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Data Mining and Machine Learning
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Market-basket analysis

- Set of items $I = \{i_1, i_2, \ldots, i_N\}$
- A transaction is a set $t \subseteq I$ of items
- Set of transactions $T = \{t_1, t_2, \ldots, t_M\}$
- Identify association rules $X \to Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - If $X \subseteq t_j$ then it is likely that $Y \subseteq t_j$
- Two thresholds
  - How frequently does $X \subseteq t_j$ imply $Y \subseteq t_j$?
  - How significant is this pattern overall?

Setting thresholds

- For $Z \subseteq I$, $Z.\text{count} = |\{t_j \mid Z \subseteq t_j\}|$
- How frequently does $X \subseteq t_j$ imply $Y \subseteq t_j$?
  - Fix a confidence level χ
  - Want $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
- How significant is this pattern overall?
  - Fix a support level σ
  - Want $\dfrac{(X \cup Y).count}{M} \geq \sigma$
- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \to Y$

$\{y_1, y_2, y_3\}$ $\{x_1, x_2\}$

$\dfrac{X\cup Y.\text{count}}{X.\text{count}} \leq 1$

$X + Y$

$X \cup Y$

$\dfrac{Z.\text{count}}{M} \geq \sigma$

Frequent - $Z.\text{count} \geq \sigma . M$

Apriori

- If Z is frequent, so is every subset $Y \subseteq Z$
- We exploit the contrapositive

Apriori observation

If Z is not a frequent itemset, no superset $Y \supseteq Z$ can be frequent

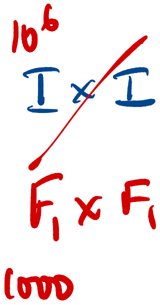
- For any frequent pair $\{x, y\}$, both $\{x\}$ and $\{y\}$ must be frequent
- Build frequent itemsets bottom up, size 1,2,. . .

Apriori algorithm

- F_i : frequent itemsets of size i — Level i

Apriori algorithm

- F_i : frequent itemsets of size i — Level i
- F_1: Scan T, maintain a counter for each $x \in I$



Apriori algorithm

- F_i : frequent itemsets of size i — Level i
- F_1: Scan T, maintain a counter for each $x \in I$
- $C_2 = \{\{x, y\} \mid x, y \in F_1\}$: Candidates in level 2

$\llcorner F_1 \times F_1$

Apriori algorithm

- F_i : frequent itemsets of size i — Level i
- F_1: Scan T, maintain a counter for each $x \in I$
- $C_2 = \{\{x, y\} \mid x, y \in F_1\}$: Candidates in level 2
- F_2: Scan T, maintain a counter for each $X \in C_2$
- $C_3 = \{\{x, y, z\} \mid \{x, y\}, \{x, z\}, \{y, z\} \in F_2\}$ $\Rightarrow x \in F_1, y \in F_1, z \in F_1$
- F_3: Scan T, maintain a counter for each $X \in C_3$
- ...
- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- F_k: Scan T, maintain a counter for each $X \in C_k$
- ...

Apriori algorithm

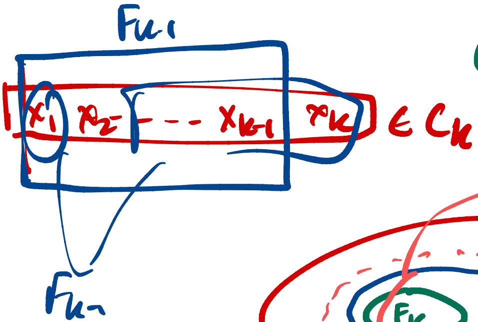
- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- How do we generate C_k?

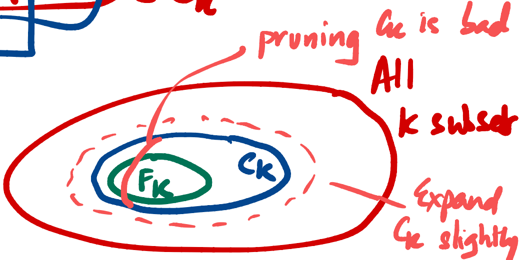
Apriori algorithm

- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- How do we generate C_k?
- Naïve: enumerate subsets of size k and check each one
  - Expensive!

C_k is an overapproximation of F_k

F_{k-1} → C_k ?





$$\{y_1, y_2, \ldots y_{k-1}\} \in F_{k-1} \quad + \quad y_k \in F_1$$

$$\{y_1, y_2 \ldots y_{k-1}, y_k\} \longrightarrow \hat{C}_k$$

$$F_{k-1} \times F_1$$

Apriori algorithm

- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- How do we generate C_k?
- Naïve: enumerate subsets of size k and check each one
  - Expensive!
- Observation: Any $C'_k \supseteq C_k$ will do as a candidate set

Apriori algorithm

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- Items are ordered: $i_1 < i_2 < \cdots < i_N$
- List each itemset in ascending order — canonical representation

Apriori algorithm

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- How do we generate C_k?
- Naïve: enumerate subsets of size k and check each one
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- Observation: Any $C'_k \supseteq C_k$ will do as a candidate set
- Items are ordered: $i_1 < i_2 < \cdots < i_N$
- List each itemset in ascending order — canonical representation
- Merge two $(k-1)$-subsets if they differ in last element
  - $X = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}\}$
  - $X' = \{i_1, i_2, \ldots, i_{k-2}, i'_{k-1}\}$
  - $\text{Merge}(X, X') = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}, i'_{k-1}\}$

k

Apriori algorithm

- $\text{Merge}(X, X') = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}, i'_{k-1}\}$
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Apriori algorithm

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  - $X = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}\}$
  - $X' = \{i_1, i_2, \ldots, i_{k-2}, i'_{k-1}\}$
- $C'_k = \{\text{Merge}(X, X') \mid X, X' \in F_{k-1}\}$
- Claim $C_k \subseteq C'_k$
  - Suppose $Y = \{i_1, i_2, \ldots, i_{k-1}, i_k\} \in C_k$
  - $X = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}\} \in F_{k-1}$ and $X' = \{i_1, i_2, \ldots, i_{k-2}, i_k\} \in F_{k-1}$
  - $Y = \text{Merge}(X, X') \in C'_k$

Apriori algorithm

- $\text{Merge}(X, X') = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}, i'_{k-1}\}$
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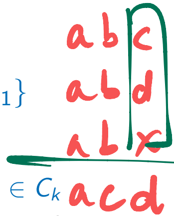
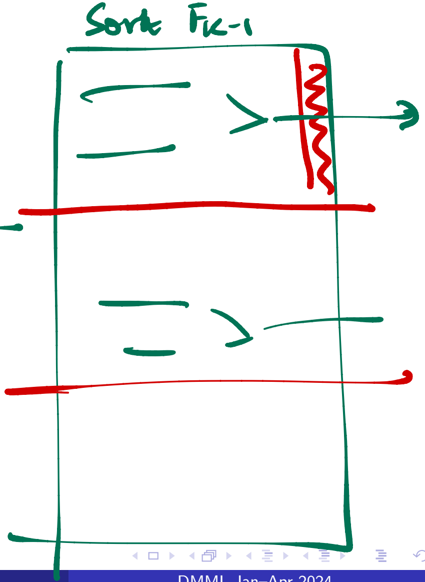
  - $Y = \text{Merge}(X, X') \in C'_k$
- Can generate C'_k efficiently
  - Arrange F_{k-1} in dictionary order
  - Split into blocks that differ on last element
  - Merge all pairs within each block





Apriori algorithm

- $C_1 = \{\{x\} \mid x \in I\}$
- $F_1 = \{Z \mid Z \in C_1, Z.\text{count} \geq \sigma \cdot M\}$
- For $k \in \{2, 3, \ldots\}$
  - $C'_k = \{\text{Merge}(X, X') \mid X, X' \in F_{k-1}\}$
  - $F_k = \{Z \mid Z \in C'_k, Z.\text{count} \geq \sigma \cdot M\}$

Apriori algorithm

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- When do we stop?

Apriori algorithm

- $C_1 = \{\{x\} \mid x \in I\}$
- $F_1 = \{Z \mid Z \in C_1, Z.\text{count} \geq \sigma \cdot M\}$
- For $k \in \{2, 3, \ldots\}$
  - $C'_k = \{\text{Merge}(X, X') \mid X, X' \in F_{k-1}\}$
  - $F_k = \{Z \mid Z \in C'_k, Z.\text{count} \geq \sigma \cdot M\}$
- When do we stop?
- k exceeds the size of the largest transaction
- F_k is empty

Z — frequent

Check if Z

contains a valid

rule $X \to Y$

Association rules

- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \to Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - $\dfrac{(X \cup Y).count}{M} \geq \sigma$

Association rules

- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \rightarrow Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - $\dfrac{(X \cup Y).count}{M} \geq \sigma$
- For a rule $X \rightarrow Y$ to be valid, $X \cup Y$ should be a frequent itemset
- Apriori algorithm finds all $Z \subseteq I$ such that $Z.\text{count} \geq \sigma \cdot M$

Association rules

Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\dfrac{(X \cup Y).count}{X.count} \geq \chi$

Association rules

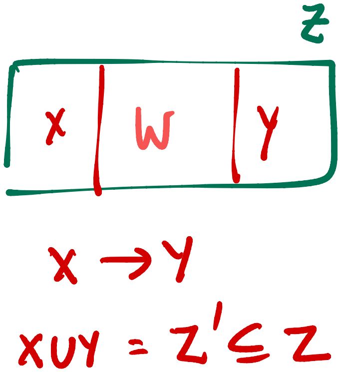
Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
- Can we do better?

Association rules

Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\frac{(X \cup Y).count}{X.count} \geq \chi$
- Can we do better?
- Sufficient to check all partitions of Z
  - If $X, Y \subseteq Z$, $X \cup Y$ is also a frequent itemset

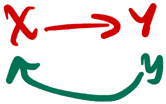


Association rules

- Sufficient to check all partitions of Z

disjoint union $X \uplus Y$

- Suppose $Z = X \uplus Y$, $X \rightarrow Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$?



Association rules

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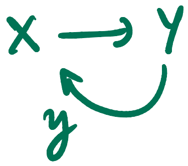
- Suppose $Z = X \uplus Y$, $X \rightarrow Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$



$X.count \geq X \cup \{y\}.count$

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \rightarrow Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$
  - X.count $\geq (X \cup \{y\})$.count, always
  - Second fraction has smaller denominator, so $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$ is also a valid rule

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \to Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \to Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$
  - $X.\text{count} \geq (X \cup \{y\}).\text{count}$, always
  - Second fraction has smaller denominator, so $(X \cup \{y\}) \to Y \setminus \{y\}$ is also a valid rule

Observation: Can use apriori principle again!

Apriori for association rules

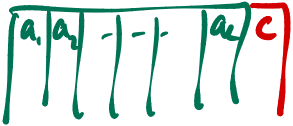
- If $X \to Y$ is a valid rule, and $y \in Y$, $(X \cup \{y\}) \to Y \setminus \{y\}$ must also be a valid rule
- If $X \to Y$ is not a valid rule, and $x \in X$, $(X \setminus \{x\}) \to Y \cup \{x\}$ cannot be a valid rule

Apriori for association rules

- If $X \to Y$ is a valid rule, and $y \in Y$, $(X \cup \{y\}) \to Y \setminus \{y\}$ must also be a valid rule
- If $X \to Y$ is not a valid rule, and $x \in X$, $(X \setminus \{x\}) \to Y \cup \{x\}$ cannot be a valid rule
- Start by checking rules with single element on the right
  - $Z \setminus z \to \{z\}$
- For $X \to \{x, y\}$ to be a valid rule, both $(X \cup \{x\}) \to \{y\}$ and $(X \cup \{y\}) \to \{x\}$ must be valid
- Explore partitions of each frequent itemset "level by level"

If $X \cup \{x\} \to \{y\}$ or $X \cup \{y\} \to \{x\}$ is not valid, skip $X \to \{x, y\}$

Supervised learning



- A set of items
  - Each item is characterized by attributes $(a_1, a_2, \ldots, a_k)$
  - Each item is assigned a class or category c
- Given a set of examples, predict c for a new item with attributes $(a'_1, a'_2, \ldots, a'_k)$

Supervised learning

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- Examples provided are called training data
- Aim is to learn a mathematical model that generalizes the training data
  - Model built from training data should extend to previously unseen inputs

Supervised learning

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- Aim is to learn a mathematical model that generalizes the training data
  - Model built from training data should extend to previously unseen inputs
- Classification problem
  - Usually assumed to binary — two classes

Association rules for classification

- Classify documents by topic
- Consider the table on the right

Words in document	**Topic**
student, teach, school	Education
student, school	Education
teach, school, city, game	Education
cricket, football	Sports
football, player, spectator	Sports
cricket, coach, game, team	Sports
football, team, city, game	Sports

Association rules for classification

- Classify documents by topic
- Consider the table on the right
- Items are regular words and topics
- Documents are transactions — set of words and one topic

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Association rules for classification

- Classify documents by topic
- Consider the table on the right
- Items are regular words and topics
- Documents are transactions — set of words and one topic
- Look for association rules of a special form
  - {student, school} $\rightarrow$ {Education}
  - {game, team} $\rightarrow$ {Sports}

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Association rules for classification

- Classify documents by topic
- Consider the table on the right
- Items are regular words and topics
- Documents are transactions — set of words and one topic
- Look for association rules of a special form
  - $\{$student, school$\} \rightarrow \{$Education$\}$
  - $\{$game, team$\} \rightarrow \{$Sports$\}$
- Right hand side always a single topic
- Class Association Rules

Words in document	**Topic**
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football, team, city, game	Sports

Summary

- Market-basket analysis searches for correlated items across transactions
- Formalized as association rules
- Apriori principle helps us to efficiently
  - identify frequent itemsets, and
  - split these itemsets into valid rules
- Class association rules — simple supervised learning model

Context – single transaction

– sequences across transactions

Uniform thresholds