The Theory of Message Sequence Charts – II

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- MSGs, a visual formalism to describe languages of MSCs.
- MSGs : regularity is not decidable, but boundedness is.
- MPAs : An operational model, distributed, ...
- Verifying implementability for MSGs is undecidable.

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- These results can be generalized further ...

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  A regular language R such that the set of MSCs generated by the words in R is L.

- Given B, we can effectively construct $Lin^B(S)$ consisiting of all the B bounded linearizations of MSCs in S.

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Corollary: Systems given as MSGs can be model-checked w.r.t. specifications presented as GC-MSGs.

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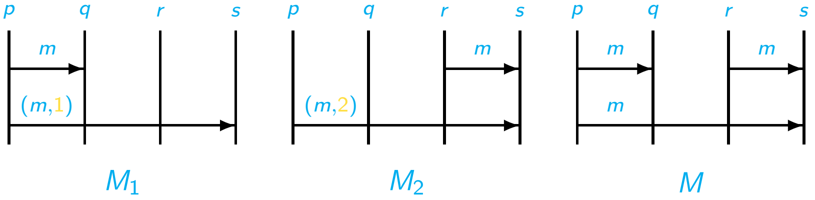

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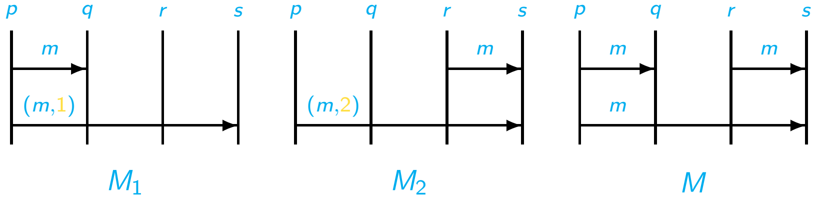

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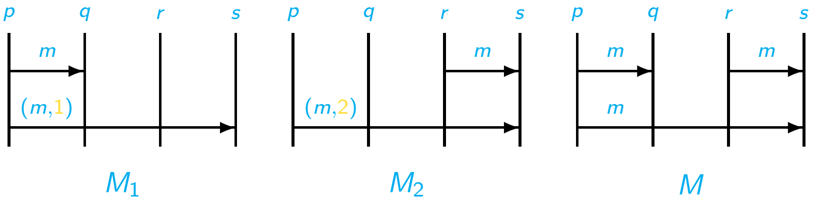

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- Explicitely construct a deterministic MPA from a FA accepting the linearizations of L.

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- A sophisticated local timestamping algorithm is needed to make all this work.

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- $<_m$ denotes the message ordering and cannot be defined using $\leq$

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Theorem: An MSC language L is regular if and only if there is a formula φ in MSO and a constant B such that

$$\mathbf{L} = \mathbf{L}(\varphi) \cap \{\mathbf{M} \mid \mathbf{M} \textbf{ is universally B-bounded}\}$$

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Theorem: Model-checking MSGs w.r.t. MSO is decidable.

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Further, MSO is strictly more expressive than MPAS w.r.t. general MSCs.

Theorem: The quantifier alternation hierarchy for MSO over MSCs is strict. In particular EMSO is strictly weaker than MSO.

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3. L is the MSO definable.

However, deterministic MPAs do not suffice.

Adding time to MSCs

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- Time constrained MSCs
  - MSCs with timing constraints between events

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- Time constrained MSCs
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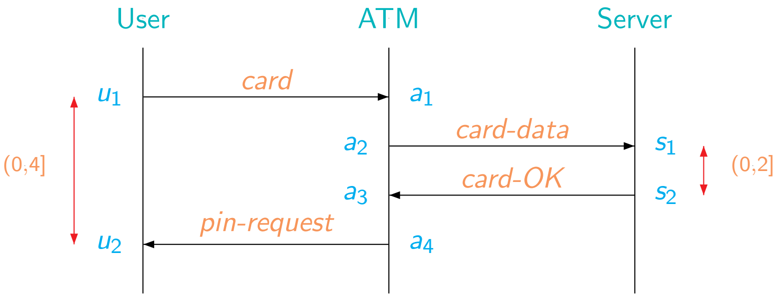
- Time constrained Message Sequence Graphs
  - Generate infinite families of time constrained MSCs

MSCs with time constraints



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  - ... or across a single message

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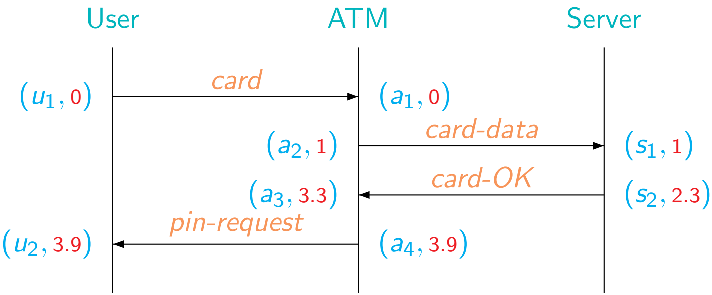
- If $(e, e') \mapsto [l, u]$, then the time between occurrence of e and e' must be between l and u
- Intervals may be open, closed, half-open
- Simplifying assumptions
  - Interval constraints are local to a process ...
    - Both e and e' lie on same process line
  - ... or across a single message
    - e is $p!q(m)$ and e' is corresponding receive $q?p(m)$

A timed behaviour



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- TC-MSC $T \Rightarrow L(T)$, set of timed MSCs that cover T

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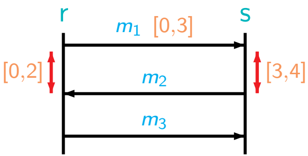
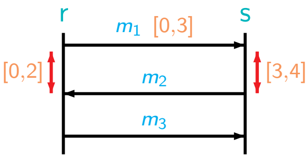


TC-MSCs and Timed MSCs

- The set of timed MSCs covering a TC-MSC may be empty.
- A TC-MSC is said to be realizable if it is covered by atleast one timed MSC.



Time Constrained Message Sequence Graphs

- States labelled by time constrained MSCs
- Local constraints for each process along edges
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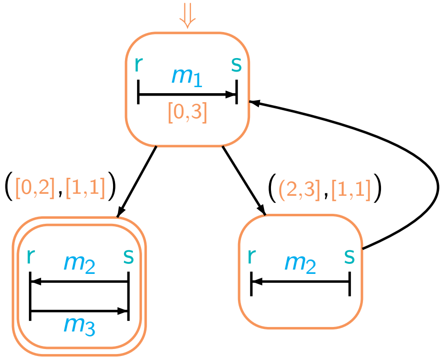

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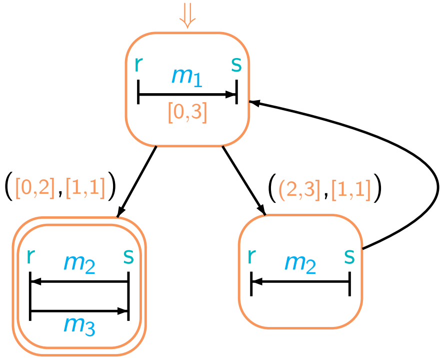

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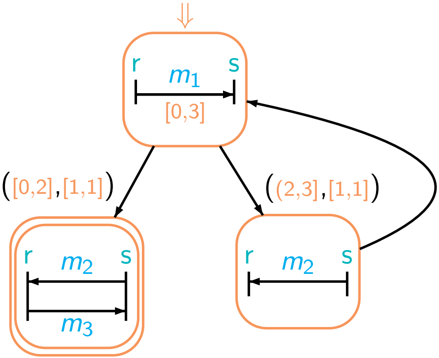

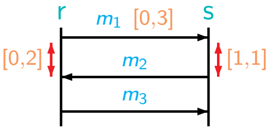

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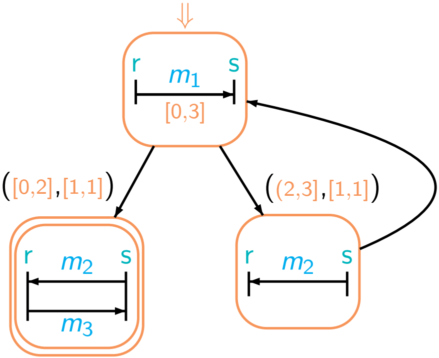

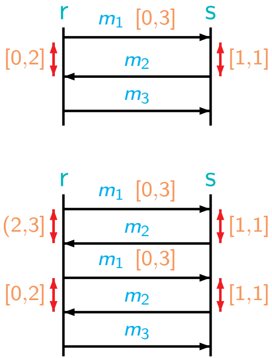

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This problem is trivial for ordinary MSGs.

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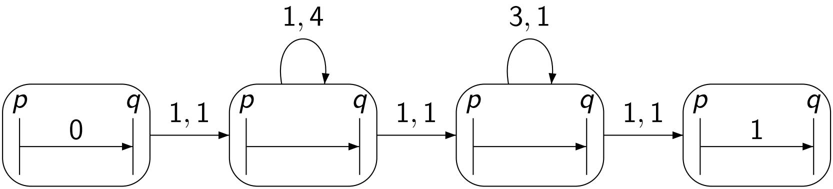

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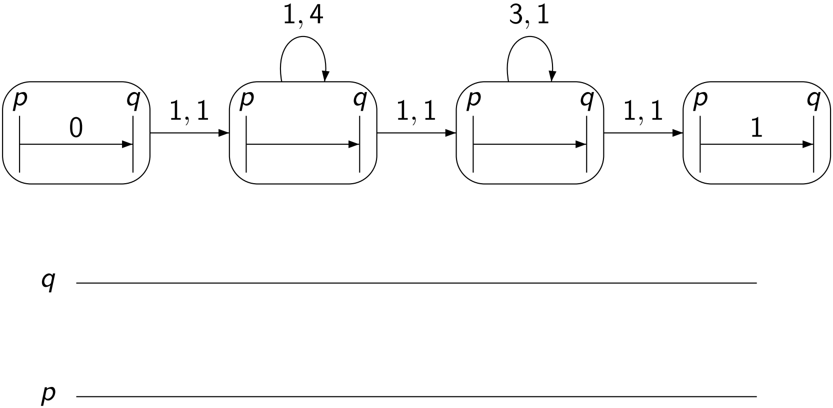

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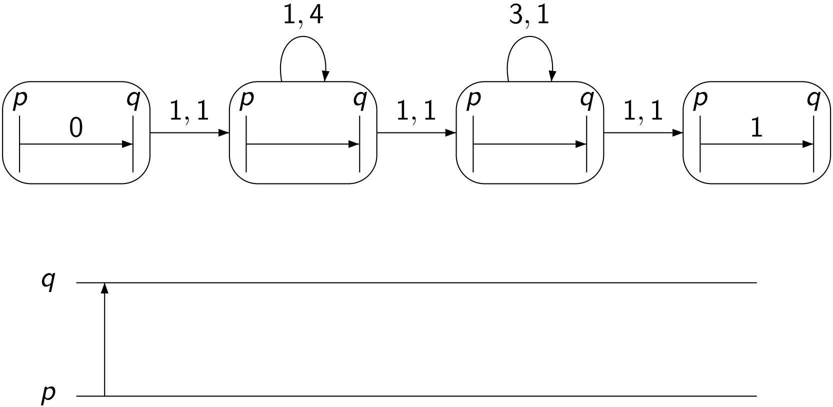

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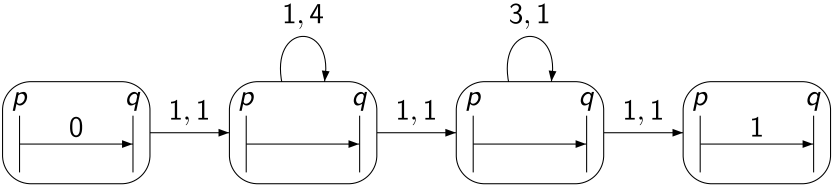

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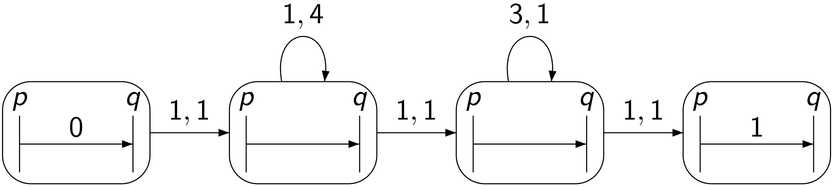

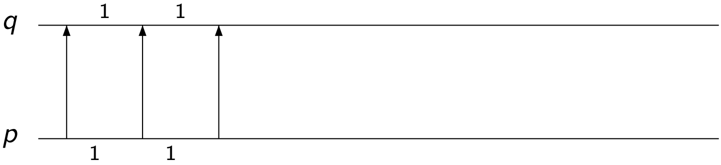

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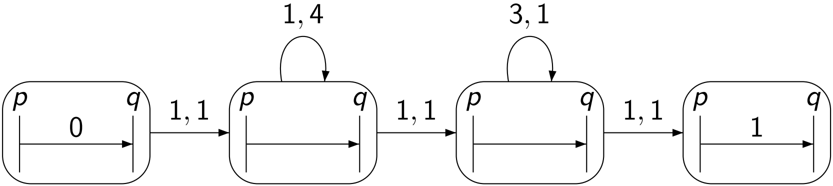

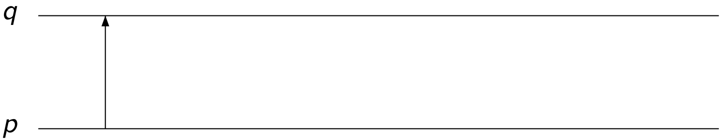

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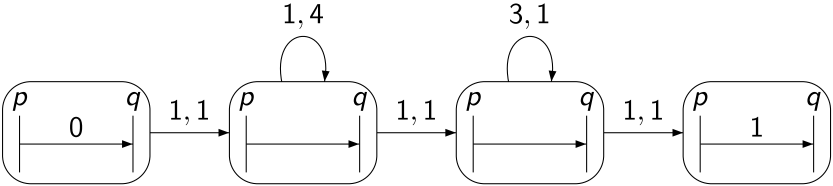

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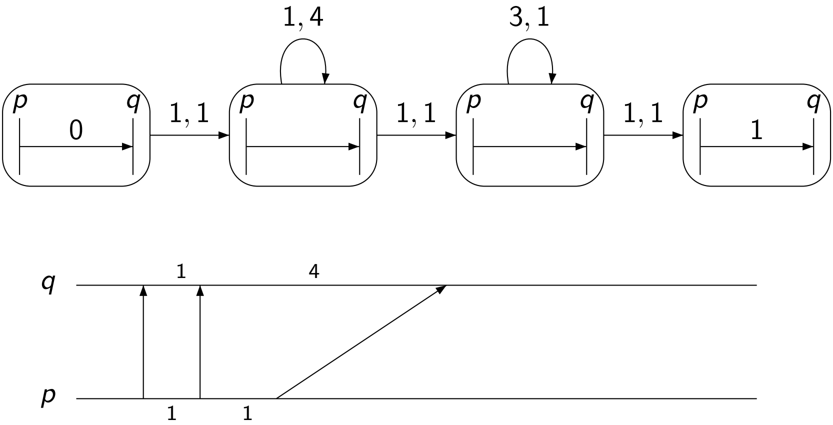

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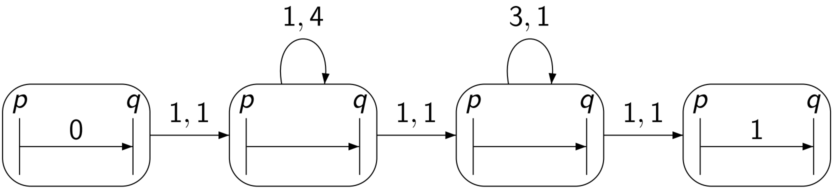

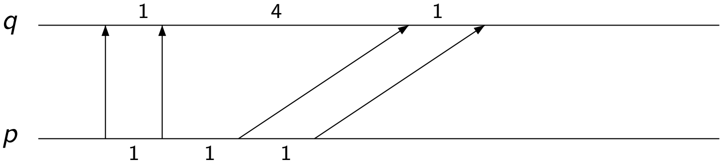

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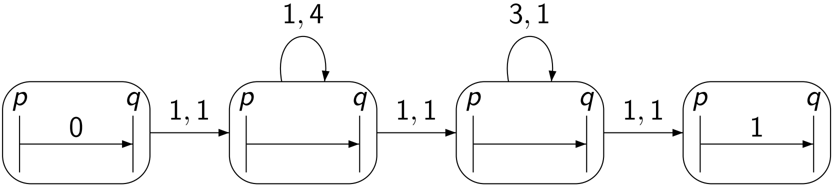

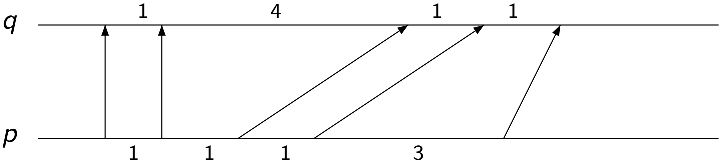

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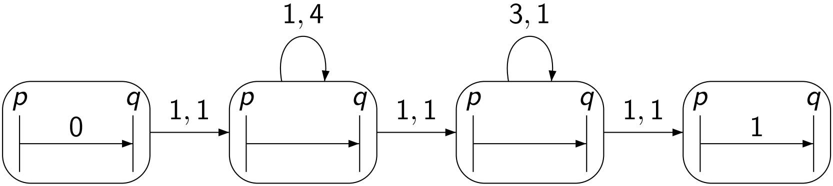

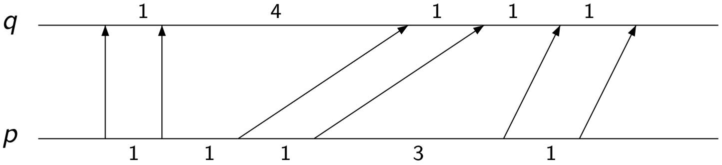

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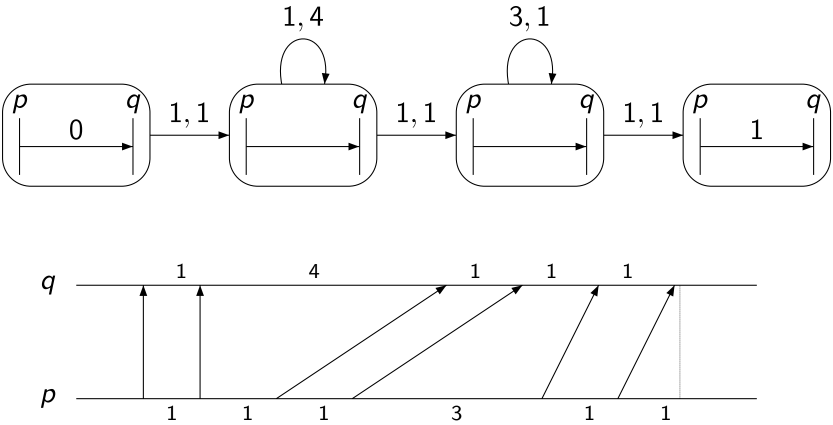

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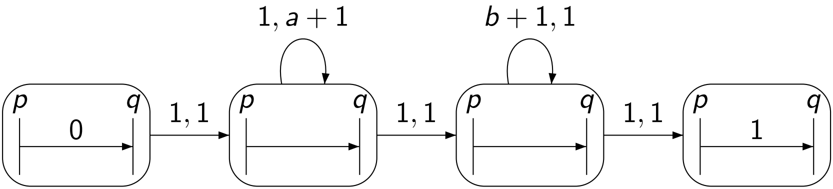

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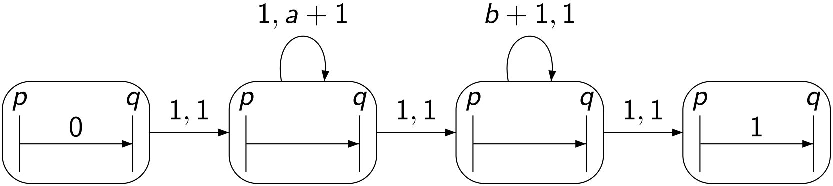

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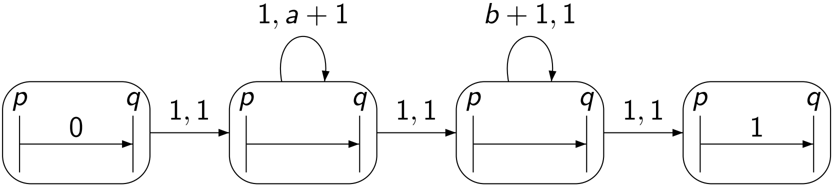

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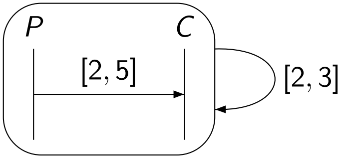

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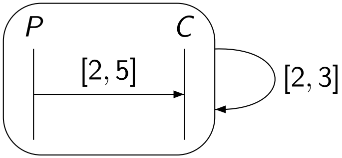

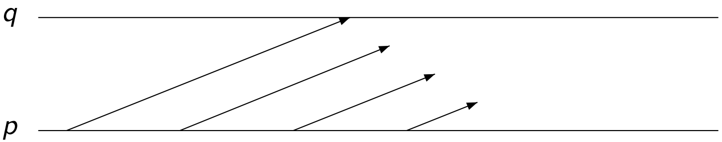

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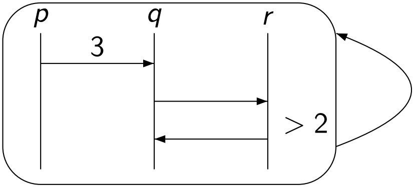

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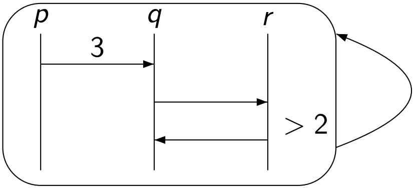

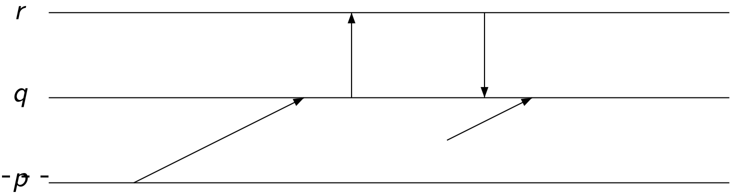

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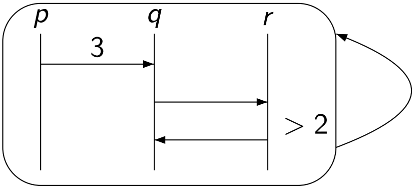

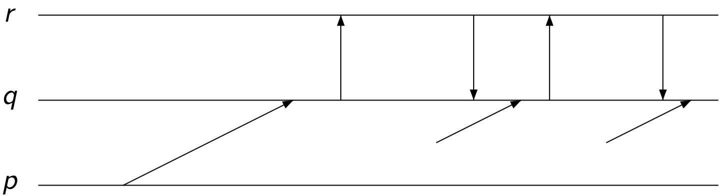

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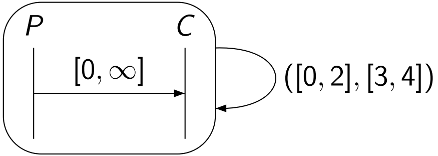

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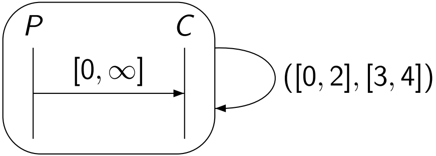

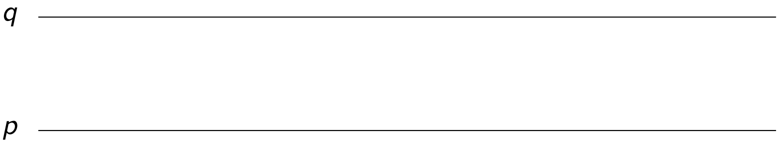

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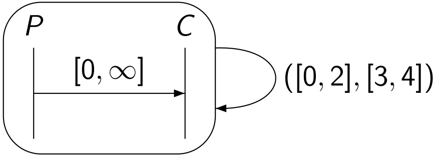

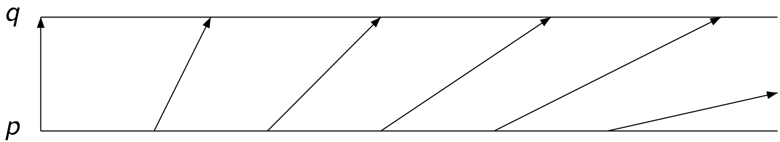

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