$(\Sigma, \underline{\underline{I}}) \rightsquigarrow \underbrace{(\Sigma_1, \Sigma_2, \ldots \Sigma_n)}$

Indep reln

$\Gamma\ \Sigma,\ loc : \Sigma \to 2^{\{1,2,\ldots,n\}}$

Regular trace languages

Direct product local presentations

$TS_i = (Q_i, \Sigma_i, \to_i, Q^i_{in}, F_i)$

Not closed wrt boolean ops

$L = shuffle(L)$

Synch product: Global final states $F \subseteq Q_1 \times \cdots \times Q_n$

Boolean closure

$[[(a|b)+(aa|bb)]c]^*$

$\langle a,c \rangle$ $\langle b,c \rangle$

$\{a$ $\{b$

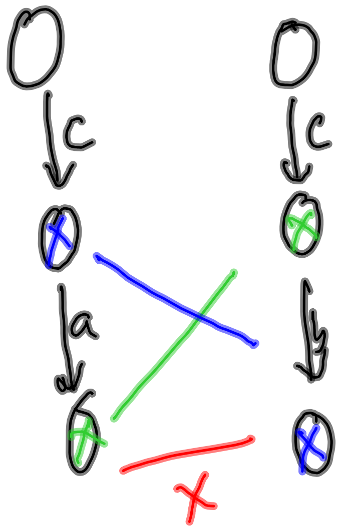
No characterization

= Finite unions of direct products

$\Sigma = \{a,b,c\}$ $I = \{(a,b),(b,a)\}$

$L = \{ca, cb\}$

$\Rightarrow cab \in L$



lang is not forward ◇ closed

$uabv \in L$

$(a,b) \in I$

$ua \in L$,

$ub \in L$, $(a,b) \in I$ $\Rightarrow ubav \in L$

$\not\Rightarrow uab \in L$

Synthesis

Asynchronous automata : $\rightarrow_a \subseteq Q_a \times Q_a$

"Traces" = equivalence of words modulo $\sim_I$

$W = a_1 a_2 \ldots a_n$

$\Sigma_1 = \{a, c\}$ $\Sigma_2 = \{b, c\}$

$a_i \lessdot a_j$:
- $a_i D a_j$
- $i < j$
- $\nexists k$ $i < k < j$

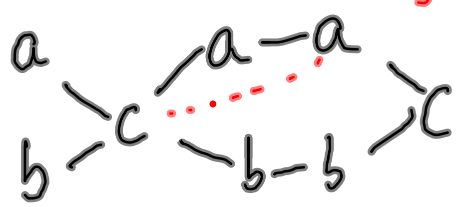
✓

$ab\boxed{c}aabbc$

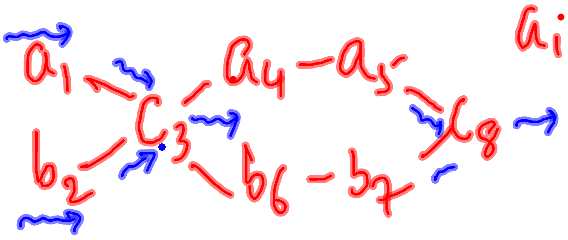
$bacaabbc$

$abab$

$bbaa$



abcaabbc
1 2 3 4 5 6 7 8



$a_i D a_k, a_k D a_j$

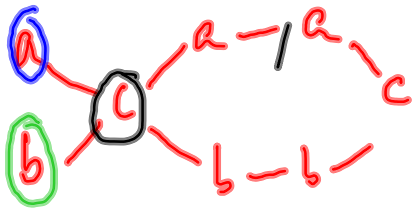
Thm (Zielonka) Every reg trace lang is recognized by a (deterministic) asynch. automaton

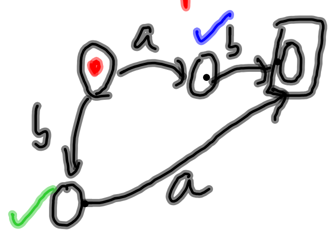
L: reg trace lang

min DFA $\mathcal{A}_L$

Distributed simulation of $\mathcal{A}_L$

On w, run $\mathcal{A}$ and check if it accepts





Information flow in an asynchronous automaton

P "processes" Periodically $X \subseteq P$ synchronize

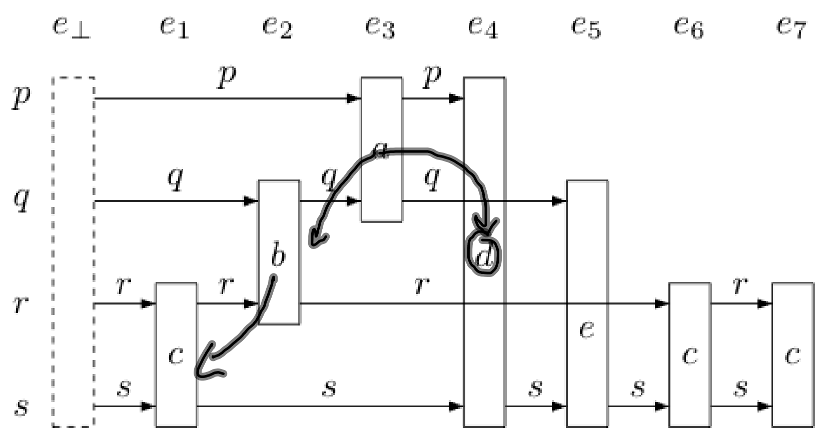
Each $p \in X$ has it's "latest" info about every $q \in P$

Determine whose information is best

"Gossip problem"

→ Timestamping

Bounded no. of states ?



$\Sigma_p = \{a, d\}$

$\Sigma_q = \{b, a, e\}$

$\Sigma_r = \{c, b\}$

$\Sigma_s = \{c, d, e\}$

c — b — a — d — e → c — c

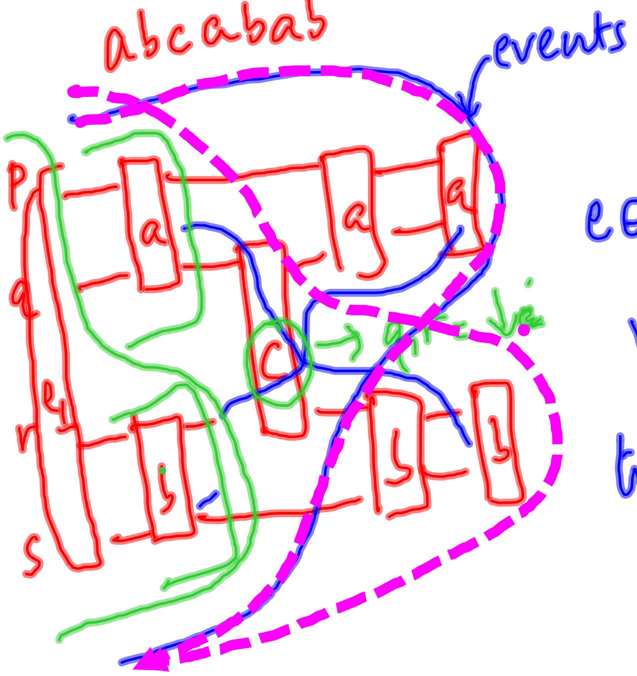
$(E, \leq, \lambda)$

$\Sigma_p = \{a\}$

$\Sigma_q = \{a, c\}$

$\Sigma_r = \{b, c\}$

$\Sigma_s = \{b\}$



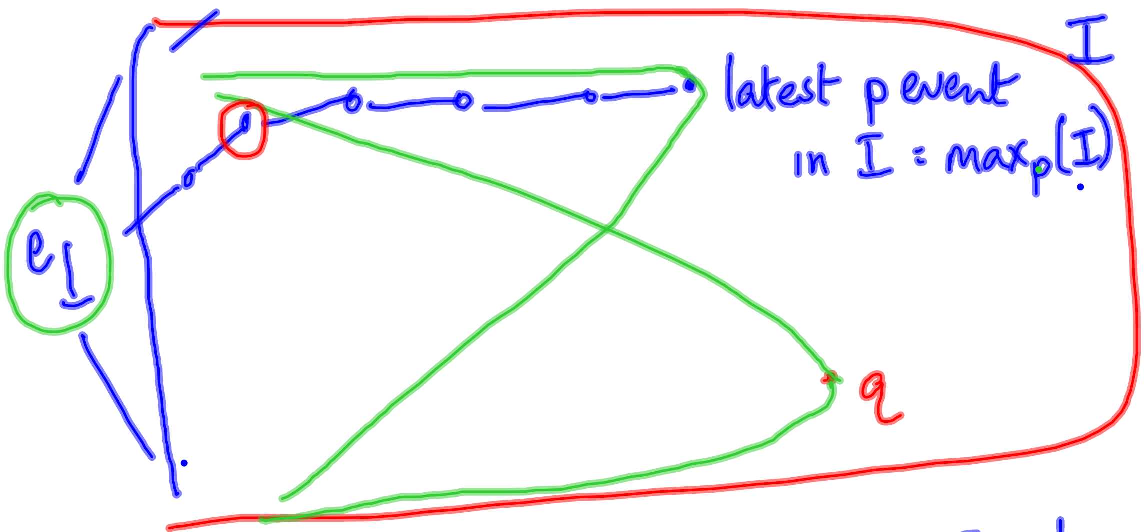
$e \in E$

$\downarrow e \triangleq \{f \mid f \in E, f \leq e\}$

When e occurs, everything in $\downarrow e$ is "known" to $p \in loc(\lambda(e))$

$\downarrow X = \bigcup_{x \in X} \downarrow x$

$I \subseteq E$ is an ideal if $I = \downarrow I$



p's info about $I = \downarrow \max_p(I)$

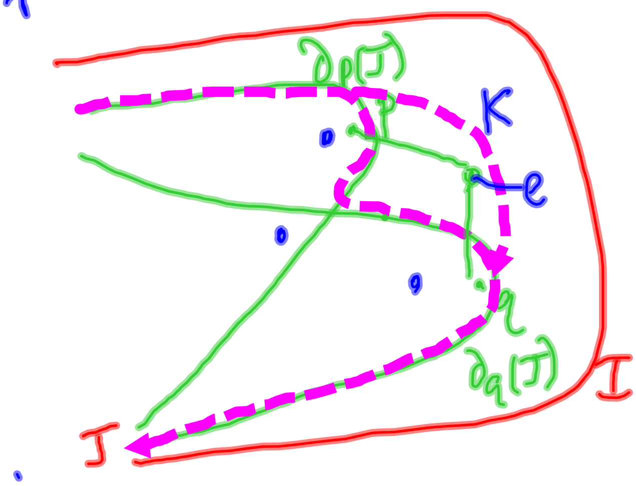
$= \partial_p(I) =$ p view

Best info q has about p in I

$$latest_{q\to p}(I) = max_p\left(\partial_q(I)\right)$$

$max_r(K)$?

$r \notin loc(e)$

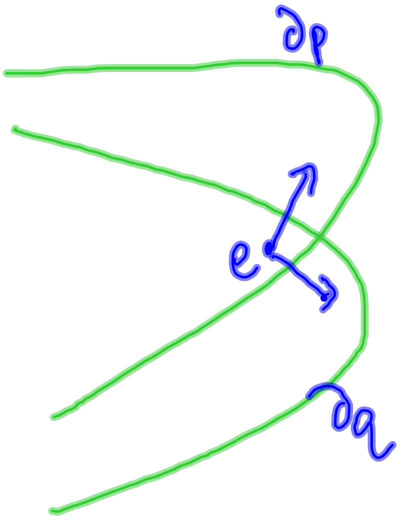


p and q should resolve if

$max_r(K) \in \partial_p(J) \cap \partial_q(J)$

$latest_{p\to r}(J)$ $\quad \partial_p(J) \setminus \partial_q(J)$

$latest_{q\to r}(J)$ $\quad \partial_q(J) \setminus \partial_p(J)$



$e \in \partial_p(I) \cap \partial_q(I)$

e is maximal in intersection

$\nexists f \quad e < f, \; f \in \partial_p(I) \cap \partial_q(I)$

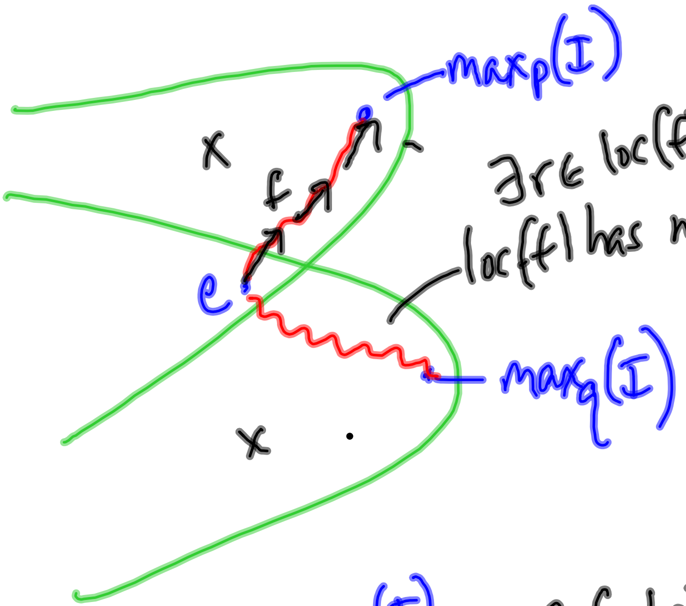
Primary information of p in I

$primary_p(I) = \{ latest_{p \to q}(I) \mid q \in P \}$

Lemma e maximal in $\partial_p(I) \cap \partial_q(I)$
implies $e \in primary_p(I) \cap primary_q(I)$

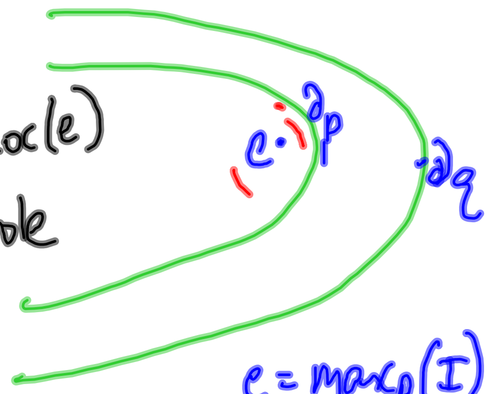
$\exists r, s \quad e = lat_{p \to r}(I) = lat_{q \to s}(I)$

$e \in \cap \Rightarrow e \in prm_p \cap prm_q$



$\exists r \in loc(f) \cap loc(e)$

$loc(f)$ has no role



$e = max_p(I)$

$e = latest_{p \to p}(I)$

$e = latest_{q \to p}(I)$

$e \leq max_p(I)$

$e \leq max_q(I)$

e, f both r-events

$e \in \partial_q(I),\ f \notin \partial_q(I) \Rightarrow e = lat_{q \to r}(I)$

$e \in prim_p(I) \cap prim_q(I) \cancel{\Rightarrow} e$ is maximal in $\partial_p(I) \cap \partial_q(I)$

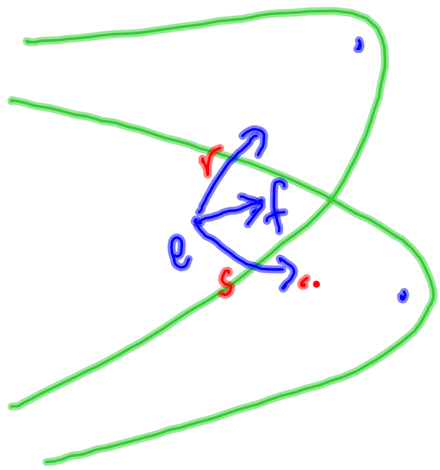
• $e \in \partial_p(I) \cap \partial_q(I)$

Given $e = lat_{p \to r}(I)$

$e' = lat_{q \to r}(I)$

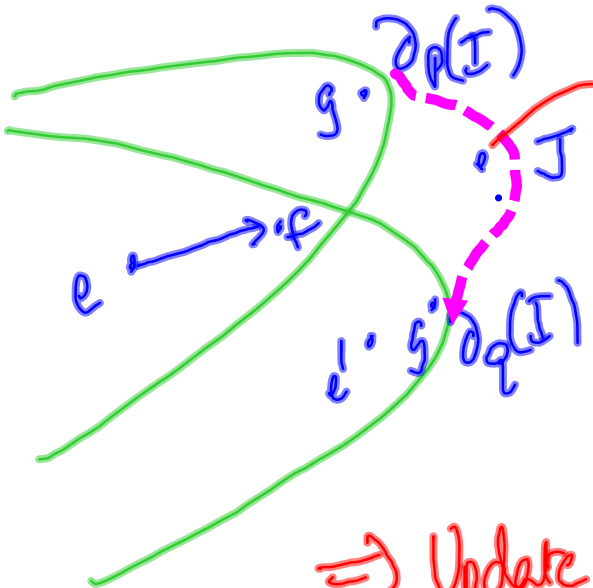
$e < e'$ or $e' < e$?

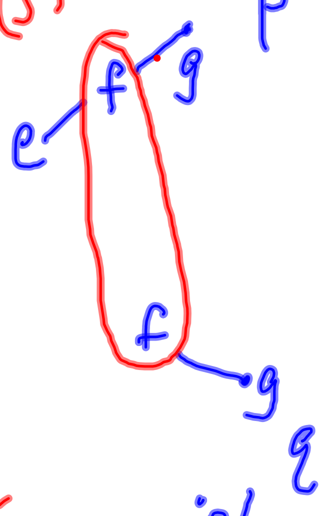
$e <$ some max event in $\cap$



Maintain primary events with $\leq$ ordering, can check if $e < e'$ or $e' < e$

$lat_{p\to p}(J) = lat_{q\to q}(J)$



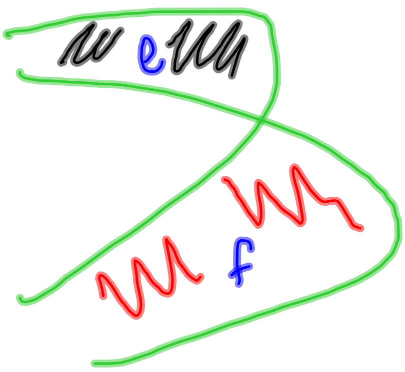


$\Rightarrow$ Update & get

same set $\stackrel{?}{=}$ edges? $primary_p(J) = primary_q(J)$

$e, f \in \text{primary}_p(J), \text{primary}_q(J)$ not $\max_p(J) < \max_q(J)$

$e < f$?



Case 1 e, f both contributed by p

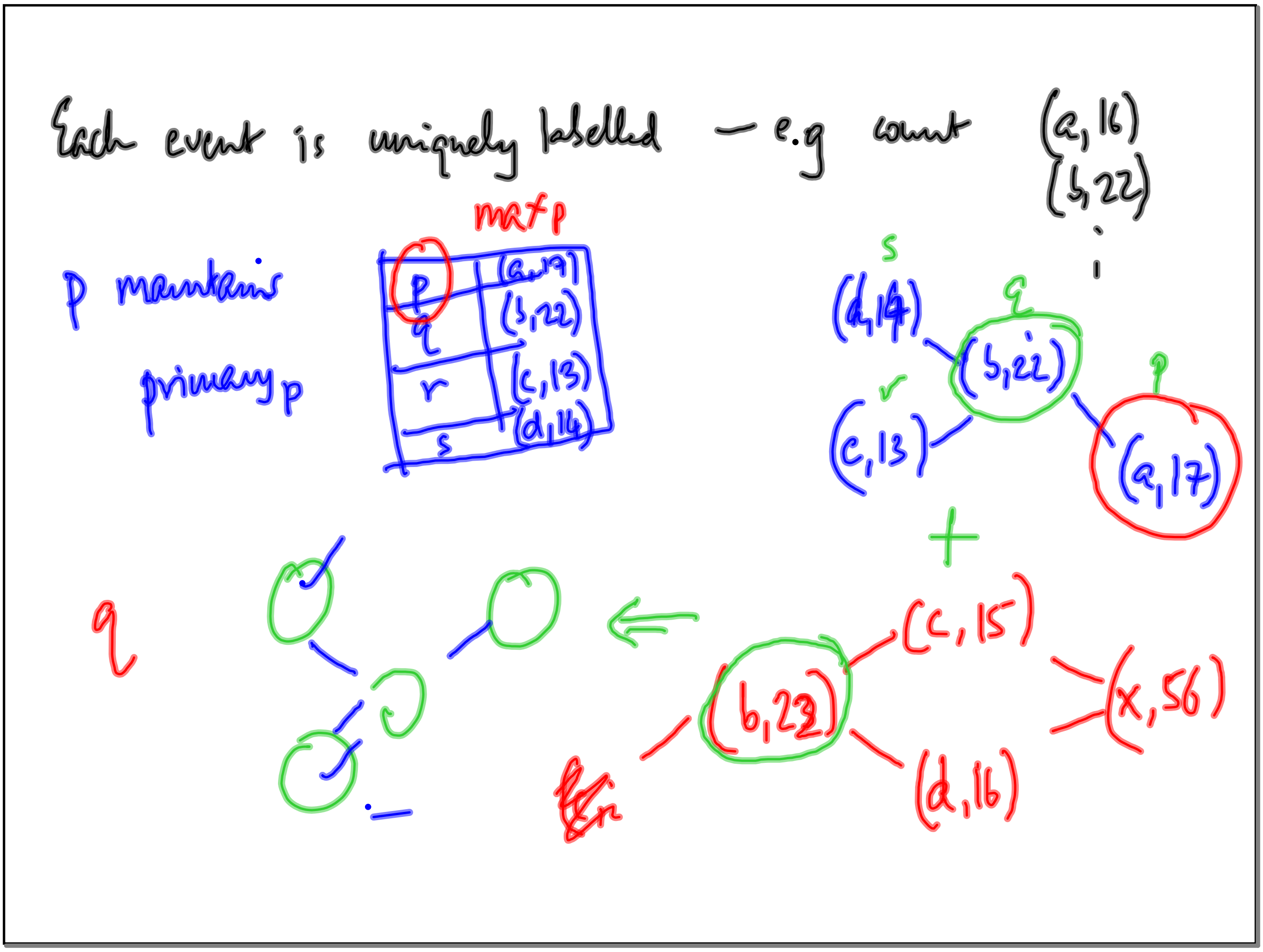
already know if $e < f$

Case 2 Symmetric

Case 3 p contributes e

q contributes f

e, f unordered


Each event is uniquely labelled — e.g count
(a,16)
(b,22)
⋮
mat p
p maintains
primary p
p
q
r
s
(a,17)
(b,22)
(c,13)
(d,14)
s
q
r
p
(d,14)
(b,22)
(c,13)
(a,17)
+
q
(c,15)
(x,56)
(b,22)
(d,16)

Reuse labels

processes participating in e label e

ensure label is not in use

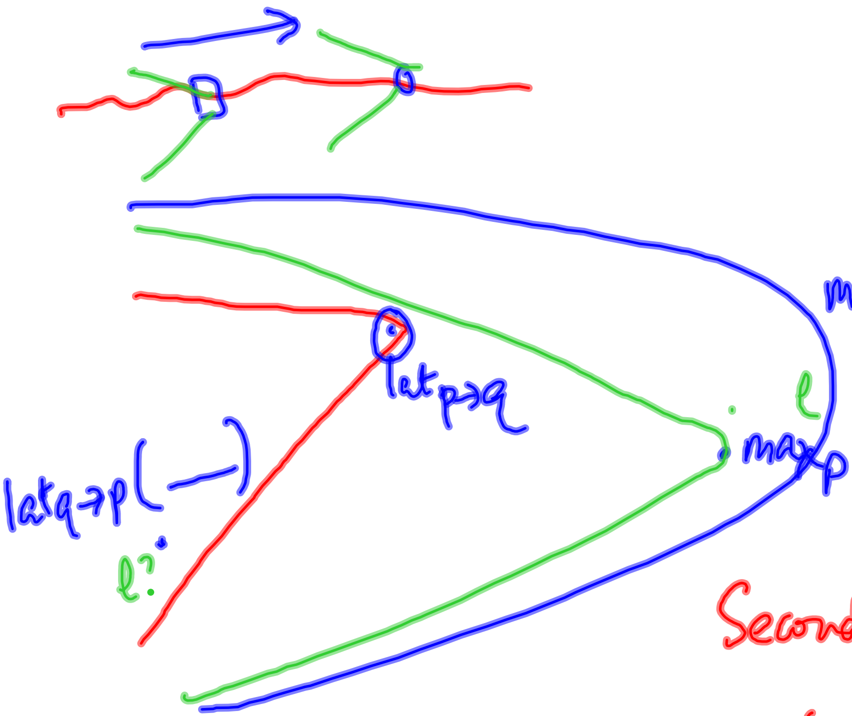
$\exists q \notin loc(e)$ s.t for q this label is

in $primary_q(\underline{I})$ wrt some $p \in loc(e)$

if a label is currently unused, it will

become "active" unless it assigned

to fresh event



$max_p(I)$

$max_p(\downarrow(max_q(\partial_p(I))))$

Secondary $lat_{p \to q \to r}(I)$

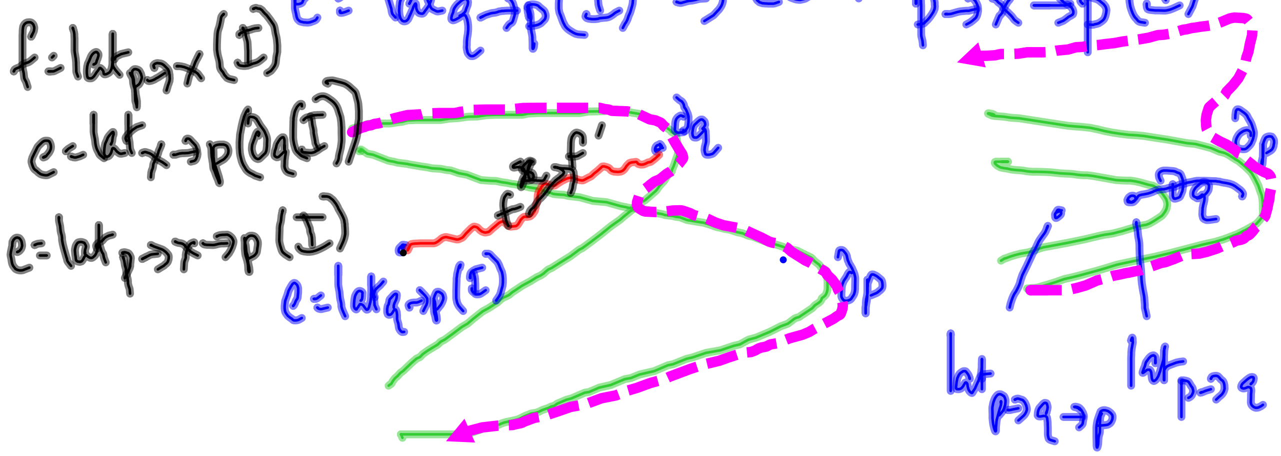
$max_r(\downarrow(max_q(\downarrow max_p(I))))$

Is a label of a p-event part of $\text{primary}_q(I)$ for some q?

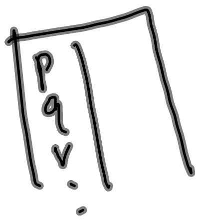
Lemma I ideal, $q \in P$, $e \in \text{primary}_q(I)$.

$e \in \text{secondary}_p(I)$ for every $p \in loc(e)$

$e = lat_{q \to p}(I) \Rightarrow e \in lat_{p \to x \to p}(I)$

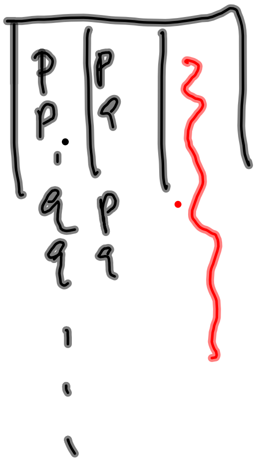


$Primary_p(I)$



N entries

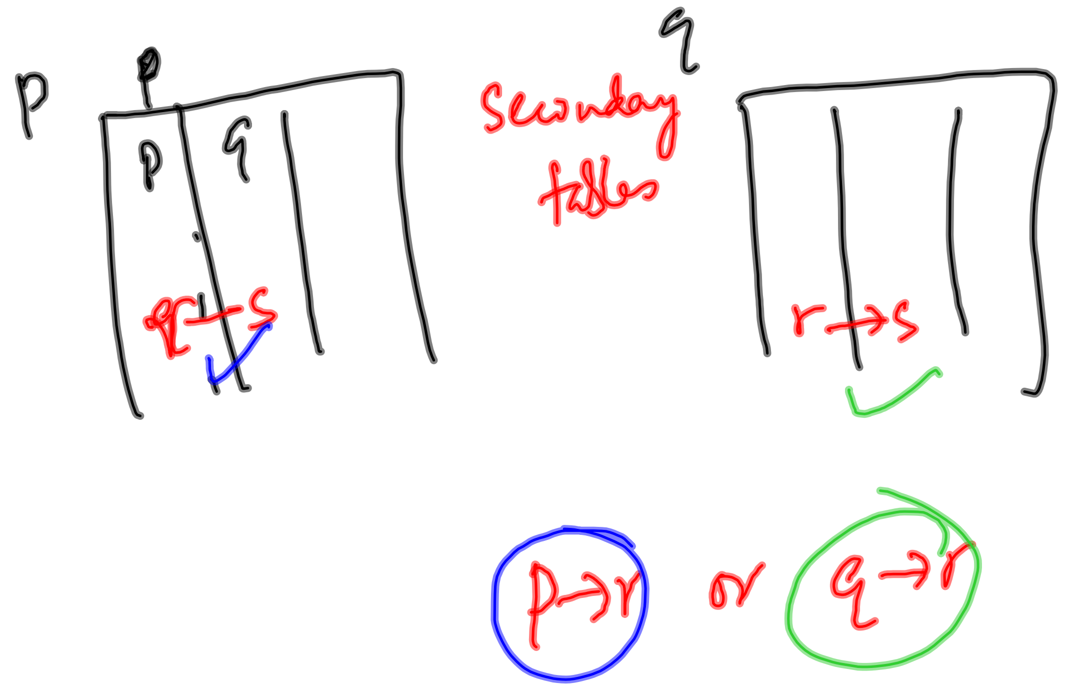
$Secondary_p(I)$



N^2 entries

$e \notin secondary_p(I) \Rightarrow e \notin primary_q(I) \quad \forall q$

does it appear in table?


p
p
p
q
q → s
q
Secondary tables
r → s
p → r
or
q → r

When $X \subseteq P$ synchronize to perform e,
choose a new label outside secondary_X

$$|\text{secondary}_X| \leq |X| \cdot N^2 \leq N^3$$

labelling $\Sigma \times N^3$

State of p: Primary Graph (~~Pr~~ Table + $\leq$)
Secondary Table
N^2

entries are
$\log N + \log \Sigma$ bits

Initially: all entries as $(e_\perp, 0)$

Assuming label set $(0, 1, \ldots, N^3)$

Deterministic transition relation

$2^{N^2 \log N \text{ bits}}$

Update all tables via gossip protocol

Assign new event smallest unused label

After update – all secondary x are <u>same</u>
N^2 labels suffice.

Distributed simulation

Regular trace lang L

Min DFA $\mathcal{A}_L$

has $\diamond$ property

$uabvx \equiv_R ubavx$

if $(a,b) \in I$

$\Rightarrow$ by induction, if $u \sim_I v$

$u \equiv_R v$

Myhill Nerode.

$w \cong_R v \triangleq$

$\forall x \; wx \in L$ iff $vx \in L$

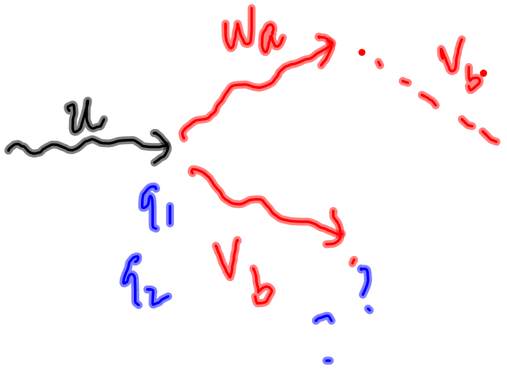
$q_{in} \xrightarrow{w} q_w$

$q_{in} \xrightarrow{v} q_v$

$q_w = q_v$ iff $w \equiv_R v$

Finite simulation of a DFA?

effect of a word as function $Q \to Q$



$u \longrightarrow f_u : Q \to Q$

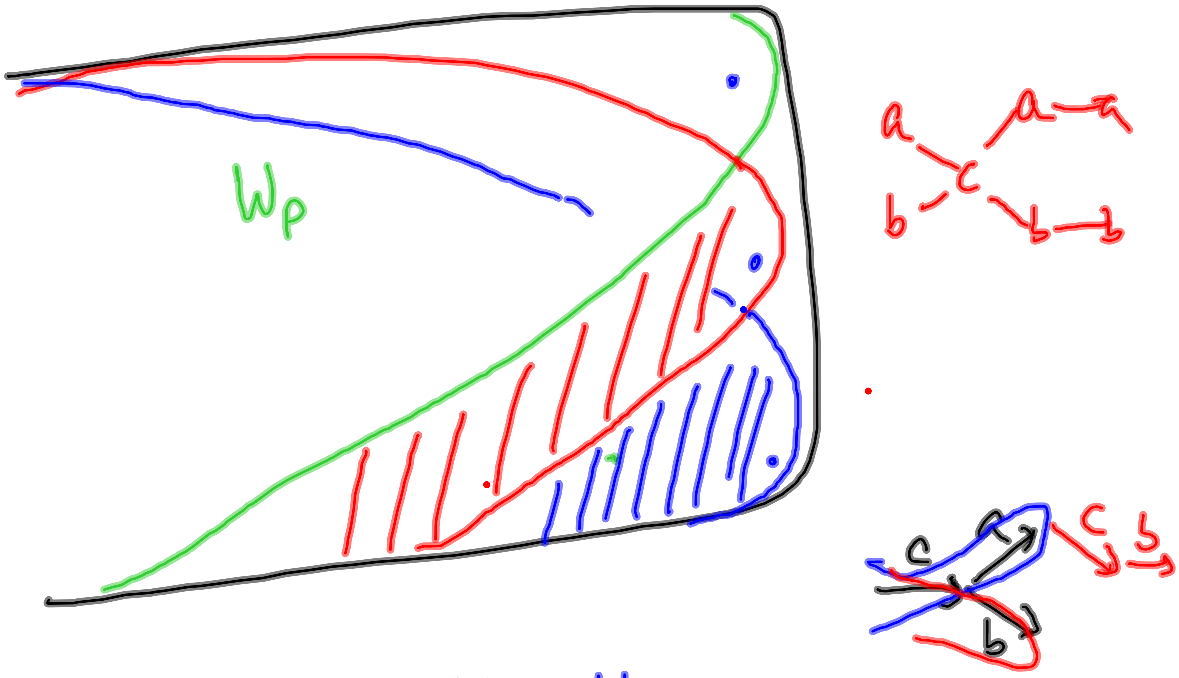
$u \equiv_{syntactic} v$ if $f_u = f_v$

$u \sim_I v \Rightarrow f_u = f_v$

$u \sim v$

$x\,u\,y \sim x\,v\,y$

$\in L \qquad \in L$



$$W = w_p \, w_{q-p} \, w_{r-\{p,q\}}$$

$$f_{w_p} \quad f_{w_{q-p}} \quad f_{w_{r-\{p,q\}}}$$

Each p maintains f_w for various $w \subseteq \partial_p(\mathcal{I})$

$$f_{u \cdot v} = f_v \circ f_u$$