Concurrency Theory, January–April 2017

Assignment 2, 13 February, 2017
Due: 19 February, 2017

Note: Only electronic submissions accepted, via Moodle.

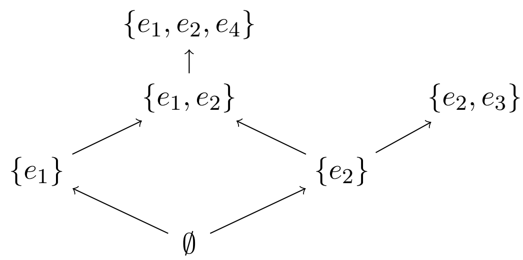
1. Let (Σ, I) be a trace alphabet with $\Sigma = \{a, b, c\}$ and $I = \{(a, b), (b, a)\}$.
   (a) Draw the trace $[abacabbccbaa]$ as a labelled partial order $(E, \leq, \ell)$.
   (b) For a trace $t = (E, \leq, \ell)$ and $a \in \Sigma$, we define the a-view of t to be the trace consisting of the events $\{e \mid e \leq \max_a(t)\}$, where $\max_a(t)$ is the maximum a-labelled event in t. Draw the a-view, b-view and c-view of the trace $[abacabbccbaa]$.

2. Let (Σ, I) be a trace alphabet. An $I\Diamond$ transition system $TS = (S, s_{in}, \rightarrow)$ is one in which the following property holds for any $(a, b) \in I$.
   - If $s \xrightarrow{a} s_a \xrightarrow{b} s_{ab}$ then there exists s_b such that $s \xrightarrow{b} s_b \xrightarrow{a} s_{ab}$.

   A finite state automaton (NFA or DFA) is said to be an $I\Diamond$ automaton if its underlying transition system is an $I\Diamond$ transition system.
   (a) Given an $I\Diamond$ automaton $\mathcal{A}$ over (Σ, I), is the language $L(\mathcal{A})$ that it accepts always a regular trace language?
   (b) Show that every regular trace language is accepted by an $I\Diamond$ automaton. (*Hint:* Consider the minimum DFA.)

3. In an event structure, a configuration c is *prime* if the following holds: whenever $c \subseteq \bigcup C$ for a subset of configurations C, then $c \subseteq c'$ for some $c' \in C$.
   Define a transition relation $c \xrightarrow{e} c'$ if $c' = c \cup \{e\}$. Show that if $c_1 \xrightarrow{e_1} c$ and $c_2 \xrightarrow{e_2} c$, where $e_1 \neq e_2$, then c cannot be a prime configuration.

4. The figure on the right below shows the configurations of an event structure ordered under inclusion.
   (a) Draw the corresponding event structure. Use $\longrightarrow$ to indicate causality and $\#$ to indicate conflict.
   (b) Draw an unlabelled 1-safe net that generates this behaviour.



5.
   (a) Construct the unfolding upto depth 3 of the net on the right. Recall that the depth of a transition t is the maximum length k among sequences of the form $t_1 < t_2 < \cdots < t_k$ where $t_k = t$.
   (b) Identify all cutoff events at this depth using the following definition of an adequate order for configurations.
   $C \prec C'$ iff
   $$\mathrm{Mark}(C) = \mathrm{Mark}(C') \text{ and } |C| < |C'|$$