Lecture 7: 30 January, 2024

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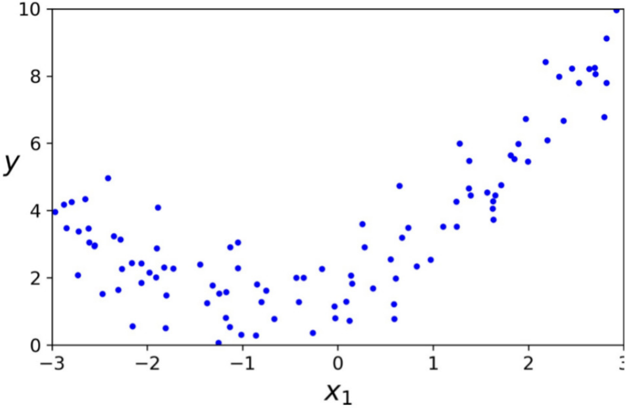
Data Mining and Machine Learning
January–April 2024

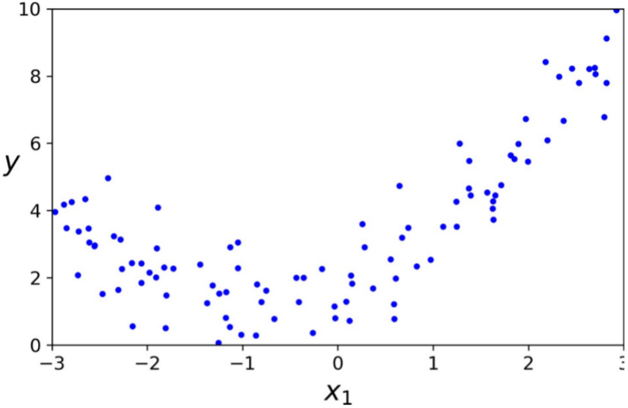
Decision trees for regression

- How do we use decision trees for regression?



Decision trees for regression

- How do we use decision trees for regression?
- Partition the input into intervals



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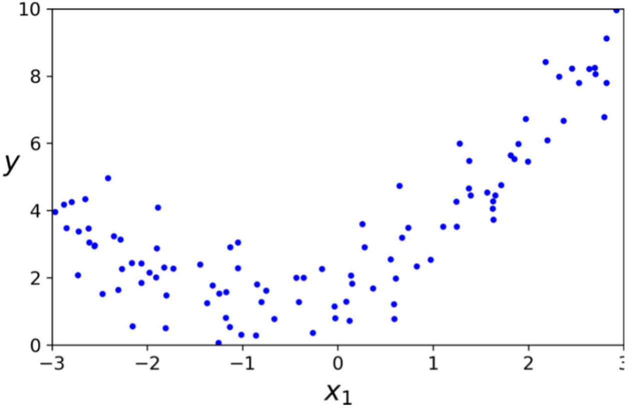

Decision trees for regression

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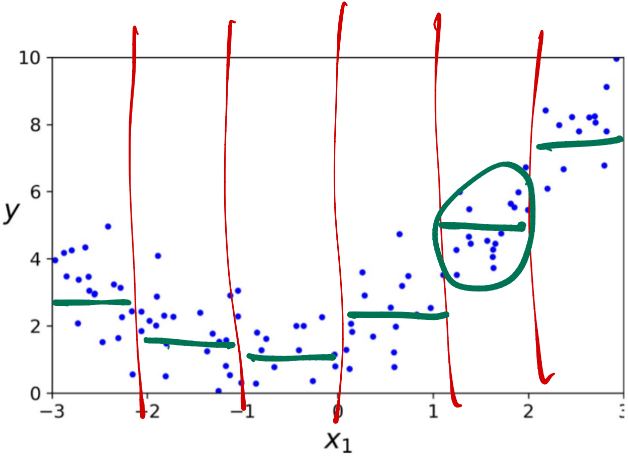
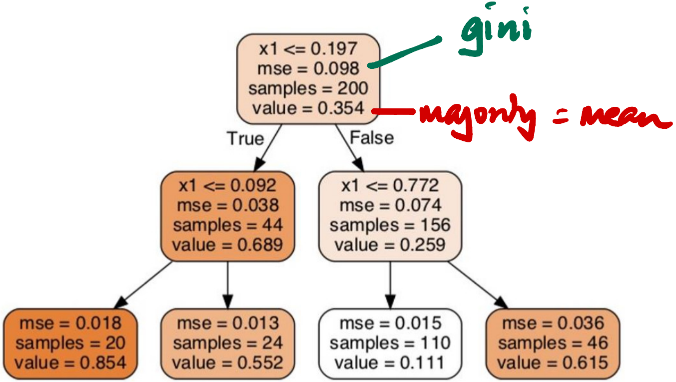
- Regression tree



Decision trees for regression

- Regression tree for noisy quadratic centered around $x_1 = 0.5$



Decision trees for regression

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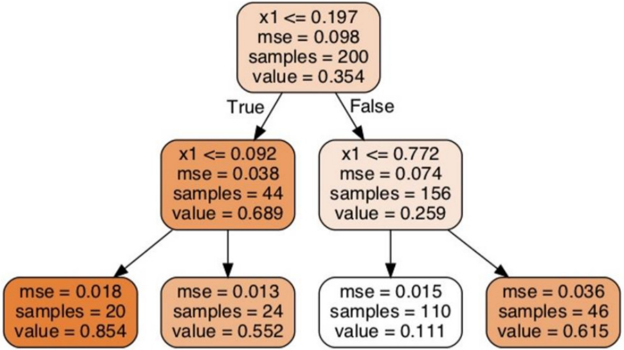

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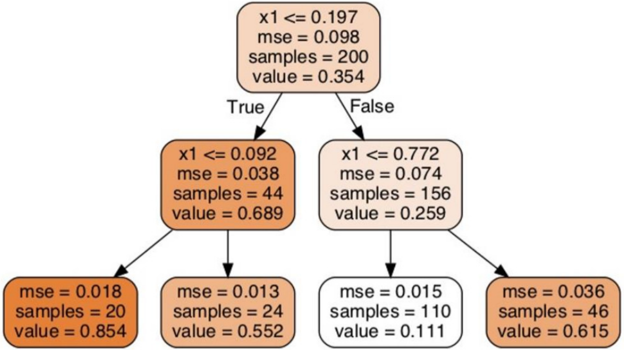
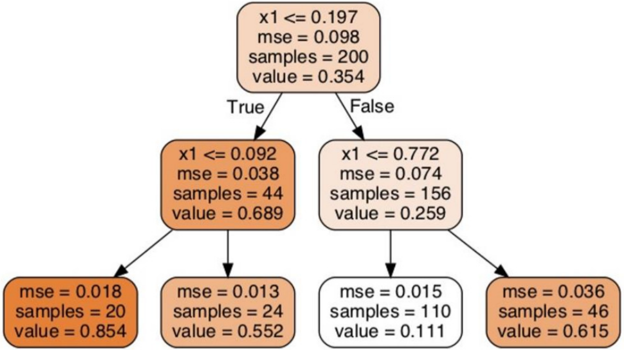
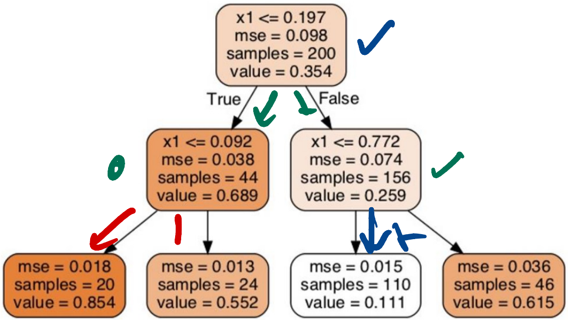


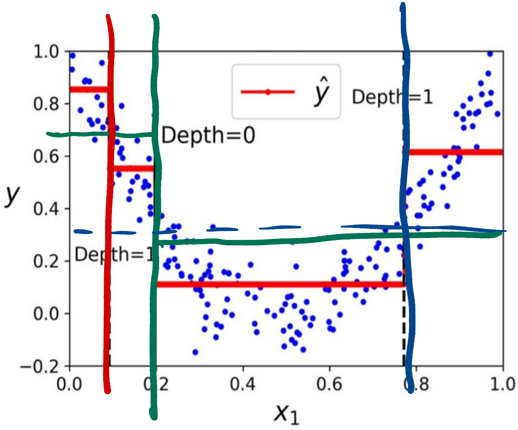
Decision trees for regression

- Regression tree for noisy quadratic centered around $x_1 = 0.5$
- For each node, the output is the mean y value for the current set of points
- Instead of impurity, use mean squared error (MSE) as cost function
- Choose a split that minimizes MSE



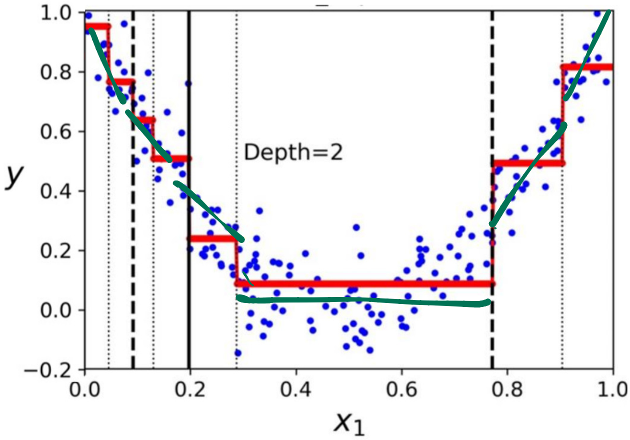
- Approximation using regression tree





Regression trees

- Extend the regression tree one more level to get a finer approximation



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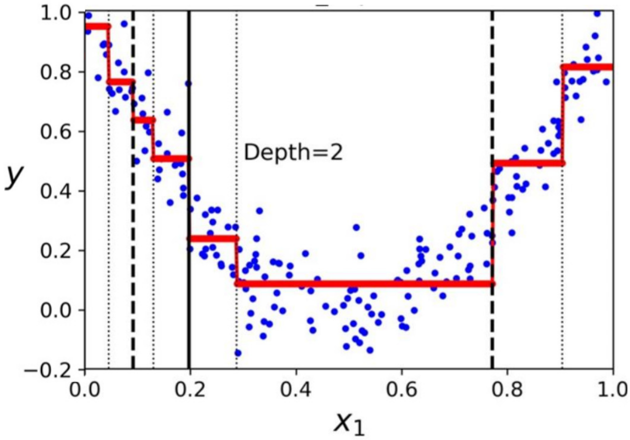
- Extend the regression tree one more level to get a finer approximation
- Set a threshold on MSE to decide when to stop



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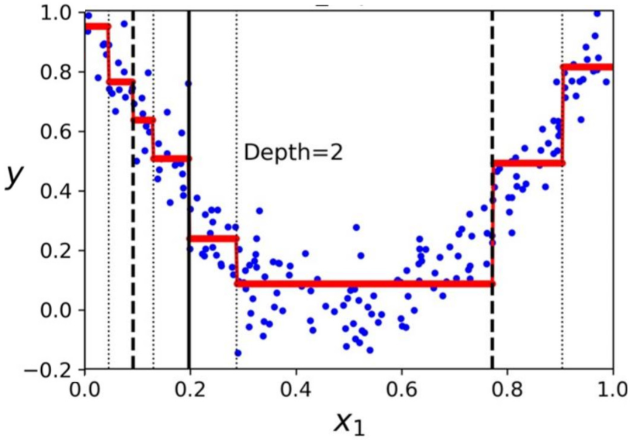

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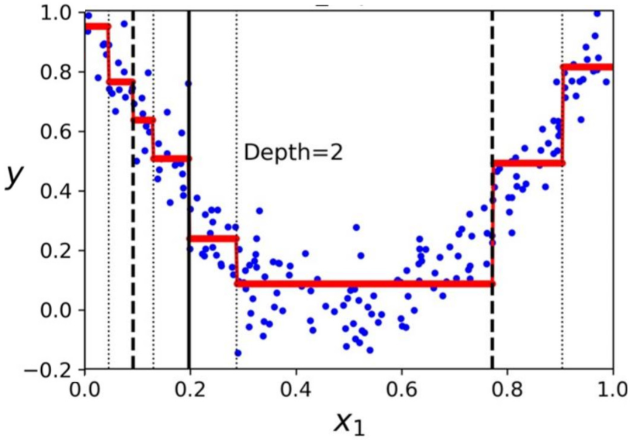

Regression trees

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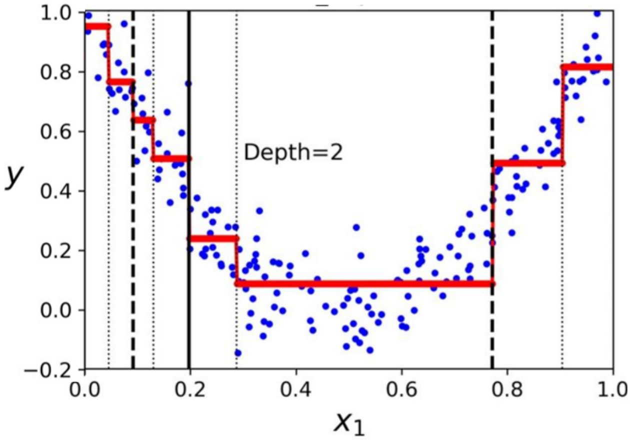
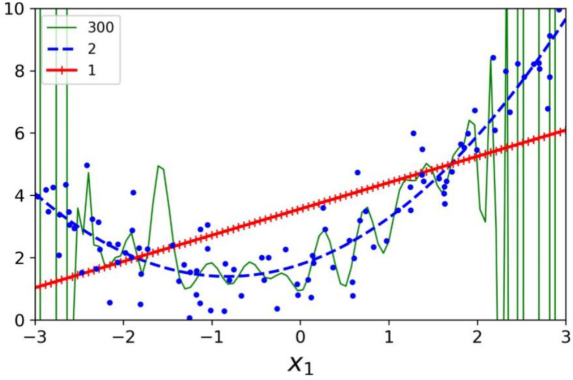
- Set a threshold on MSE to decide when to stop
- Classification and Regression Trees (CART)
  - Combined algorithm for both use cases
- Programming libraries typically provide CART implementation



Overfitting

- Overfitting: model too specific to training data, does not generalize well



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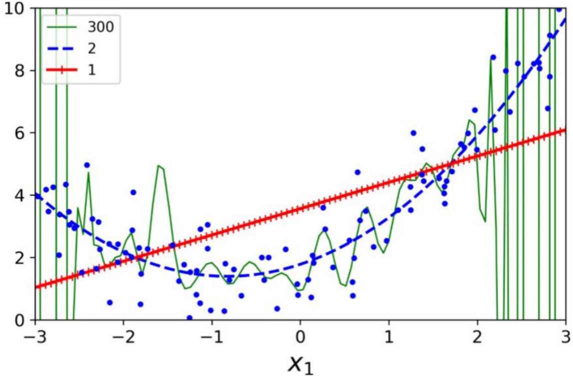

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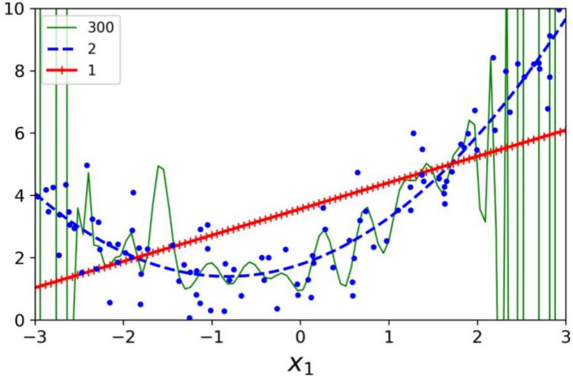

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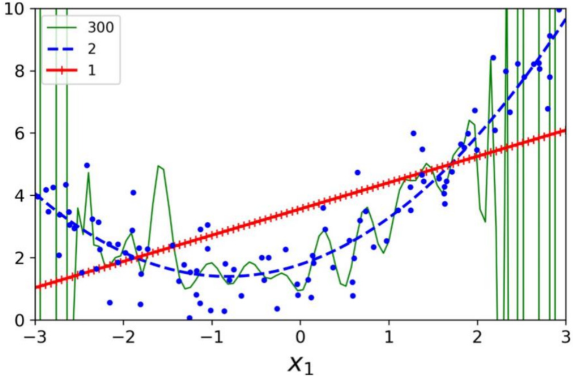

Overfitting

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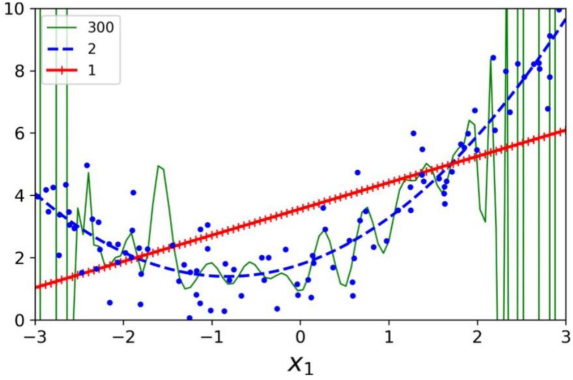
- Regression — use regularization to penalize model complexity
- What about decision trees?
- Deep, complex trees ask too many questions
- Prefer shallow, simple trees



Tree pruning

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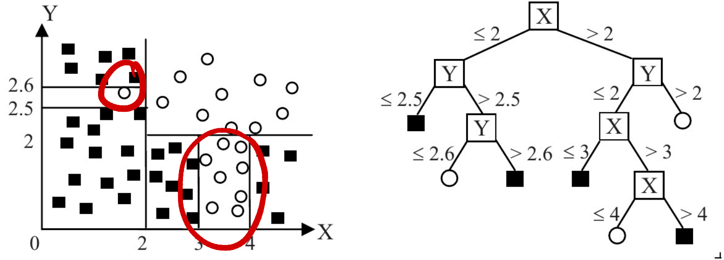
- Top-down pruning
  - Fix a maximum depth when building the tree
  - How to decide the depth in advance?
- Bottom-up pruning
  - Build the full tree
  - Remove a leaf if the reduced tree generalizes better
  - How do we measure this?

Tree pruning

Overfitted tree



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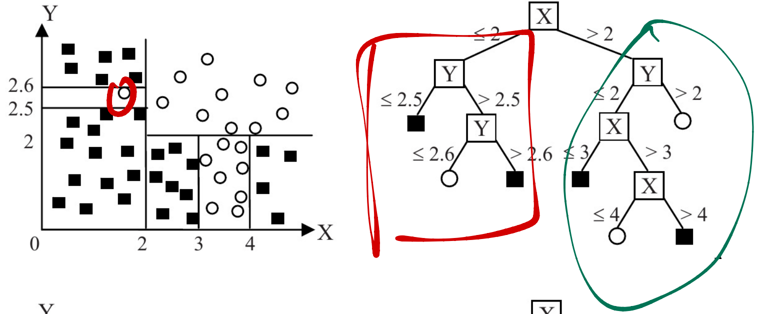
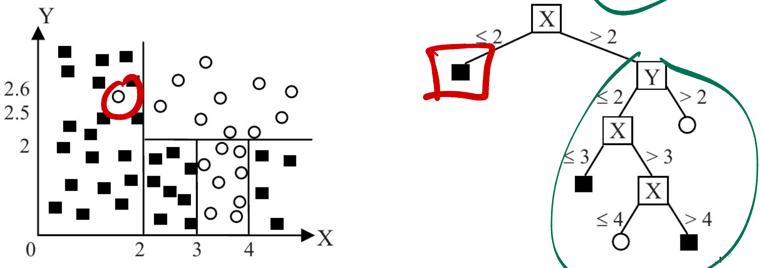


Pruned tree



Bottom up tree pruning

- Build the full tree, remove leaf if the reduced tree generalizes better
- How do we measure this?

Quinlan

= Entropy for trees

Bottom up tree pruning

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- Pruning leaves creates a larger impure sample one level above
- Does the confidence interval decrease (improve)?

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right

physican fee

y n u

```
physician fee freeze = n:
|   adoption of the budget resolution = y: democrat (151)
|   adoption of the budget resolution = u: democrat (1)
|   adoption of the budget resolution = n:
|   |   education spending = n: democrat (6)
|   |   education spending = y: democrat (9)
|   |   education spending = u: republican (1)
physician fee freeze = y:
|   synfuels corporation cutback = n: republican (97/3)
|   synfuels corporation cutback = u: republican (4)
|   synfuels corporation cutback = y:
|   |   duty free exports = y: democrat (2)
|   |   duty free exports = u: republican (1)
|   |   duty free exports = n:
|   |   |   education spending = n: democrat (5/2)
|   |   |   education spending = y: republican (13/2)
|   |   |   education spending = u: democrat (1)
physician fee freeze = u:
|   water project cost sharing = n: democrat (0)
|   water project cost sharing = y: democrat (4)
|   water project cost sharing = u:
|   |   mx missile = n: republican (0)
|   |   mx missile = y: democrat (3/1)
|   |   mx missile = u: republican (2)
```

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right
- After pruning, drastically simplified tree

conf. interval

```
physician fee freeze = n: democrat (168/2.6)
physician fee freeze = y: republican (123/13.9)
physician fee freeze = u:
|     mx missile = n: democrat (3/1.1)
|     mx missile = y: democrat (4/2.2)
|     mx missile = u: republican (2/1)
```

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right
- After pruning, drastically simplified tree
- Quinlan's comment on his use of sampling theory for post-pruning

  Now, this description does violence to statistical notions of sampling and confidence limits, so the reasoning should be taken with a large grain of salt. Like many heuristics with questionable underpinnings, however, the estimates it produces seem frequently to yield acceptable results.

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