random 1.1455828940088395
ascending 0.00039821400423534214
descending 2.23140712500026
```

## Insertion sort, recursive

```
In [10]: def Insert(L,v):
n = len(L)
if n == 0:
    return([v])
if v >= L[-1]:
    return(L+[v])
else:
    return(Insert(L[:-1],v)+L[-1:])

def ISort(L):
n = len(L)
if n < 1:
    return(L)
L = Insert(ISort(L[:-1]),L[-1])
return(L)
```

```
In [11]: import random
random.seed(2023)
inputlists = {}
inputlists["random"] = [random.randrange(100000) for i in range(5000)]
inputlists["ascending"] = [i for i in range(5000)]
inputlists["descending"] = [i for i in range(4999,-1,-1)]
for k in inputlists.keys():
    tmlist = inputlists[k][:]
    starttime = time.perf_counter()
    ISort(tmlist)
    elapsed = time.perf_counter() - starttime
    print(k,elapsed)
```

-----  
RecursionError: maximum recursion depth exceeded while calling a Python object  
Traceback (most recent call last)

```
Cell In [11], line 10
      8 tmlist = inputlists[k][:]
      9 starttime = time.perf_counter()
----> 10 ISort(tmlist)
      11 elapsed = time.perf_counter() - starttime
      12 print(k,elapsed)
```

```
Cell In [10], line 14, in ISort(L)
      12 if n < 1:
      13     return(L)
----> 14 L = Insert(ISort(L[:-1]),L[-1])
      15 return(L)
```

```
Cell In [10], line 14, in ISort(L)
      12 if n < 1:
      13     return(L)
----> 14 L = Insert(ISort(L[:-1]),L[-1])
      15 return(L)
```

[... skipping similar frames: ISort at line 14 (2969 times)]

```
Cell In [10], line 14, in ISort(L)
      12 if n < 1:
      13     return(L)
----> 14 L = Insert(ISort(L[:-1]),L[-1])
      15 return(L)
```

```
Cell In [10], line 11, in ISort(L)
      10 def ISort(L):
----> 11     n = len(L)
      12     if n < 1:
      13         return(L)
```

RecursionError: maximum recursion depth exceeded while calling a Python object

## Setup

- Set recursion limit to  $2^{31} - 1$ 
  - This is the highest value Python allows

```
In [12]: import sys
sys.setrecursionlimit(2**31-1)
```

## Recursive insertion sort is slower than iterative

- Input of 2000 (40%) takes more time than 5000 for iterative
  - Overhead of recursive calls
- Performance pattern between unsorted, sorted and random is similar

```
In [13]: import random
random.seed(2023)

inputlists = {}
inputlists["random"] = [random.randrange(100000) for i in range(2000)]
inputlists["ascending"] = [i for i in range(2000)]
inputlists["descending"] = [i for i in range (1999,-1,-1)]
for k in inputlists.keys():
    tmlist = inputlists[k][:]
    starttime = time.perf_counter()
    ISort(tmlist)
    elapsed = time.perf_counter() - starttime
    print(k,elapsed)
```

```
random 5.81613647499762
ascending 0.017258646999835037
descending 10.139818099996774
```

### Merge sort

```
In [14]: def merge(A,B):
(m,n) = (len(A),len(B))
(C,i,j,k) = ([],0,0,0)
while k < m+n:
    if i == m:
        C.extend(B[j:])
        k = k + (n-j)
    elif j == n:
        C.extend(A[i:])
        k = k + (n-i)
    elif A[i] < B[j]:
        C.append(A[i])
        (i,k) = (i+1,k+1)
    else:
        C.append(B[j])
        (j,k) = (j+1,k+1)
return(C)
```

```
In [15]: def mergesort(A):
n = len(A)

if n <= 1:
    return(A)

L = mergesort(A[:n//2])
R = mergesort(A[n//2:])

B = merge(L,R)

return(B)
```

### A simple input to check correctness

```
In [16]: mergesort([i for i in range(0,1000,2)]+[j for j in range (1,1000,2)])
```

```
318,
319,
320,
321,
322,
323,
324,
325,
326,
327,
328,
329,
330,
331,
332,
333,
334,
335,
336,
337
```

## Performance on large inputs, $10^6$ , random and sorted

```
In [17]: import random
random.seed(2023)
inputlists = {}
inputlists["random"] = [random.randrange(10000000) for i in range(1000000)]
inputlists["ascending"] = [i for i in range(1000000)]
inputlists["descending"] = [i for i in range(999999,-1,-1)]
for k in inputlists.keys():
    tmlist = inputlists[k][:]
    starttime = time.perf_counter()
    mergesort(tmlist)
    elapsed = time.perf_counter() - starttime
    print(k,elapsed)
```

```
random 4.532351800997276
ascending 2.5078760080068605
descending 2.5784778259985615
```

## Iterative implementation of binary search

- Each recursive call halves the interval to search
- Maintain the endpoints explicitly and update with each iteration

```
In [18]: def binarysearchiterative(v,L):
if len(L) == 0:
    return(True)
left = 0
right = len(L)
while (right - left > 0):
    mid = (left + right) // 2
    if v == L[mid]:
        return(True)
    if v < L[mid]:
        right = mid
    else:
        left = mid+1
return(False)
```

## Checking correctness

```
In [19]: evenlist = [n for n in range(50) if n%2 == 0]
print(evenlist)
for i in range(0,60,3):
    print(i,binarysearchiterative(i,evenlist))
```

```
0 True
3 False
6 True
9 False
12 True
15 False
18 True
21 False
24 True
27 False
30 True
33 False
36 True
39 False
42 True
45 False
48 True
51 False
54 False
57 False
```

```
In [21]: l = list(range(0,200000,2))
starttime = time.perf_counter()
for i in range(3001,23000,2):
    v = binarysearchiterative(i,l)
elapsed = time.perf_counter() - starttime
print()
print("Binary search", elapsed)
```

```
Binary search 0.047113468986935914
```