## Database Management Systems, Aug-Dec 2023

Solution sheet, 29 September 2023

Problem 1 Consider the following functional dependencies over the attributes $(A, B, C, D, E)$.

$$
\begin{gathered}
A \rightarrow B C \\
C D \rightarrow E \\
B \rightarrow D \\
E \rightarrow A
\end{gathered}
$$

Compute the attribute closure $X^{+}$for each attribute $X \in\{A, B, C, D, E\}$.

## Solution

- $A+=\{A\}$

Using $A \rightarrow B C$, expand $A^{+}$to $\{A, B, C\}$
Using $B \rightarrow D$, expand $A^{+}$to $\{A, B, C, D\}$
Using $C D \rightarrow E$, expand $A^{+}$to $\{A, B, C, D, E\}$

- $B^{+}=\{B\}$

Using $B \rightarrow D$, expand $B^{+}$to $\{B, D\}$
No further dependencies have LHS in $B^{+}$, so stop.

- $C^{+}=\{C\}$

No dependency has LHS $C$, so stop.

- $D^{+}=\{D\}$

No dependency has LHS $D$, so stop.

- $E^{+}=\{E\}$

Using $E \rightarrow A$, expand $E^{+}$to $\{A, E\}$
(We know $A^{+}=\{A, B, C, D, E\}$ so we can directly conclude $E^{+}=\{A, B, C, D, E\}$.)
Using $A \rightarrow B C$, expand $E^{+}$to $\{A, B, C, E\}$
Using $B \rightarrow D$, expand $E^{+}$to $\{A, B, C, D, E\}$

Problem 2 Consider the following tables for an online book seller.

```
CREATE TABLE Books (
    isbn CHAR(10),
    title CHAR(80),
    author CHAR(80),
    qty_in_stock INTEGER,
    price REAL,
    year_published INTEGER,
)
CREATE TABLE Customers (
    cid INTEGER,
    cname CHAR(80),
    address CHAR(200)
)
```

```
CREATE TABLE Orders (
    ordernum INTEGER,
    isbn CHAR(10),
    cid INTEGER,
    cardnum CHAR(16),
    qty INTEGER,
    order_date DATE,
    ship_date DATE
)
```

We have the following assumptions about these tables.

- isbn is a unique identifier for each book published.
- A book has only one title but may have multiple authors.
- cid is a unique customer id for each customer.
- ordernum is a unique identifier for each order.
- An order is placed by a single customer cid, paid by a single card cardnum on a single order date order_date.
- An order may consist of several books (distinct isbn) each with its own order quantity (qty).
- Each book is shipped (ship_date) as soon as the quantity required is ready.
- Hence each order is split in several rows, one per isbn orderedn.

Questions:

1. Enumerate the functional dependencies that you can infer from this information.
2. For each table, determine if it in BCNF or 3NF. If not, suggest a decomposition and check if the decomposition is dependency preserving.

## Solution

1. Functional dependencies

- isbn $\rightarrow$ title, qty_in_stock, price, year_published
- cid $\rightarrow$ cname, caddress
- ordernum $\rightarrow$ cid, cardnum, order_date
- ordernum, isbn $\rightarrow$ qty, ship_date

2. Normal forms

- isbn is not a key for Books (we can have multiple authors for a book). Since we have the dependency isbn $\rightarrow$ title, qty_in_stock, price, year_published, Books is not in BCNF. Books is also not in 3NF, since the only candidate key is (isbn,author).

Split Books as (isbn, author) and (isbn, title, qty_in_stock, price, year_published') to get a decomposition in BCNF.
The only dependency that applies can be checked locally in the second table, so the decomposition is dependency preserving.

- Customer is in BCNF since cid is a key for the dependency cid $\rightarrow$ cname, caddress
- Orders is neither in BCNF nor in 3NF.

Split as (ordernum,cid,cardnum,order_date) and (ordernum,isbn,qty,ship_date).
In the first relation, ordernum is a key, so it is in BCNF with respect to the dependency ordernum $\rightarrow$ cid, cardnum, order_date.

In the second relation (ordernum,isbn) is a key, so it is in BCNF with respect to the dependency ordernum, isbn $\rightarrow$ qty, ship_date

The two dependencies that apply can be checked locally in the two tables, so the decomposition is dependency preserving.

Problem 3 Suppose we have dependencies $\{A \rightarrow B C, B \rightarrow C A, C \rightarrow A B\}$ on attributes $(A, B, C)$.
Questions:

1. Show that $C$ is extraneous on the right hand side of the first dependency.
2. If we replace the first dependency by $A \rightarrow C$, show that both $A$ and $B$ are (separately) extraneous in $C \rightarrow A B$.
3. If we replace $C \rightarrow A B$ by $C \rightarrow A$, so that the dependencies are $\{A \rightarrow B, B \rightarrow A C, C \rightarrow A\}$, show that $A$ is extraneous in $B \rightarrow A C$.
4. If we replace $C \rightarrow A B$ by $C \rightarrow B$, so that the dependencies are $\{A \rightarrow B, B \rightarrow A C, C \rightarrow B\}$, show that neither $A$ nor $C$ are extraneous in $B \rightarrow A C$.

## Solution

1. To show that $C$ is extraneous in $A \rightarrow B C$, we need to check if $A \rightarrow B C$ can be derived from $\{A \rightarrow B, B \rightarrow C A, C \rightarrow A B\}$. Compute $A^{+}$with respect to the new set of dependencies.
$A^{+}=\{A\}$
Using $A \rightarrow B$, expand $A^{+}$to $\{A, B\}$
Using $B \rightarrow C A$, expand $A^{+}$to $\{A, B, C\}$
From this, we deduce the dependency $A \rightarrow B C$.
2. Consider the dependencies $\{A \rightarrow B, B \rightarrow C A, C \rightarrow A B\}$.

- To show that $A$ is extraneous in $C \rightarrow A B$, we need to check that we can derive $C \rightarrow A B$ from $\{A \rightarrow B, B \rightarrow C A, C \rightarrow B\}$. Compute $C^{+}$with respect to the new sett of dependencies.
$C^{+}=\{C\}$
Using $C \rightarrow B$, expand $C^{+}$to $\{B, C\}$
Using $B \rightarrow C A$, expand $C^{+}$to $\{A, B, C\}$
From this, we deduce the dependency $C \rightarrow A B$.
- To show that $B$ is extraneous in $C \rightarrow A B$, we need to check that we can derive $C \rightarrow A B$ from $\{A \rightarrow B, B \rightarrow C A, C \rightarrow A\}$. Compute $C^{+}$with respect to the new sett of dependencies.
$C^{+}=\{C\}$
Using $C \rightarrow A$, expand $C^{+}$to $\{A, C\}$
Using $A \rightarrow B$, expand $C^{+}$to $\{A, B, C\}$
From this, we deduce the dependency $C \rightarrow A B$.

3. Consider the dependencies $\{A \rightarrow B, B \rightarrow A C, C \rightarrow A\}$. We wish to show that $A$ is extraneous in $B \rightarrow A C$.

Check that we can derive $B \rightarrow A C$ from $\{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$. Compute $B^{+}$with respect to the new sett of dependencies.
$B^{+}=\{B\}$
Using $B \rightarrow C$, expand $B^{+}$to $\{B, C\}$
Using $C \rightarrow A$, expand $B^{+}$to $\{A, B, C\}$
From this, we deduce the dependency $B \rightarrow A C$..
4. Consider the dependencies $\{A \rightarrow B, B \rightarrow A C, C \rightarrow B\}$. We wish to show that neither $A$ not $C$ is extraneous in $B \rightarrow A C$.

- To show that $A$ is not extraneous, check if we can derive $B \rightarrow A C$ from $\{A \rightarrow B, B \rightarrow C, C \rightarrow$ $B\}$. Compute $B^{+}$with respect to the new sett of dependencies.
$B^{+}=\{B\}$
Using $B \rightarrow C$, expand $B^{+}$to $\{B, C\}$
The only other dependency whose LHS is in $B^{+}$is $B \rightarrow C$ which adds nothing to $B^{+}$, so we stop with $B^{+}=\{B, C\}$

Thus we do not have $B \rightarrow A$, and hence $B \rightarrow A C$ is not derivable.

- To show that $C$ is not extraneous, check if we can derive $B \rightarrow A C$ from $\{A \rightarrow B, B \rightarrow A, C \rightarrow$ $B\}$. Compute $B^{+}$with respect to the new sett of dependencies.
$B^{+}=\{B\}$
Using $B \rightarrow A$, expand $B^{+}$to $\{A, B\}$
The only other dependency whose LHS is in $B^{+}$is $A \rightarrow B$ which adds nothing to $B^{+}$, so we stop with $B^{+}=\{A, B\}$

Thus we do not have $B \rightarrow C$, and hence $B \rightarrow A C$ is not derivable.

