Concurrency Theory, January–April 2017

Assignment 1, 26 January, 2017
Due: 5 February, 2017

Note: Only electronic submissions accepted, via Moodle.

Definitions

- A Petri net $(N = (P, T, F), M_{in})$ is *k-safe* if $M(p) \leq k$ for each place $p \in P$ and every reachable marking M. We usually abbreviate 1-safe as just *safe*.
- A Petri net $(N = (P, T, F), M_{in})$ is *live* if for each transition $t \in T$ and every reachable marking M, there is a marking M' reachable from M where t becomes enabled.
- A Petri net $(N = (P, T, F), M_{in})$ is *dead* if no transition $t \in T$ is enabled at M_{in}.
- A safe Petri net $(N = (P, T, F), M_{in})$ is *sequential* if it exhibits no concurrency. That is, for every reachable marking M, if $M \xrightarrow{t_1}$ and $M \xrightarrow{t_2}$, then ${}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset$.
- A safe Petri net $(N = (P, T, F), M_{in})$ is *deterministic* if it exhibits no choice. That is, for every reachable marking M, there is at most one transition enabled.

1. Construct a safe net $(N = (P, T, F), M_{in})$ whose marking graph is as follows. Initially, two independent transitions t_1 and t_2 (i.e., ${}^\bullet t_1{}^\bullet \cap {}^\bullet t_2{}^\bullet = \emptyset$) are enabled concurently, with $M_{in} \xrightarrow{t_1} M_1 \xrightarrow{t_2} M$ and $M_{in} \xrightarrow{t_2} M_2 \xrightarrow{t_1} M$ such that (N, M) is sequential, deterministic and live.

2. Construct a safe net $(N = (P, T, F), M_{in})$ whose marking graph is as follows. Initially, two transitions t_L and t_D are enabled with $M_{in} \xrightarrow{t_L} M_L$ and $M_{in} \xrightarrow{t_D} M_D$. The net (N, M_L) is live, sequential and deterministic. The net (N, M_D) is dead. Note that liveness of (N, M_L) is with respect to all of T, including t_L and t_D.

3. Construct a family of sequential Petri nets with $O(n)$ places and transistions and $2^{O(n)}$ reachable markings, for $n = 1, 2, \ldots$. Note that the trivial example with n independent transitions is not a sequential net. *Hint:* Model an asynchronous (ripple) counter.

4. Construct a Petri net with two transitions $\{a, b\}$ whose (unlabelled) prefix-closed language is $\{a, ab, aba, b\}$. You can use arc weights, if needed.

5. A Petri net (P, T, F) is *weakly connected* if the underlying undirected graph (that is, ignore the directions of the arrows in F) is a connected graph.

   Let (P, T, F) be a weakly connected net such that the net system $((P, T, F), M_{in})$ is *live* and *bounded*. Show that the net is, in fact, *strongly connected*. In other words, for every pair $(x, y) \in (P \cup T) \times (P \cup T)$, there is a directed path via F from x to y.

   Note: This is Theorem 11 (Section 3.5) in the survey paper by Desel and Reisig, but the proof in the paper is wrong.