Typed λ **-calculus**

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- What is the theoretical foundation for such languages?

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 - We will only learn Curry typing

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 - data BTree a = Nil | Node (BTree a) a (BTree a)

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where $x \in Var, M, N \in \Lambda$

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- When constructing expressions, build up the type from the types of the parts

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- σ, τ, \ldots stand for arbitrary types
- \rightarrow is right associative: $\sigma \rightarrow \tau \rightarrow \theta$ is short for $\sigma \rightarrow (\tau \rightarrow \theta)$

Adding types to λ -calculus: Curry typing

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- The typing rules:

 $\begin{array}{c} \Gamma, x: \tau \vdash x: \tau \\ \hline \Gamma \vdash (\lambda x \cdot M): \sigma \to \tau \end{array} \quad \begin{array}{c} \Gamma \vdash M: \sigma \to \tau \quad \Gamma \vdash N: \sigma \\ \hline \Gamma \vdash (MN): \tau \end{array}$

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 $[\Gamma, x : \tau \vdash x : \tau] \xrightarrow{\Gamma \vdash (\lambda x \cdot M) : \tau} [\Gamma \vdash (\lambda x \cdot M) : \sigma \to \tau] \xrightarrow{\Gamma \vdash M : \sigma \to \tau} [\Gamma \vdash (MN) : \tau]$

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• Types match



$$\frac{x:p\vdash x:p}{\vdash \lambda x \cdot x:p \longrightarrow p}$$

$$\frac{x:p,y:q\vdash x:p}{x:p\vdash\lambda y\cdot x:q\rightarrow p}$$
$$\vdash\lambda xy\cdot x:p\rightarrow (q\rightarrow p)$$

• Let
$$\Gamma = \{x : p \to q \to r, y : p \to q, z : p\}$$

$$\frac{\Gamma \vdash x : p \to q \to r \quad \Gamma \vdash z : p}{\Gamma \vdash xz : q \to r} \quad \frac{\Gamma \vdash y : p \to q \quad \Gamma \vdash z : p}{\Gamma \vdash yz : q}$$

$$\frac{\Gamma \vdash xz(yz) : r}{x : p \to q \to r, y : p \to q \vdash \lambda z \cdot xz(yz) : p \to r}$$

$$\frac{x : p \to q \to r \vdash \lambda yz \cdot xz(yz) : (p \to q) \to (p \to r)}{\vdash \lambda xyz \cdot xz(yz) : (p \to q \to r) \to (p \to r)}$$

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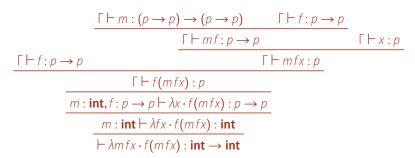
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- Define int := $(p \rightarrow p) \rightarrow (p \rightarrow p)$
- For all $n \in \mathbb{N}$, $\vdash \ll n \gg$: int

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- For example, letting $\tau := p \rightarrow p$,

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- *f* is definable in typed λ -calculus iff it is essentially a polynomial function!

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 - Same proof as for untyped λ -calculus

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- The typed λ -calculus is both strongly and weakly normalizing

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Principal type

- a type for a term *M* such that every other type for *M* is got by uniformly replacing each variable by a type
- unique for each typable term modulo renaming of variables!