Concurrency Theory, January–April 2018

Assignment 1, 8 February, 2018
Due: 18 February, 2018

Note: Only electronic submissions accepted, via Moodle.

Definitions

- A Petri net $(N = (P, T, F), M_{in})$ is *k-safe* if $M(p) \leq k$ for each place $p \in P$ and every reachable marking M. We usually abbreviate 1-safe as just *safe*. A net is *bounded* if it is k-safe for some k.
- A safe Petri net $(N = (P, T, F), M_{in})$ is *sequential* if it exhibits no concurrency. That is, for every reachable marking M, if $M \xrightarrow{t_1}$ and $M \xrightarrow{t_2}$, then ${}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset$.
- A Petri net $(N = (P, T, F), M_{in})$ is *live* if for each transition $t \in T$ and every reachable marking M, there is a marking M' reachable from M where t becomes enabled.

1. Construct a family of sequential Petri nets with $O(n)$ places and transistions and $2^{O(n)}$ reachable markings, for $n = 1, 2, \ldots$. Note that the trivial example with n independent transitions is not a sequential net. *Hint:* Model an asynchronous (ripple) counter.

2. Construct a Petri net with two transitions $\{a, b\}$ whose (unlabelled) prefix-closed language is $\{a, ab, aba, b\}$. You can use arc weights, if needed.

3. A Petri net (P, T, F) is *weakly connected* if the underlying undirected graph (that is, ignore the directions of the arrows in F) is a connected graph.

   Let (P, T, F) be a weakly connected net such that the net system $((P, T, F), M_{in})$ is *live* and *bounded*. Show that the net is, in fact, *strongly connected*. In other words, for every pair $(x, y) \in (P \cup T) \times (P \cup T)$, there is a directed path via F from x to y.

   Note: This is Theorem 11 (Section 3.5) in the survey paper by Desel and Reisig, but the proof in the paper is wrong.

4. In an event structure, a configuration c is *prime* if the following holds: whenever $c \subseteq \bigcup C$ for a subset of configurations C, then $c \subseteq c'$ for some $c' \in C$.

   Define a transition relation $c \xrightarrow{e} c'$ if $c' = c \cup \{e\}$. Show that if $c_1 \xrightarrow{e_1} c$ and $c_2 \xrightarrow{e_2} c$, where $e_1 \neq e_2$, then c cannot be a prime configuration.

5. The figure on the right below shows the configurations of an event structure ordered under inclusion.

   (a) Draw the corresponding event structure. Use $\longrightarrow$ to indicate causality and $\#$ to indicate conflict.

   (b) Draw an unlabelled 1-safe net that generates this behaviour.

$$\begin{tikzcd}
& \{e_1, e_2, e_4\} & & \\
& \{e_1, e_2\} \arrow[u] & & \{e_2, e_3\} \\
\{e_1\} \arrow[ur] & & \{e_2\} \arrow[ul] \arrow[ur] & \\
& \emptyset \arrow[ul] \arrow[ur] & &
\end{tikzcd}$$