How to get decidability of distributed synthesis for asynchronous systems

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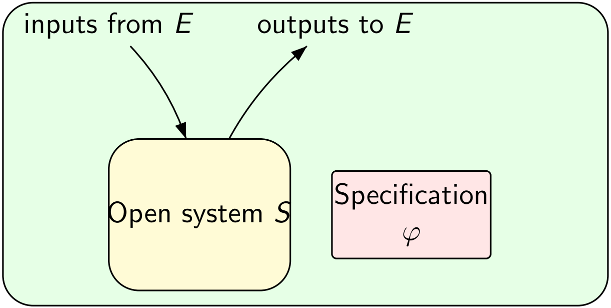
Joint work with Thomas Chatain and Nathalie Sznajder

January 29-31, 2009
Workshop ACTS

Outline

Synthesis of a reactive system



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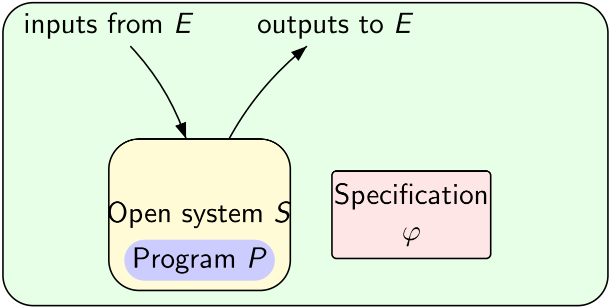
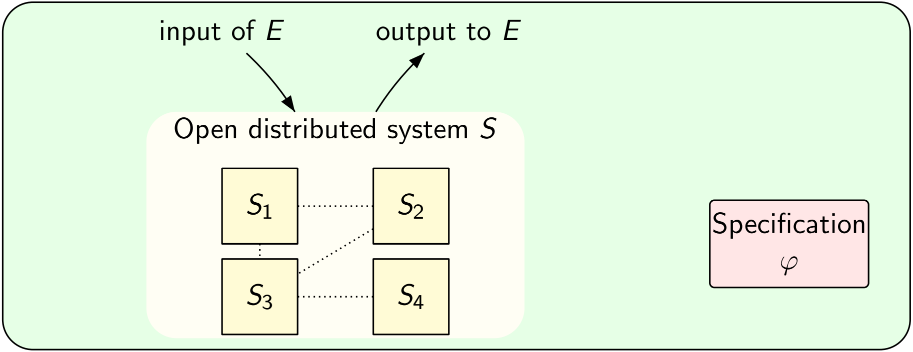


Two problems

- Decide whether there exists a program st. $P || E \models \varphi, \quad \forall E$.
- Synthesis: If so, compute such a program.

For reasonable systems and specifications, the problems are decidable.

Distributed synthesis



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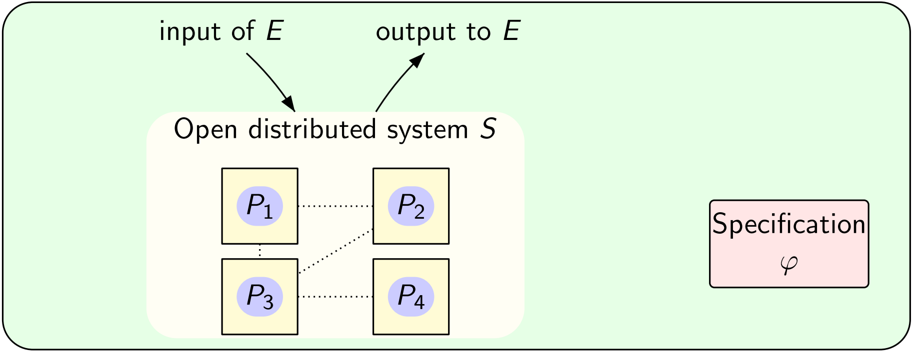

Two problems

- Decide the existence of a distributed program such that their joint behavior $P_1||P_2||P_3||P_4||E$ satisfies φ, for all E.
- Synthesis : If it exists, compute such a distributed program.

Distributed synthesis
Synchronous or asynchronous semantics?

Synchronous semantics

- At each tick of a global clock, all processes and the environment output their new value
- Introduced in [PnueliRosner90].
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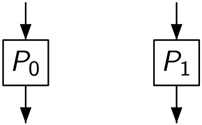

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P.G., Benjamin Lerman, Marc Zeitoun

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- Specification : regular over Mazurkiewicz traces

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Theorem

Synthesis problem is decidable for co-graph dependence alphabets, i.e., for series-parallel systems.

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Our model

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- Specifications :
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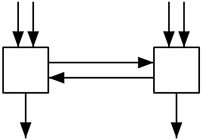
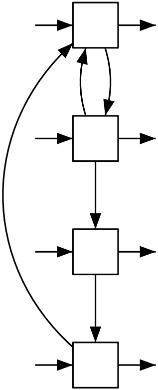
  - will not restrain communication abilities

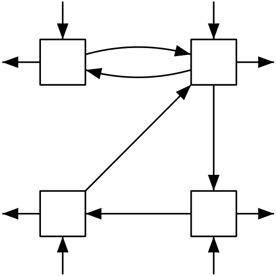
Decidability Results

Theorem

Synthesis problem is decidable for strongly-connected architectures







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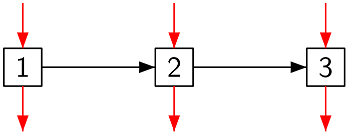

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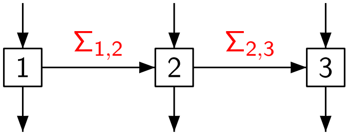

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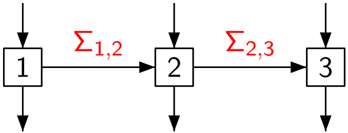

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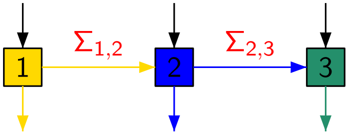
- $\Sigma = \Gamma \cup \bigcup_{(i,j) \in E} \Sigma_{i,j}$
- For each process i, Σ_i is the set of signals it can send or receive, and $\Sigma_i^c = \mathrm{Out}_i \cup \bigcup_{j,(i,j) \in E} \Sigma_{i,j}$



The model: runs

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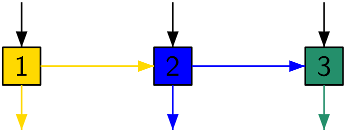
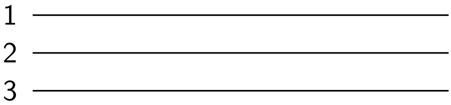
A run is a Mazurkiewicz trace $t = (V, \lambda, \leq)$ over (Σ, D)
where $a\ D\ b$ if there is a process that takes part both in a and b





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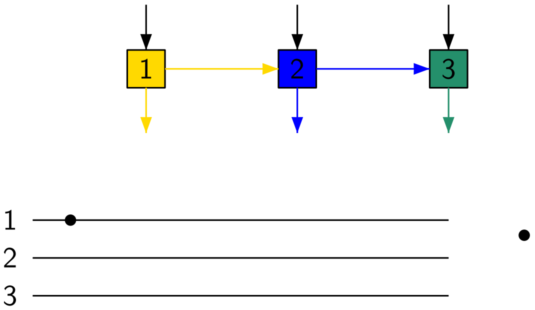

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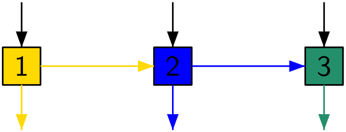

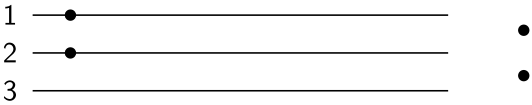

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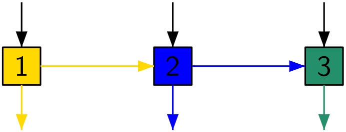

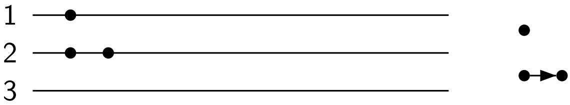

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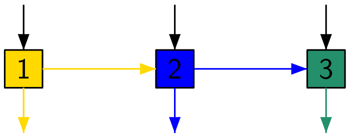

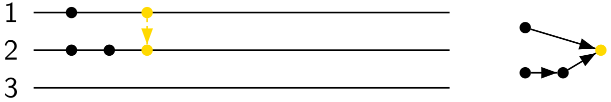

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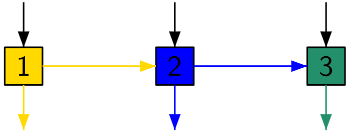

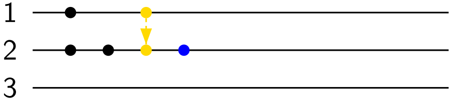

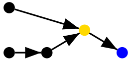

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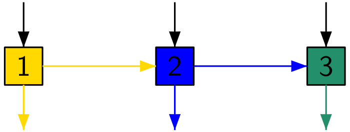

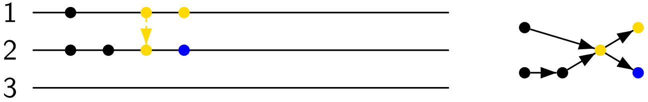

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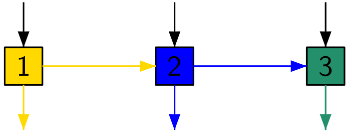

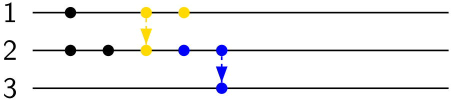

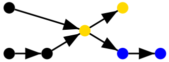

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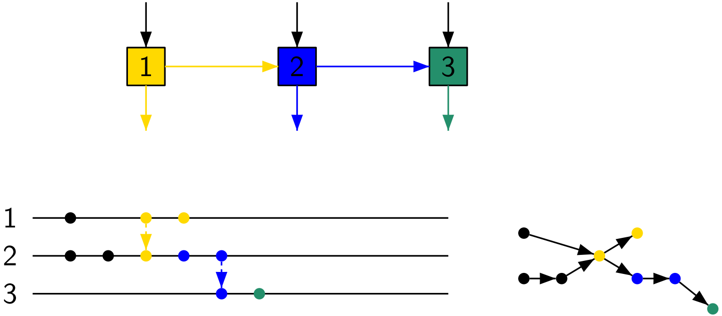

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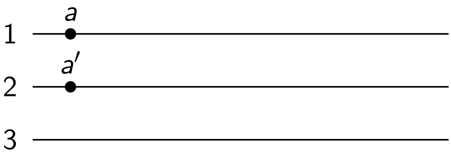

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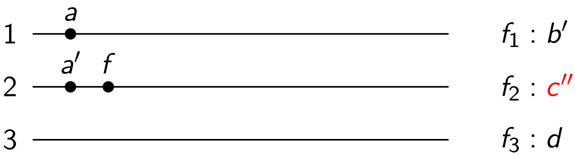

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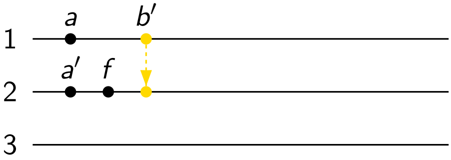

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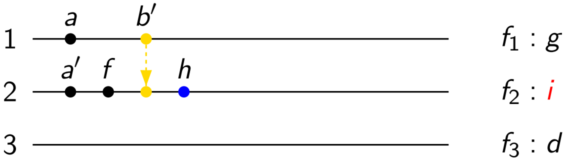

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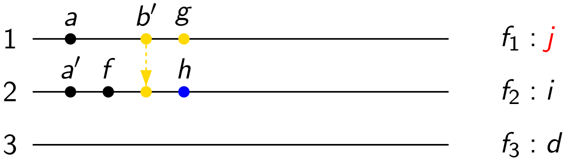

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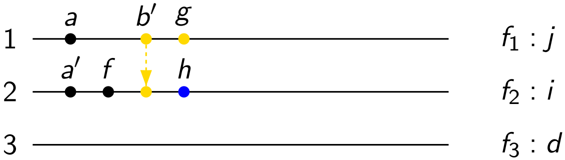

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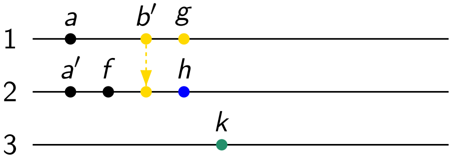

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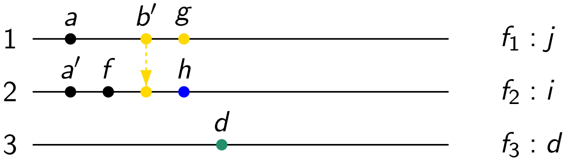

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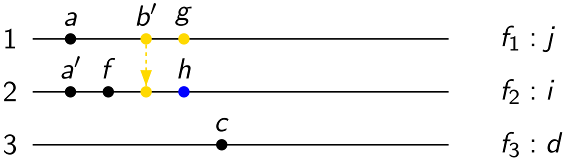

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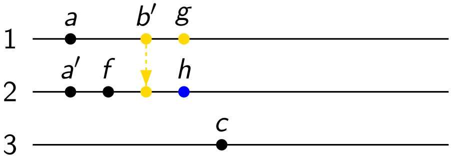
- A run $t = (V, \lambda, \leq)$ is f-maximal if for each process i either $V_i = \lambda^{-1}(\Sigma_i)$ is infinite, or f_i is undefined on the maximal event of V_i.



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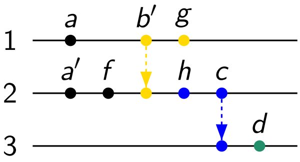

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If so, compute them

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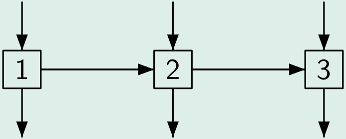
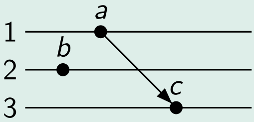
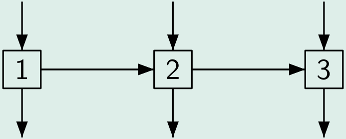
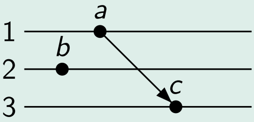


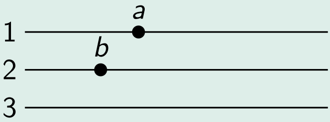


Specifications

Communication induces order relation







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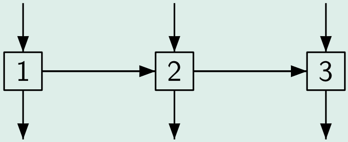

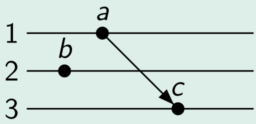

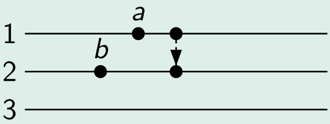

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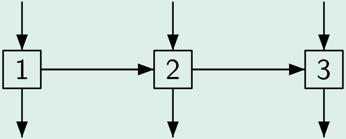

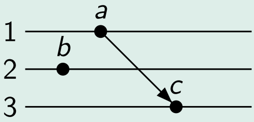

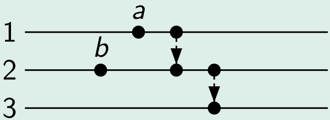

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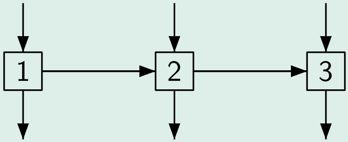

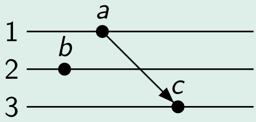

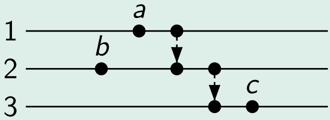

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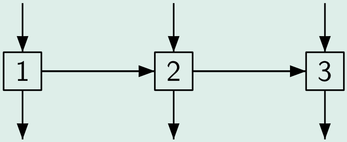

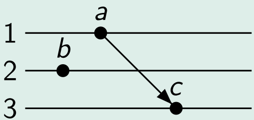

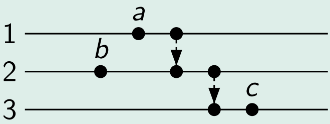

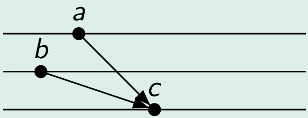

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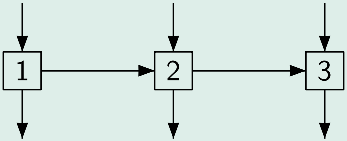

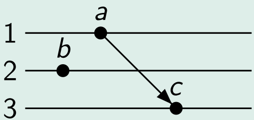

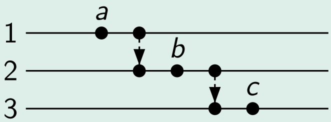

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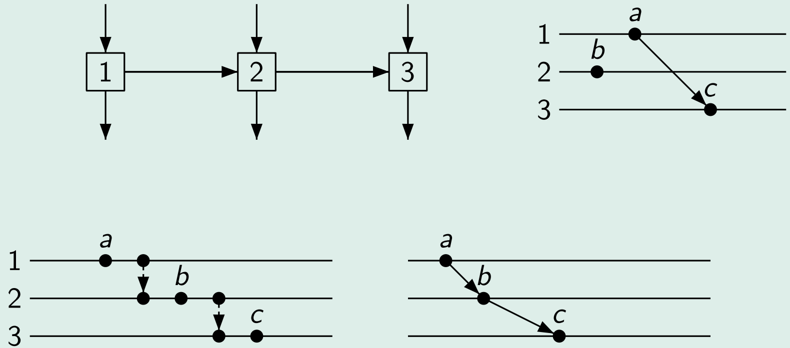

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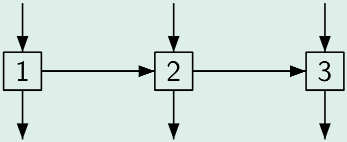

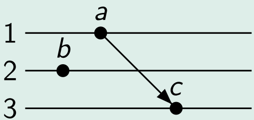

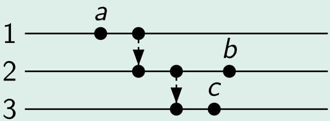

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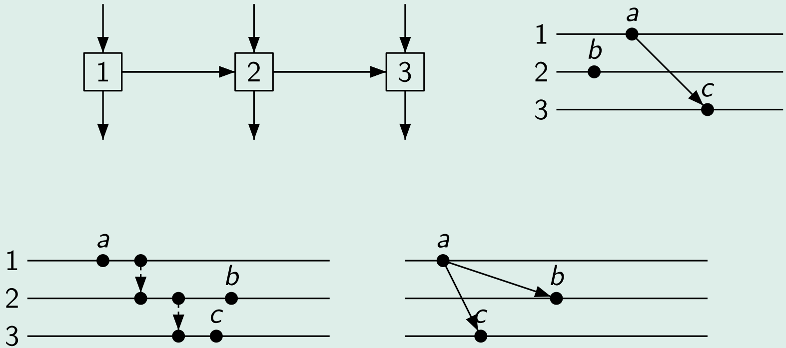

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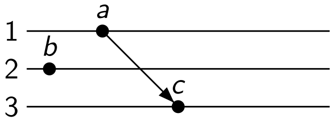

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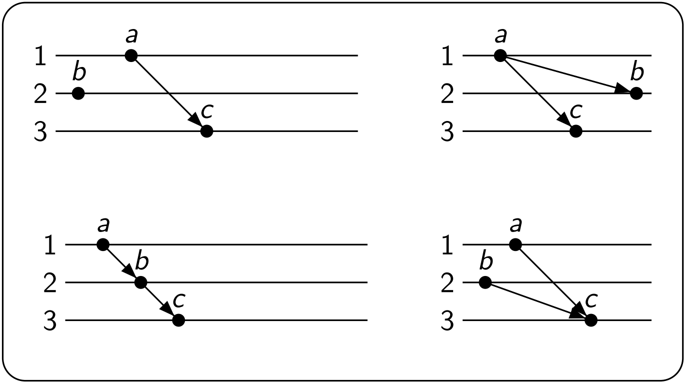

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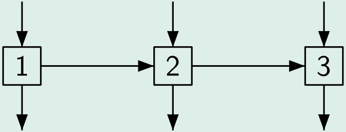

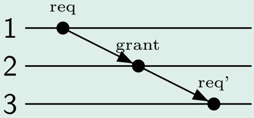

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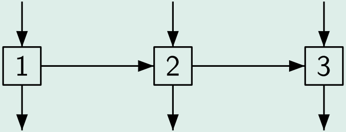

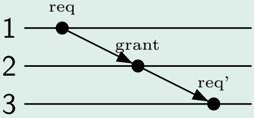

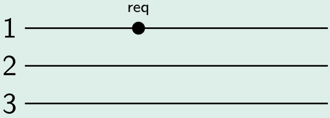

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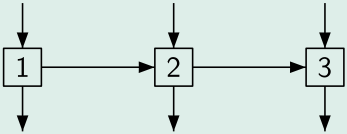

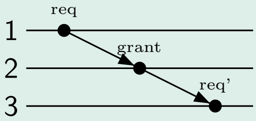

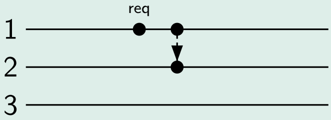

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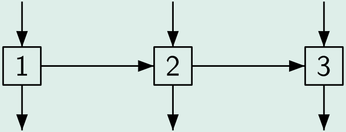

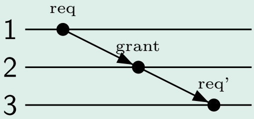

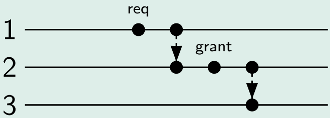

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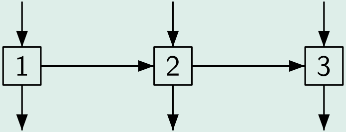

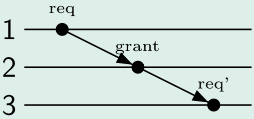

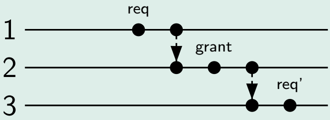

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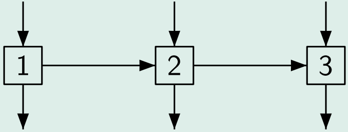

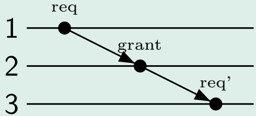

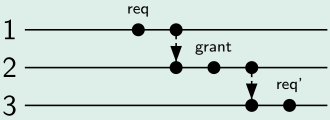

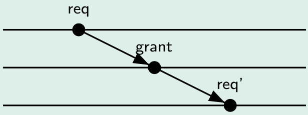

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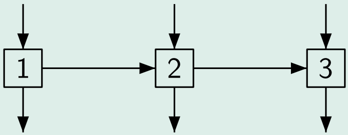

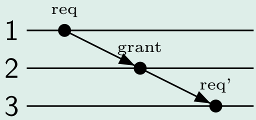

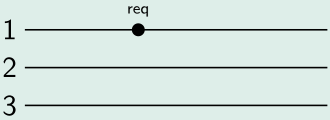

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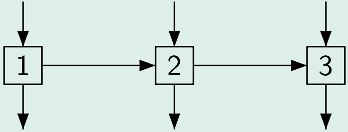

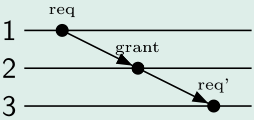

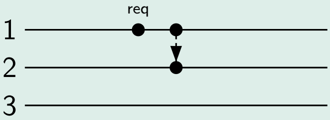

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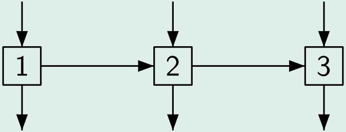

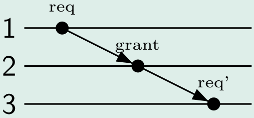

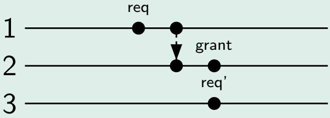

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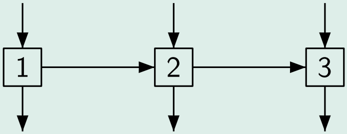

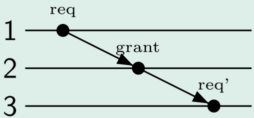

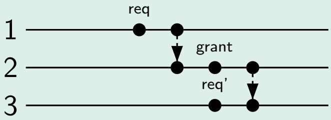

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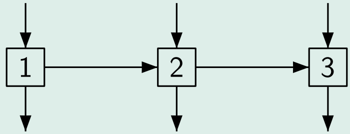

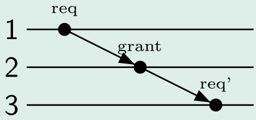

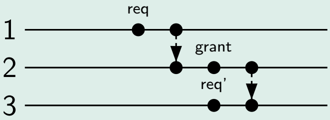

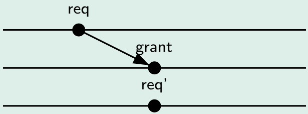

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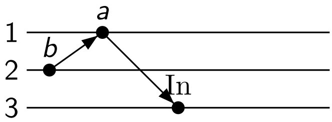

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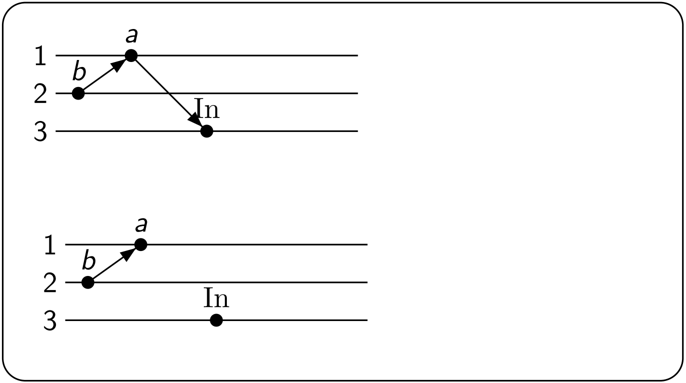

Example of a logic closed by extension and weakening

AlocTL

$$\begin{aligned} \varphi ::= {} & a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \\ & \mid \mathsf{X}_i\, \varphi \mid \varphi\, \mathsf{U}_i\, \varphi \mid \neg\, \mathsf{X}_i\, \top \mid \varphi\, \widetilde{\mathsf{U}}_i\, \varphi \\ & \mid \mathsf{Y}_i\, \varphi \mid \varphi\, \mathsf{S}_i\, \varphi \mid \neg\, \mathsf{Y}_i\, \top \mid \varphi\, \widetilde{\mathsf{S}}_i\, \varphi \\ & \mid \mathsf{F}_{i,j}(\mathrm{Out} \wedge \varphi) \mid \mathrm{Out} \wedge \mathsf{H}_{i,j}\, \varphi \end{aligned}$$

with $a \in \Gamma$ and $i, j \in \mathrm{Proc}$

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$$\varphi ::= a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi$$
$$\mid \mathsf{X}_i\, \varphi \mid \varphi\, \mathsf{U}_i\, \varphi \mid \neg\, \mathsf{X}_i \top \mid \varphi\, \widetilde{\mathsf{U}}_i\, \varphi$$
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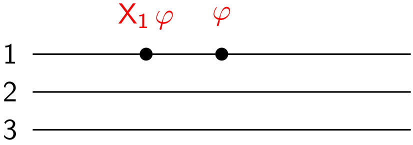

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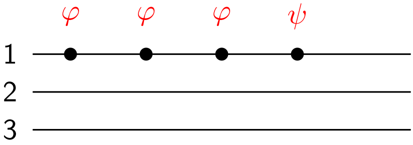

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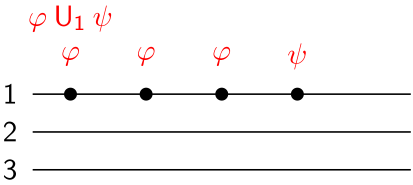

Example of a logic closed by extension and weakening

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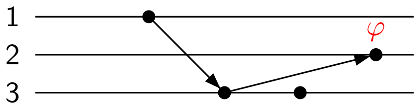
$$\begin{aligned}\varphi ::= {} & a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \\ & \mid \mathsf{X}_i\, \varphi \mid \varphi\, \mathsf{U}_i\, \varphi \mid \neg\, \mathsf{X}_i\, \top \mid \varphi\, \widetilde{\mathsf{U}}_i\, \varphi \\ & \mid \mathsf{Y}_i\, \varphi \mid \varphi\, \mathsf{S}_i\, \varphi \mid \neg\, \mathsf{Y}_i\, \top \mid \varphi\, \widetilde{\mathsf{S}}_i\, \varphi \\ & \mid \mathsf{F}_{i,j}(\mathrm{Out} \wedge \varphi) \mid \mathrm{Out} \wedge \mathsf{H}_{i,j}\, \varphi\end{aligned}$$

with $a \in \Gamma$ and $i, j \in \mathrm{Proc}$



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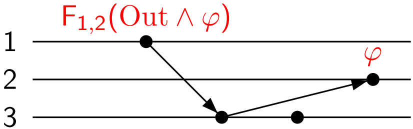

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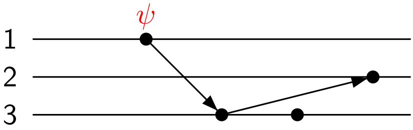

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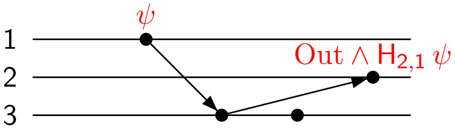

Example of a logic closed by extension and weakening

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$$\varphi ::= a \mid \neg a \mid \varphi \vee \varphi \mid \varphi \wedge \varphi$$
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Formulae

- $\mathsf{G}_1(\texttt{request} \longrightarrow \mathsf{F}_{1,2}(\mathrm{Out} \wedge \texttt{grant}))$
- $\mathsf{G}_2(\texttt{grant} \longrightarrow (\mathrm{Out} \wedge \mathsf{H}_{2,1}\, \texttt{request}))$

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Theorem

AlocTL is closed under extension and weakening

Closure by extension

- $\neg \mathsf{F}_{i,j}\, \varphi$ forbidden!

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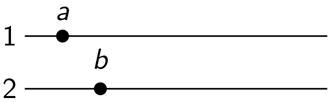

$a \wedge \neg \mathsf{F}_{1,2}\, b$

OK

Closure by extension

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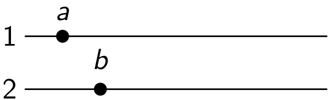
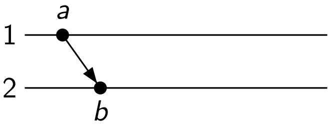


OK



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- $\neg \mathsf{F}_{i,j}\,\varphi$ forbidden!
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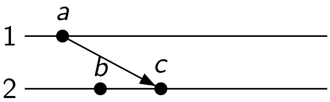

$a \wedge \mathsf{X}_{1,2}\, c$

OK

Closure by extension

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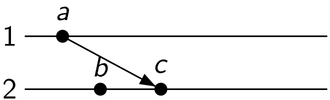

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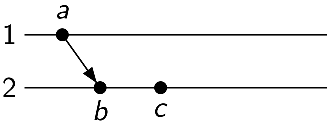

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Closure by extension

- $\neg\, \mathsf{F}_{i,j}\, \varphi$ forbidden!
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Specification is not allowed to require concurrency

Closure by extension

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Specification is not allowed to require concurrency

Closure by weakening

Ensured by $\mathsf{F}_{i,j} \wedge \mathrm{Out}$ and $\mathrm{Out} \wedge \mathsf{H}_{i,j}\, \varphi$.

Outline

Decidability Results

Theorem

The synthesis problem over singleton architectures is decidable for regular specifications.

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Theorem

The distributed synthesis problem over strongly connected architectures is decidable for AlocTL specifications.

Decidability Results

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The synthesis problem over singleton architectures is decidable for regular specifications.

Theorem

The distributed synthesis problem over strongly connected architectures is decidable for AlocTL specifications.

Proof

By reduction to the singleton case.

Strongly connected architectures (2)

Proposition

If there are communication sets $\Sigma_{i,j}$ for $(i,j) \in E$ and a winning distributed strategy on the strongly connected architecture, then there is a winning strategy on the singleton.

Proof

Easy.

Strongly connected architectures

Proposition

If there is a winning strategy f over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected architecture.

Strongly connected architectures

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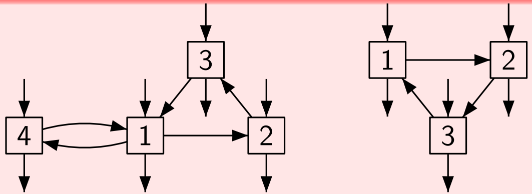

- We select a master process and a cycle.

Strongly connected architectures

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If there is a winning strategy f over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected architecture.

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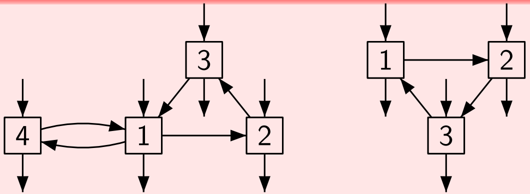

- We select a master process and a cycle.
- The master process will centralize information in order to simulate f and tell other processes which value to output

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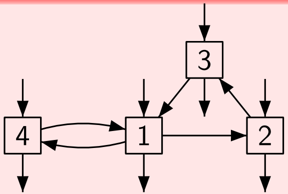
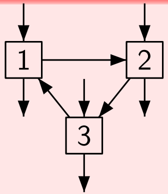




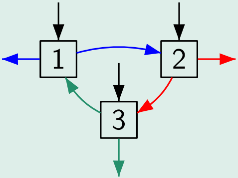
- We select a master process and a cycle.
- The master process will centralize information in order to simulate f and tell other processes which value to output
- Aim: create a run that will be a weakening of some f-run over the singleton

Centralize information

Example

Specification: $\mathrm{req}_3 \rightarrow \mathsf{F}_{32}(\neg\,\mathsf{Y}_2\,\mathrm{alert} \leftrightarrow \mathrm{grant})$

Strategy for the singleton: $f(\sigma) = \mathrm{grant}$ iff σ contains req_3 but no alert



Master collect information by sending a signal Msg through the cycle

Centralize information

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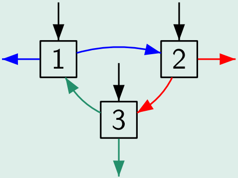

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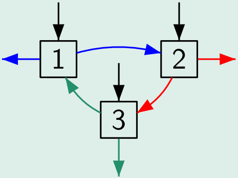

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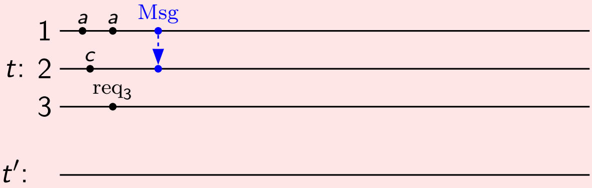

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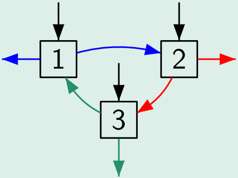

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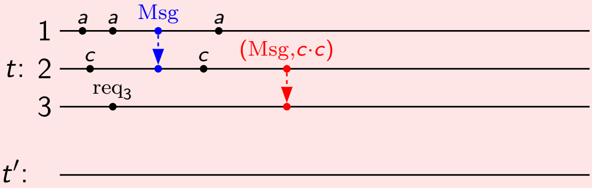

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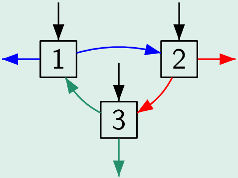

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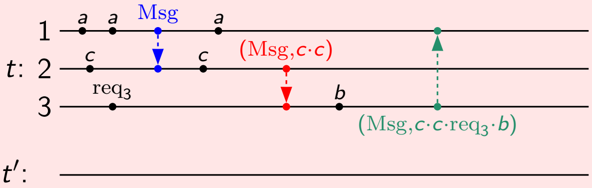

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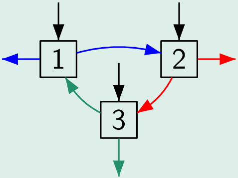

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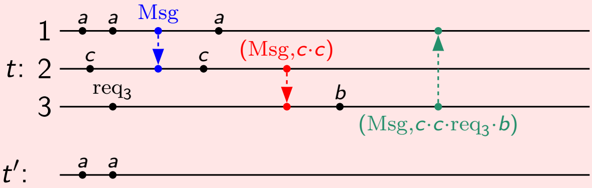

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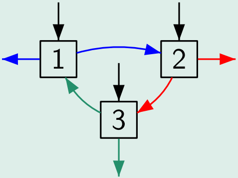

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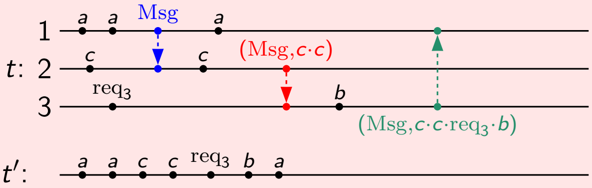

Tell processes what to output

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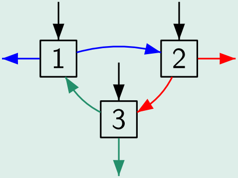
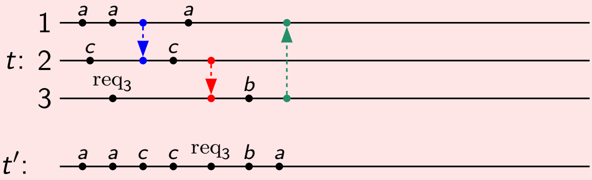


Master sends orders to other processes to simulate the strategy f



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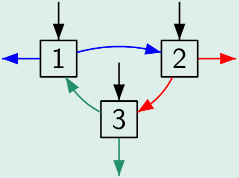

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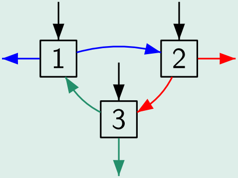

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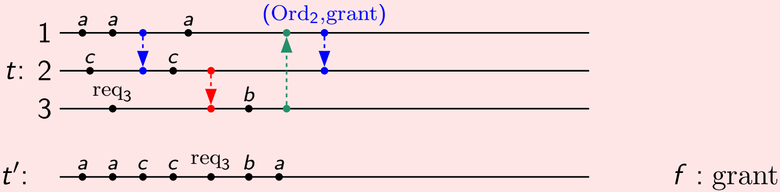

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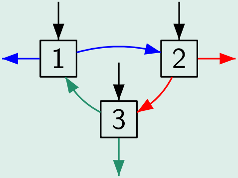

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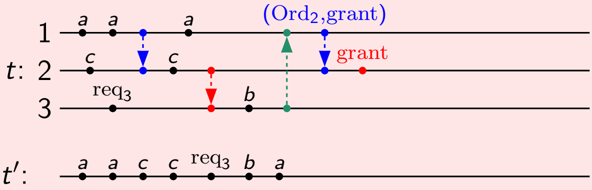

f : grant

Tell processes what to output

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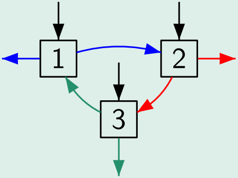

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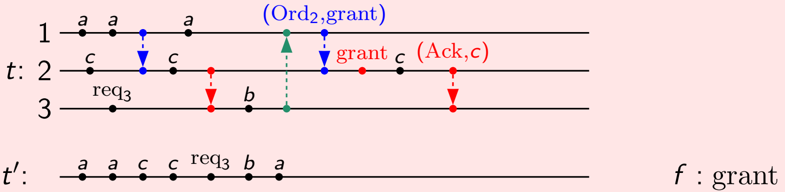

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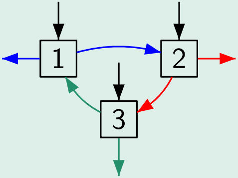

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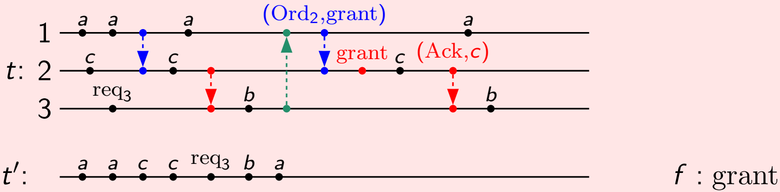

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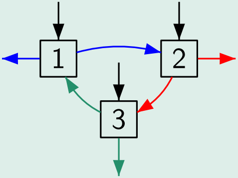

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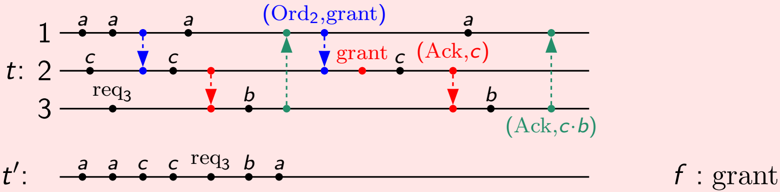

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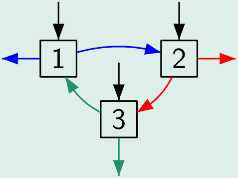

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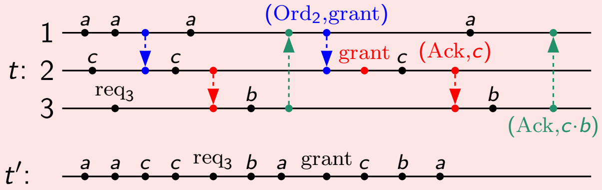

f : grant

Tell processes what to ouptut (2)

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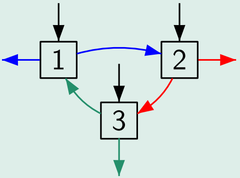

Master sends orders to other processes to simulate the strategy f

Tell processes what to ouptut (2)

Example

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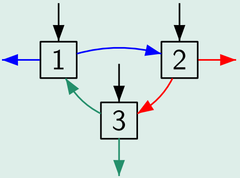

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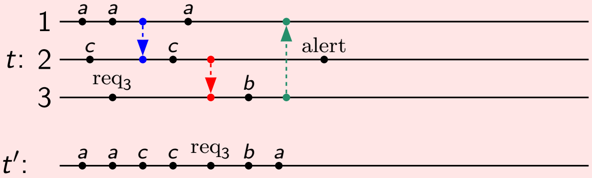

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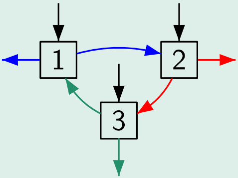

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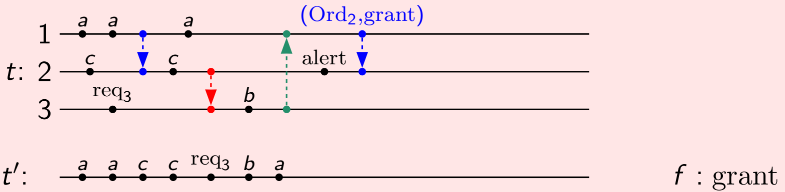

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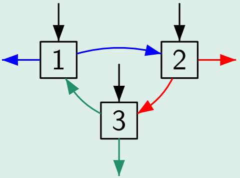

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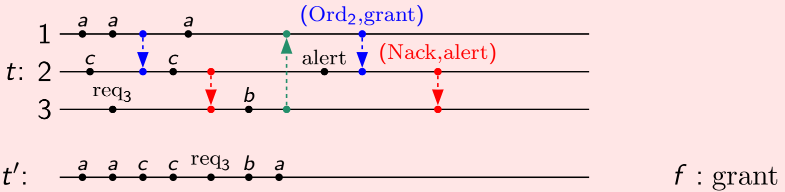

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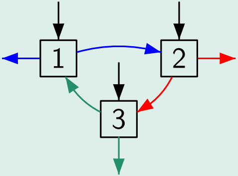

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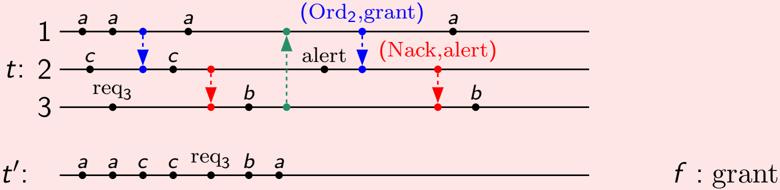

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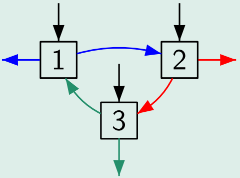

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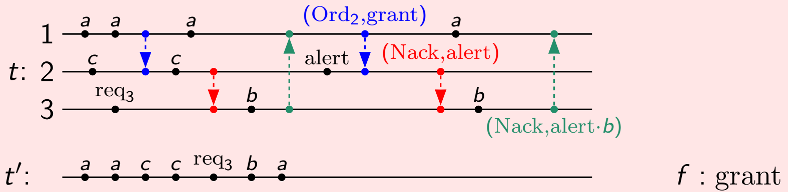

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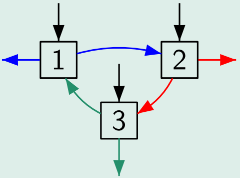

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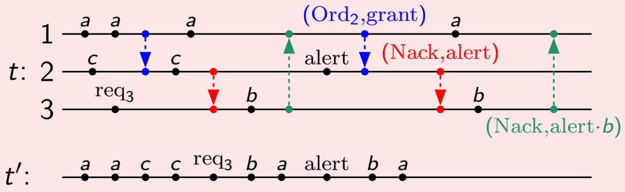

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If $x <' y$ in t' and $x \parallel y$ in $\pi_\Gamma(t)$ then $\lambda(y) \in \mathrm{In}$.

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Corollary

$\pi_\Gamma(t)$ is a weakening of t'.

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Corollary

$\pi_\Gamma(t)$ is a weakening of t'.

Conclusion

Then $t' \models \varphi$ and, by closure property $\pi_\Gamma(t) \models \varphi$.

Conclusion

- Asynchrony removes undecidability causes
- We have defined a new model of communication
- We have identified a class of decidable architectures
- Hopefully, many more to come!