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Data Mining and Machine Learning
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Bayesian classifiers

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  - Attributes $\{A_1, A_2, \ldots, A_k\}$ and
  - Classes $C = \{c_1, c_2, \ldots c_\ell\}$

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- Given a data item $d = (a_1, a_2, \ldots, a_k)$, identify the best class c for d
- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$

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- Generative model has associated parameters $\theta = (\theta_1, \ldots, \theta_m)$
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- We need to estimate these parameters

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- $\hat{\theta} = H/N$ maximizes this likelihood — $\arg\max_{\theta} L(\theta) = \hat{\theta} = H/N$
  - Maximum Likelihood Estimator (MLE)

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— Indep of c_i

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- Denominator is the same for all c_i, so sufficient to maximize

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)$$

$$P(C|A) = \frac{P(A|C)\,P(C)}{P(A)}$$

Bayes

$$P(C|A) = \frac{P(A \wedge C)}{P(A)}$$

Example

- To classify $A = g, B = q$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
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- $Pr(C = t) = 5/10 = 1/2$
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- $Pr(A = g, B = q \mid C = f) = 1/5$
- $Pr(A = g, B = q \mid C = f) \cdot Pr(C = f) = 1/10$

P(C|A) = P(A|C) . P(C)

Numerator

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- $Pr(C = f) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = f) = 1/5$
- $Pr(A = g, B = q \mid C = f) \cdot Pr(C = f) = 1/10$

- Hence, predict $C = t$

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Example ...

- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$

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- What if we want to classify $A = m, B = q$?
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- $Pr(A = m, B = q \mid C = t) = 0$
- Also $Pr(A = m, B = q \mid C = f) = 0$!
- To estimate joint probabilities across all combinations of attributes, we need a much larger set of training data

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Naïve Bayes classifier

- Strong simplifying assumption: attributes are pairwise independent

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) = \prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)$$

  - $Pr(C = c_i)$ is fraction of training data with class c_i
  - $Pr(A_j = a_j \mid C = c_i)$ is fraction of training data labelled c_i for which $A_j = a_j$

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- Final classification is

$$\arg\max_{c_i} \; Pr(C = c_i) \prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)$$

Common term

Naïve Bayes classifier . . .

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- For instance, text classification
  - Items are documents, attributes are words (absent or present)
  - Classes are topics
  - Conditional independence says that a document is a set of words: ignores sequence of words
  - Meaning of words is clearly affected by relative position, ordering

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  - Meaning of words is clearly affected by relative position, ordering
- However, naive Bayes classifiers work well in practice, even for text classification!
  - Many spam filters are built using this model

Example revisited

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- $Pr(A = m, B = q \mid C = t) = Pr(A = m, B = q \mid C = f) = 0$

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- $Pr(A = m \mid C = t) \cdot Pr(B = q \mid C = t) \cdot Pr(C = t) = 2/25$

$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{1}{2}$

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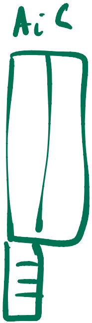
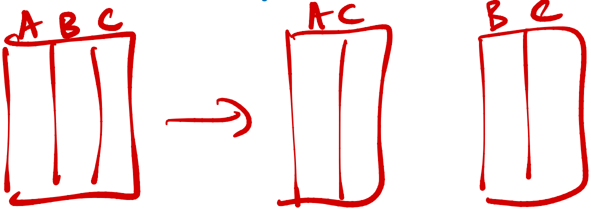
- "Pad" training data with one sample for each value a_j — m_i extra data items





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- Assume A_i takes m_i values $\{a_{i1}, \ldots, a_{im_i}\}$
- "Pad" training data with one sample for each value a_j — m_i extra data items
- Adjust $Pr(A_i = a_i \mid C = c_j)$ to $\dfrac{n_{ij} + 1}{n_j + m_i}$ where
  - n_{ij} is number of samples with $A_i = a_i$, $C = c_j$
  - n_j is number of samples with $C = c_j$

number of (a_i, c_j)

Smoothing

- Laplace's law of succession

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + 1}{n_j + m_i}$$

fudge factor

Smoothing

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- More generally, Lidstone's law of succession, or smoothing

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- $\lambda = 1$ is Laplace's law of succession

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- Want to use a naïve Bayes classifier
- Need to define a generative model
- How do we represent documents?

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- Each topic c has probability $Pr(c)$
- Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$

Toss m coins

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- Generating a random document d
  - Choose a topic c with probability $Pr(c)$
  - For each $w \in V$, toss a coin, include w in d with probability $Pr(w \mid c)$

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  - Choose a topic c with probability $Pr(c)$
  - For each $w \in V$, toss a coin, include w in d with probability $Pr(w \mid c)$
- $Pr(d \mid c) = \prod_{w_i \in d} Pr(w_i \mid c) \prod_{w_i \notin d} (1 - Pr(w_i \mid c))$

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- Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$
- Generating a random document d
  - Choose a topic c with probability $Pr(c)$
  - For each $w \in V$, toss a coin, include w in d with probability $Pr(w \mid c)$
- $Pr(d \mid c) = \prod_{w_i \in d} Pr(w_i \mid c) \prod_{w_i \notin d} (1 - Pr(w_i \mid c))$
- $Pr(d) = \sum_{c \in C} Pr(d \mid c)$

Naïve Bayes classifier

- Training set $D = \{d_1, d_2, \ldots, d_n\}$
  - Each $d_i \subseteq V$ is assigned a unique label from C

$V = w_1 \; w_2 \; - - \; w_m$

$0 \; 1 \; 1 \; \cdots 0 \cdots \; 1$

$d : V \rightarrow \{0, 1\}$

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Bag of words model

- Each document is a multiset or bag of words over a vocabulary $V = \{w_1, w_2, \ldots, w_m\}$
  - Count multiplicities of each word

Set $f: V \rightarrow \{0,1\}$

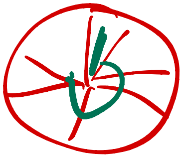
Multiset/bag $f: V \rightarrow \mathbb{N}_0$

Bag of words model

- Each document is a multiset or bag of words over a vocabulary $V = \{w_1, w_2, \ldots, w_m\}$
  - Count multiplicities of each word
- As before
  - Each topic c has probability $Pr(c)$
  - Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$ (but we will estimate these differently)
  - Note that $\sum_{i=1}^{m} Pr(w_i \mid c_j) = 1$
  - Assume document length is independent of the class

Bag of words model

- Generating a random document d
  - Choose a document length ℓ with $Pr(\ell)$
  - Choose a topic c with probability $Pr(c)$
  - Recall $|V| = m$.
    - To generate a single word, throw an m-sided die that displays w with probability $Pr(w \mid c)$
    - Repeat ℓ times



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- Let n_j be the number of occurrences of w_j in d
- $$Pr(d \mid c) = Pr(\ell)\ \ell! \prod_{j=1}^{m} \frac{Pr(w_j \mid c)^{n_j}}{n_j!}$$

Order

Parameter estimation

- Training set $D = \{d_1, d_2, \ldots, d_n\}$
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- $Pr(w_i \mid c_j)$ — fraction of occurrences of w_i over documents $D_j \subseteq D$ labelled c_j
  - n_{id} — occurrences of w_i in d
  - $Pr(w_i \mid c_j) = \dfrac{\sum\limits_{d \in D_j} n_{id}}{\sum\limits_{t=1}^{m} \sum\limits_{d \in D_j} n_{td}}$

— Seeing w_i

— all words in c_j

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  - $$Pr(w_i \mid c_j) = \frac{\displaystyle\sum_{d \in D_j} n_{id}}{\displaystyle\sum_{t=1}^{m} \sum_{d \in D_j} n_{td}} = \frac{\displaystyle\sum_{d \in D} n_{id}\ Pr(c_j \mid d)}{\displaystyle\sum_{t=1}^{m} \sum_{d \in D} n_{td}\ Pr(c_j \mid d)},$$

    $$\text{since } Pr(c_j \mid d) = \begin{cases} 1 & \text{if } d \in D_j, \\ 0 & \text{otherwise} \end{cases}$$

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- Recall, $Pr(d \mid c) = Pr(\ell)\; \ell! \prod_{j=1}^{m} \dfrac{Pr(w_j \mid c)^{n_j}}{n_j!}$, where $|d| = \ell$
- Discard $Pr(\ell), \ell!$ since they do not depend on c
- Compute $\arg\max_{c}\; Pr(c) \prod_{j=1}^{m} \dfrac{Pr(w_j \mid c)^{n_j}}{n_j!}$

Subset of words model

Bag of words model