The Theory of Message Sequence Charts

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Chennai Mathematical Institute
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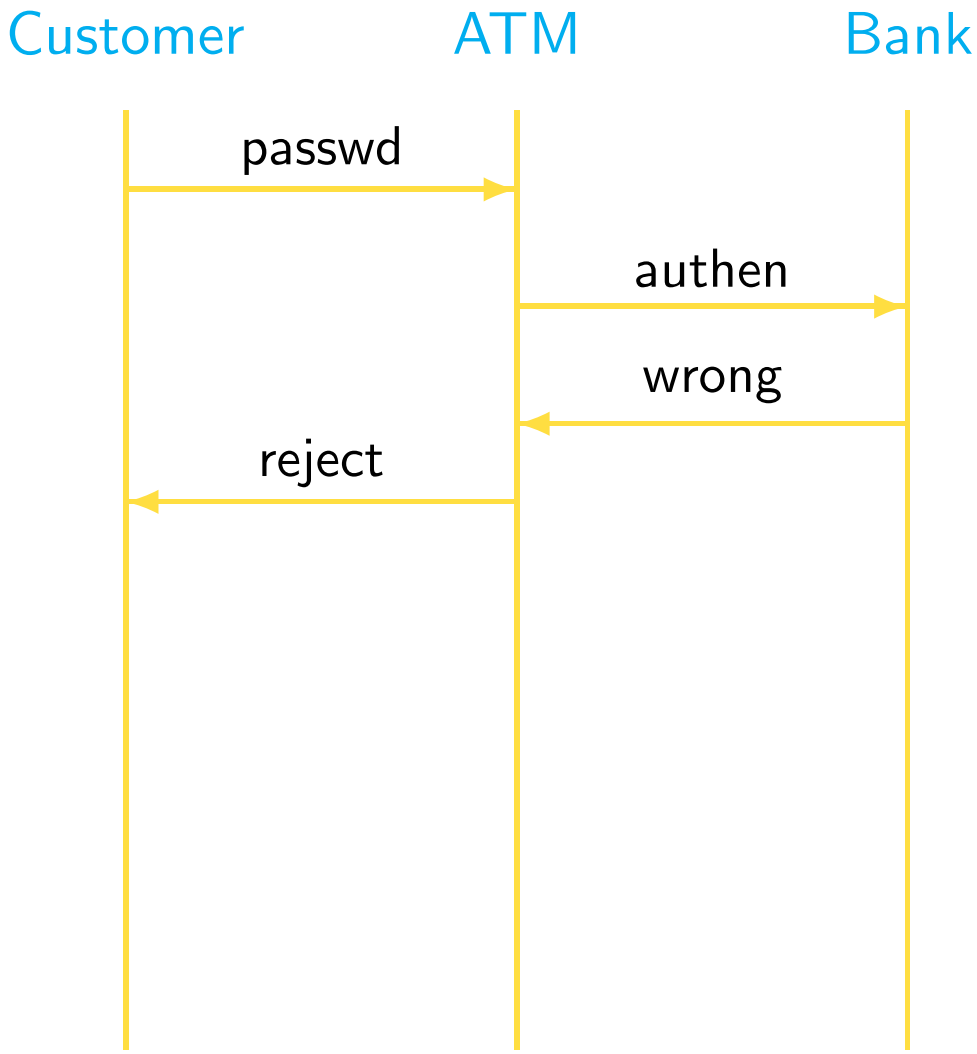
TIFR, Mumbai, 28 April, 2009

An ATM



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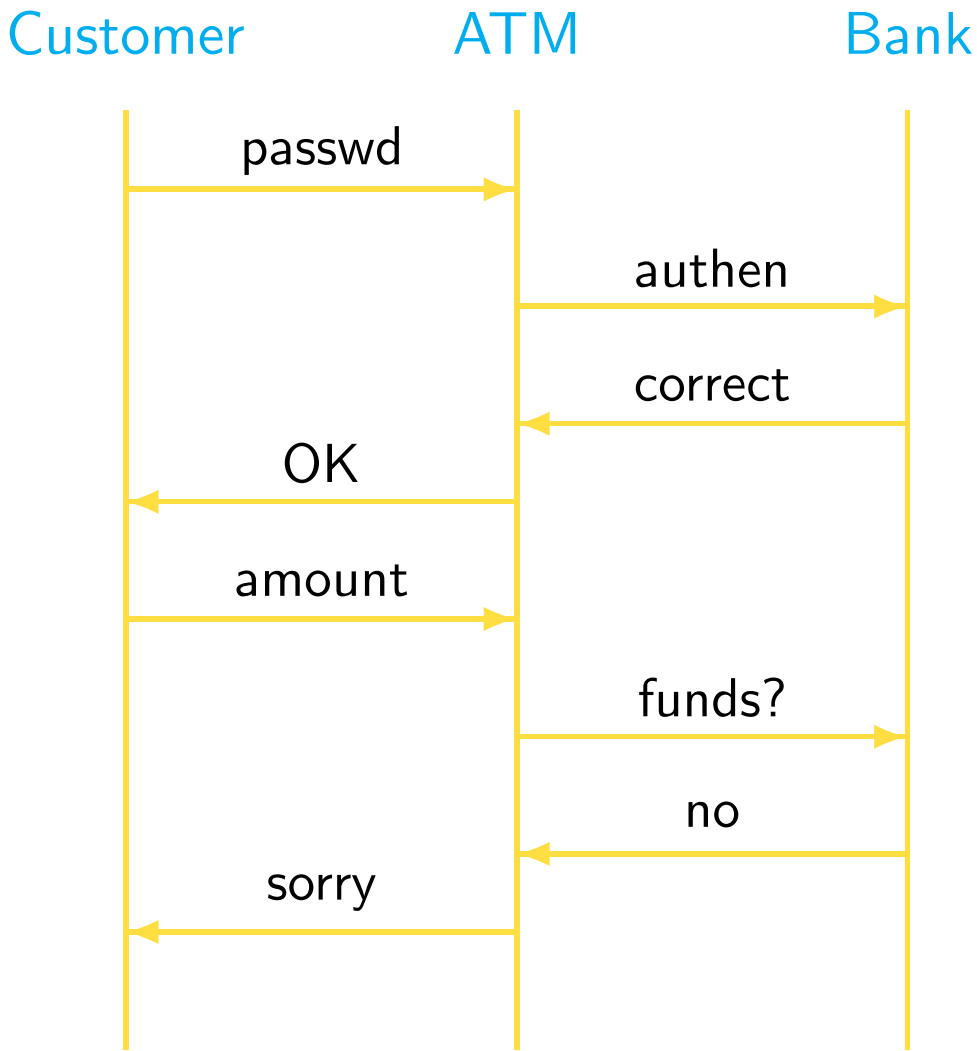
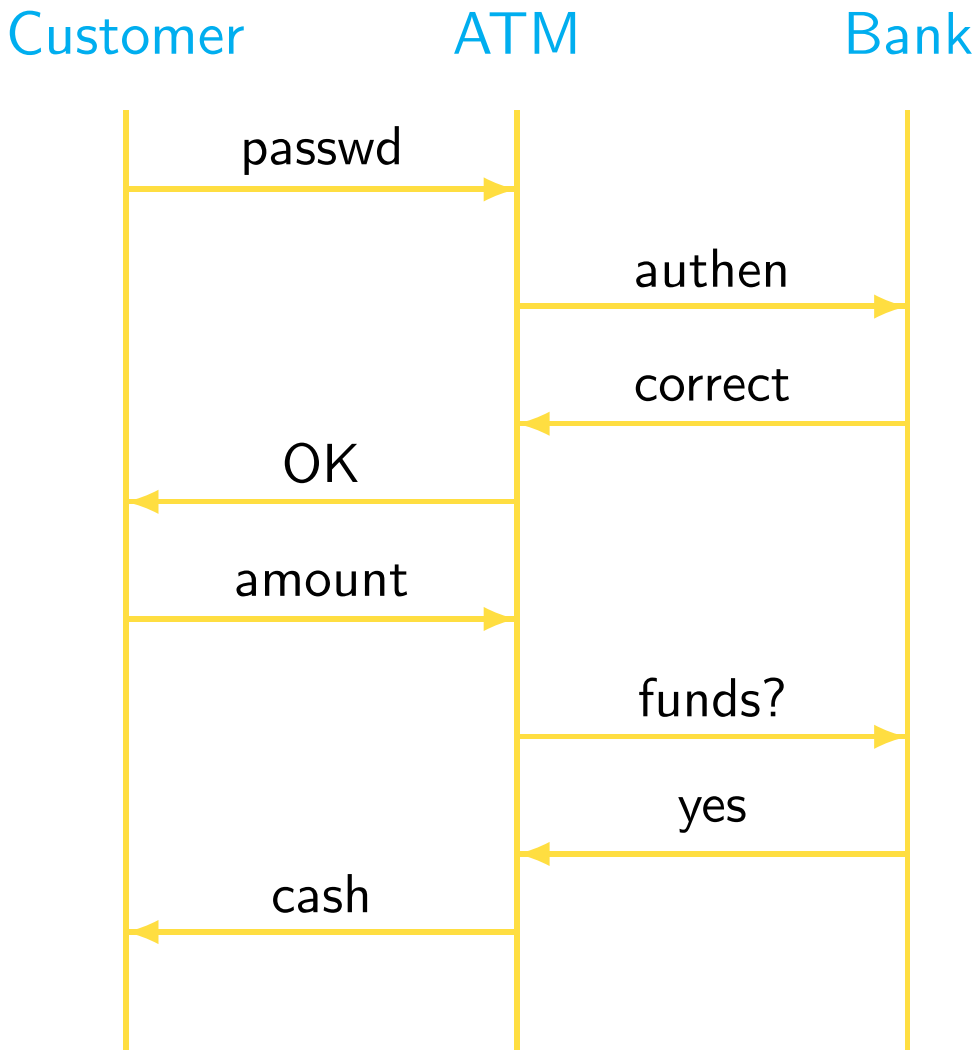

Customer
ATM
Bank
passwd
authen
correct
OK
amount
funds?
no
sorry

An ATM


Customer
ATM
Bank
passwd
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funds?
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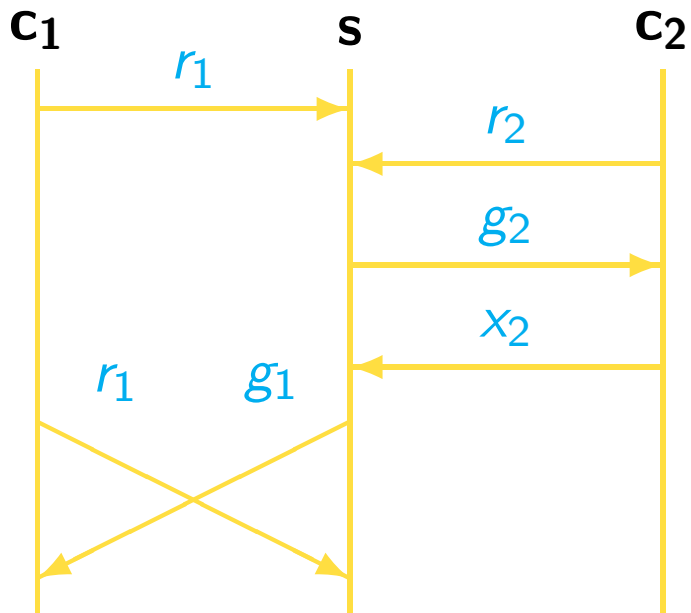
cash

Message Sequence Charts

Two clients and a server



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- Visual formalism for specifying scenarios.

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- Formalize these pictures as ...
- Formulate natural (and relevant) questions ...

MSCs

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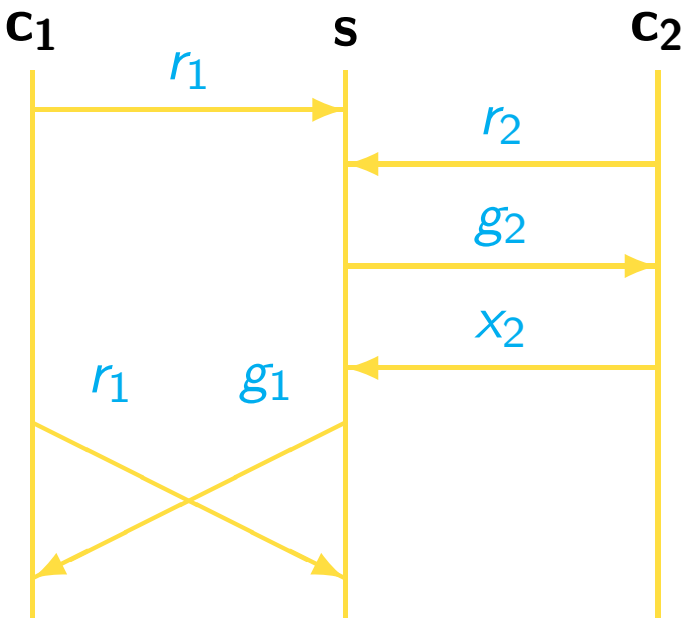

MSCs as Labelled Partial Orders

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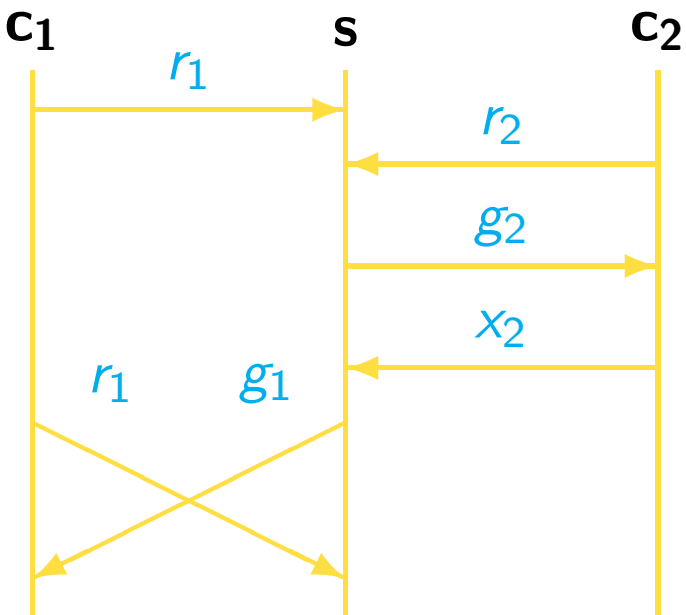
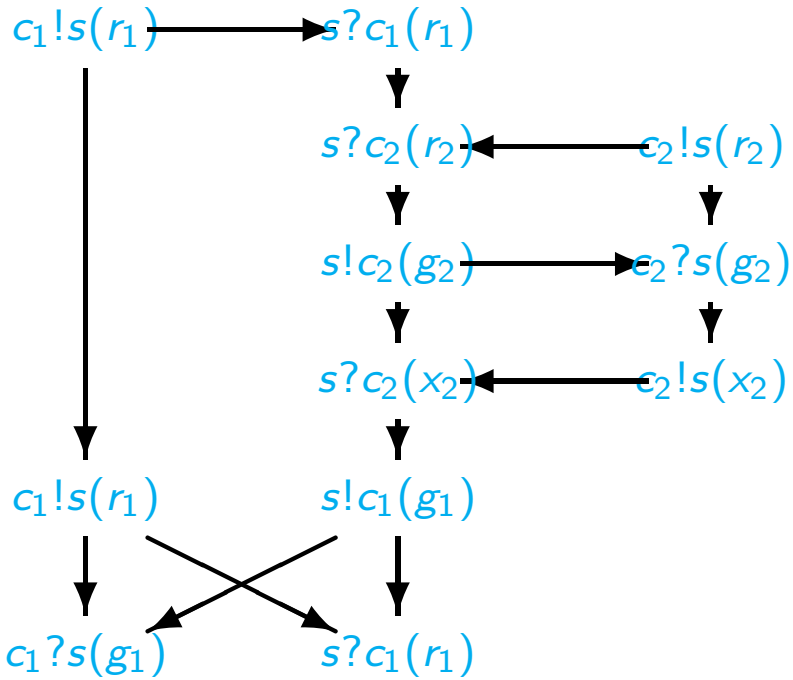


and a partial order representation



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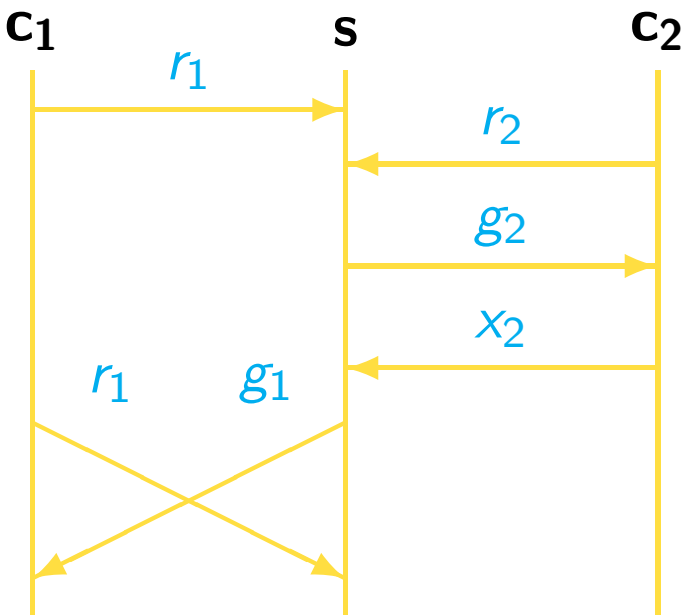

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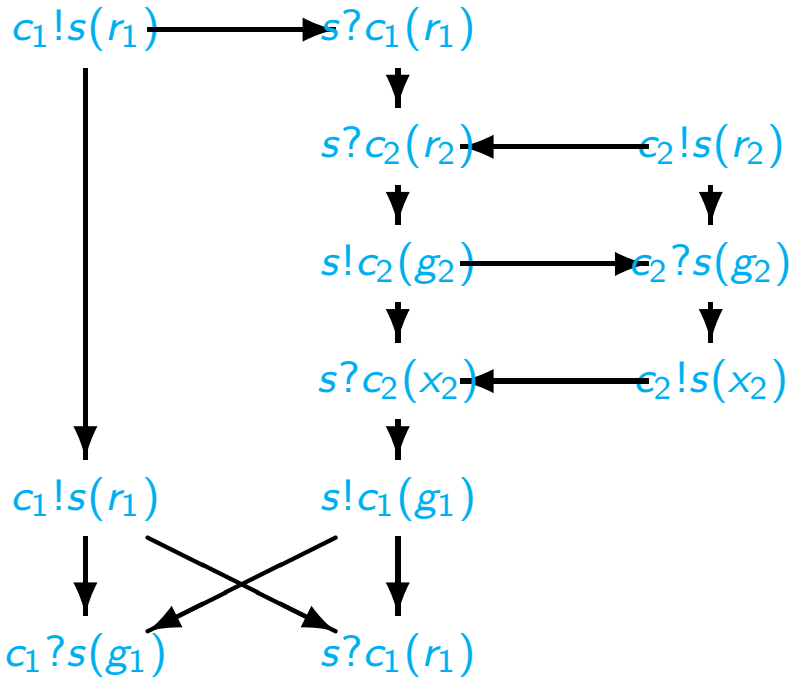

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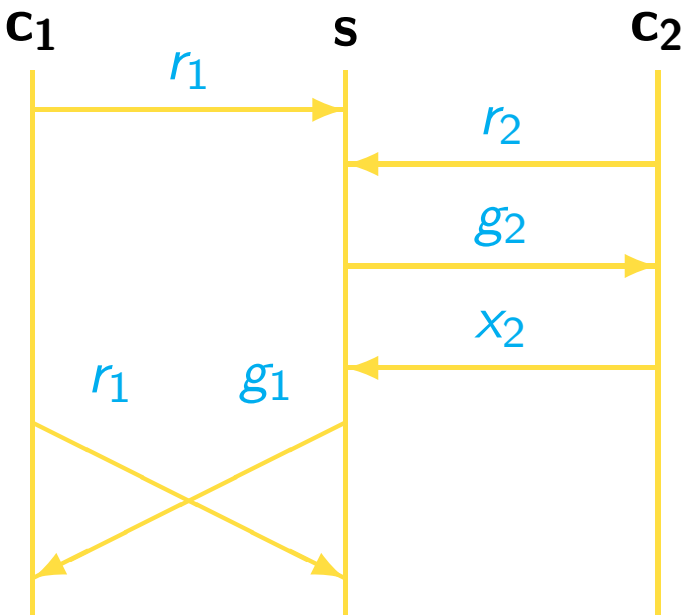

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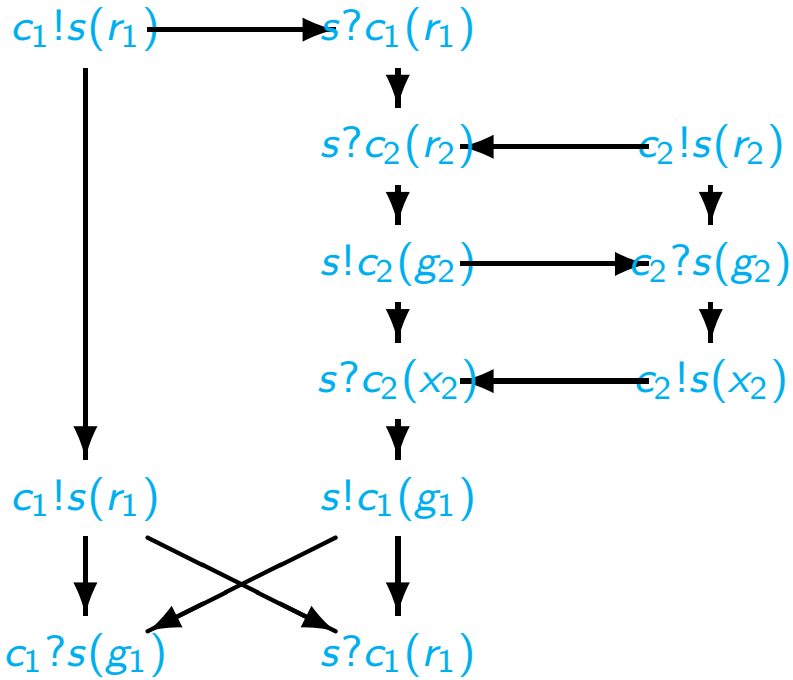

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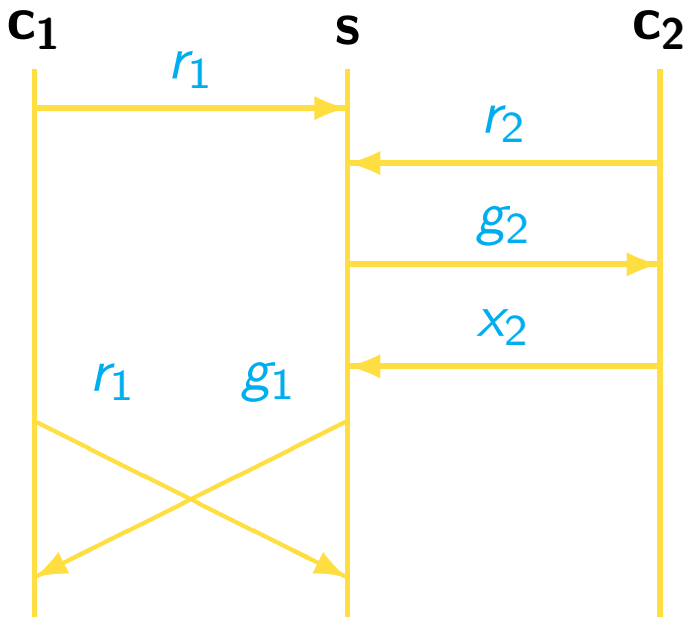

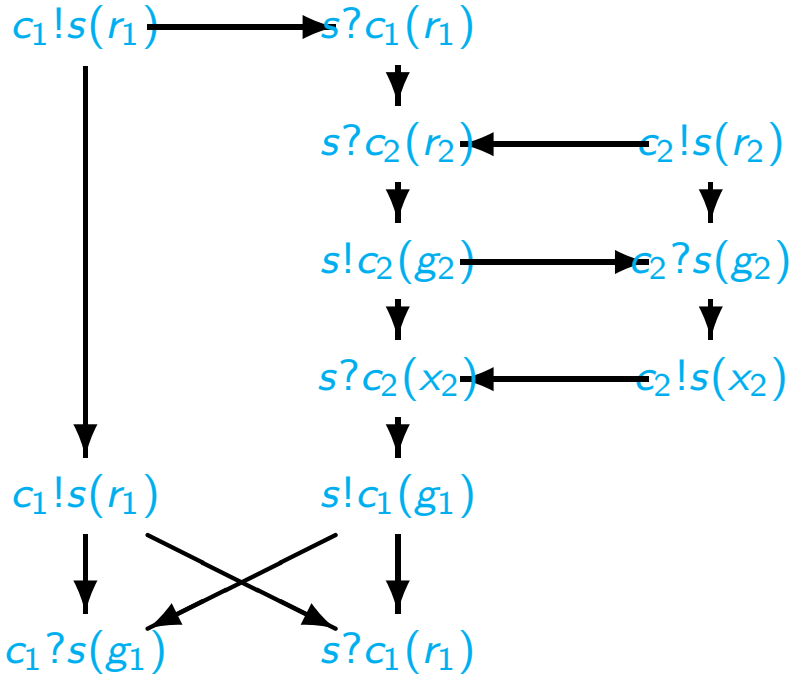

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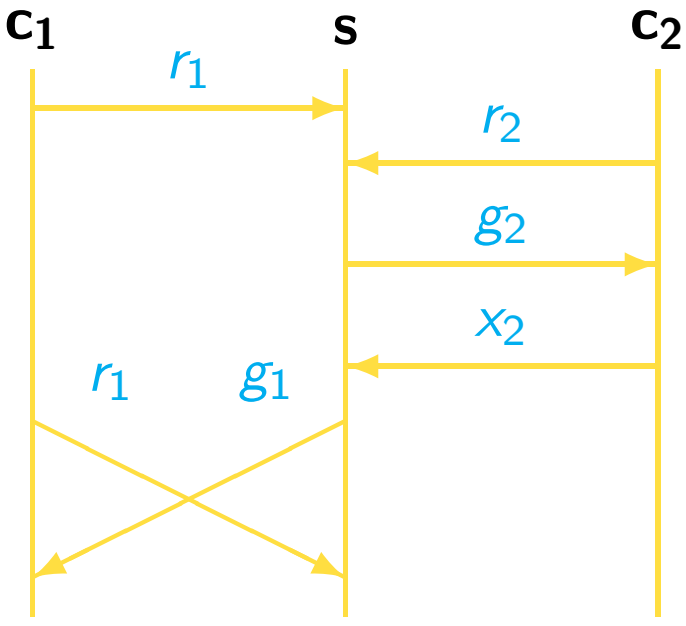

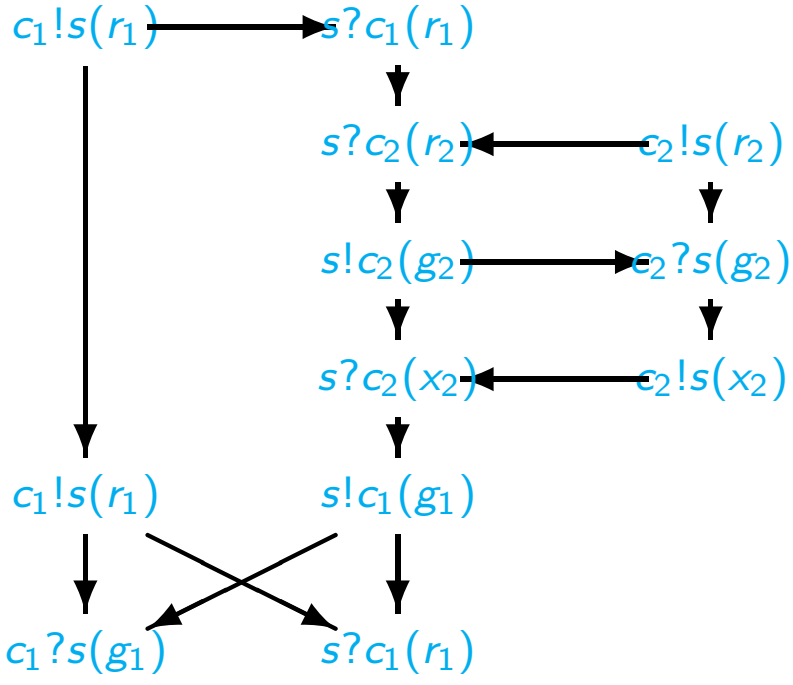

$c_2!s(r_2)\ c_1!s(r_1)\ c_1!s(r_1)\ s?c_1(r_1)\ s?c_2(r_2)\ s!c_2(g_2)\ c_2?s(g_2)\ c_2!s(x_2)$
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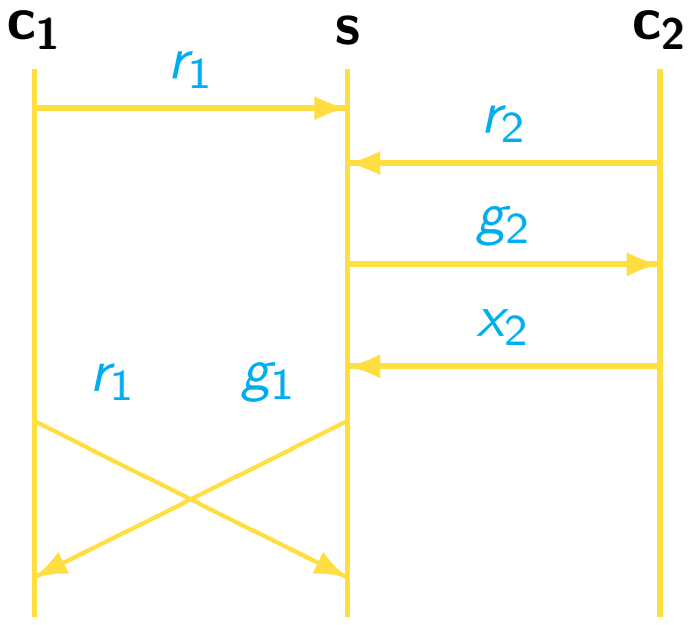

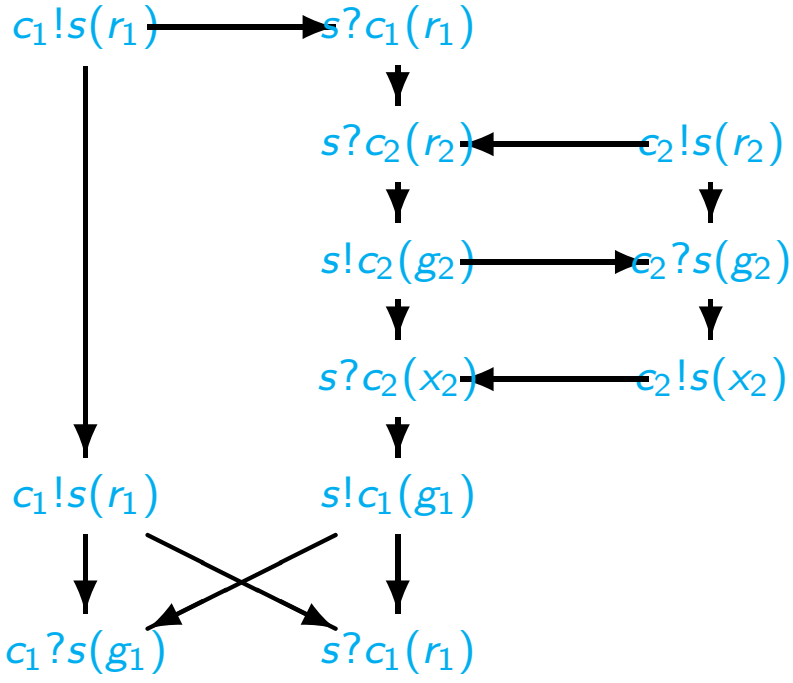

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As a matter of fact, under the FIFO assumption, any word satisfying these two properties is the linearization of a unique MSC.

Linearizations to MSCs

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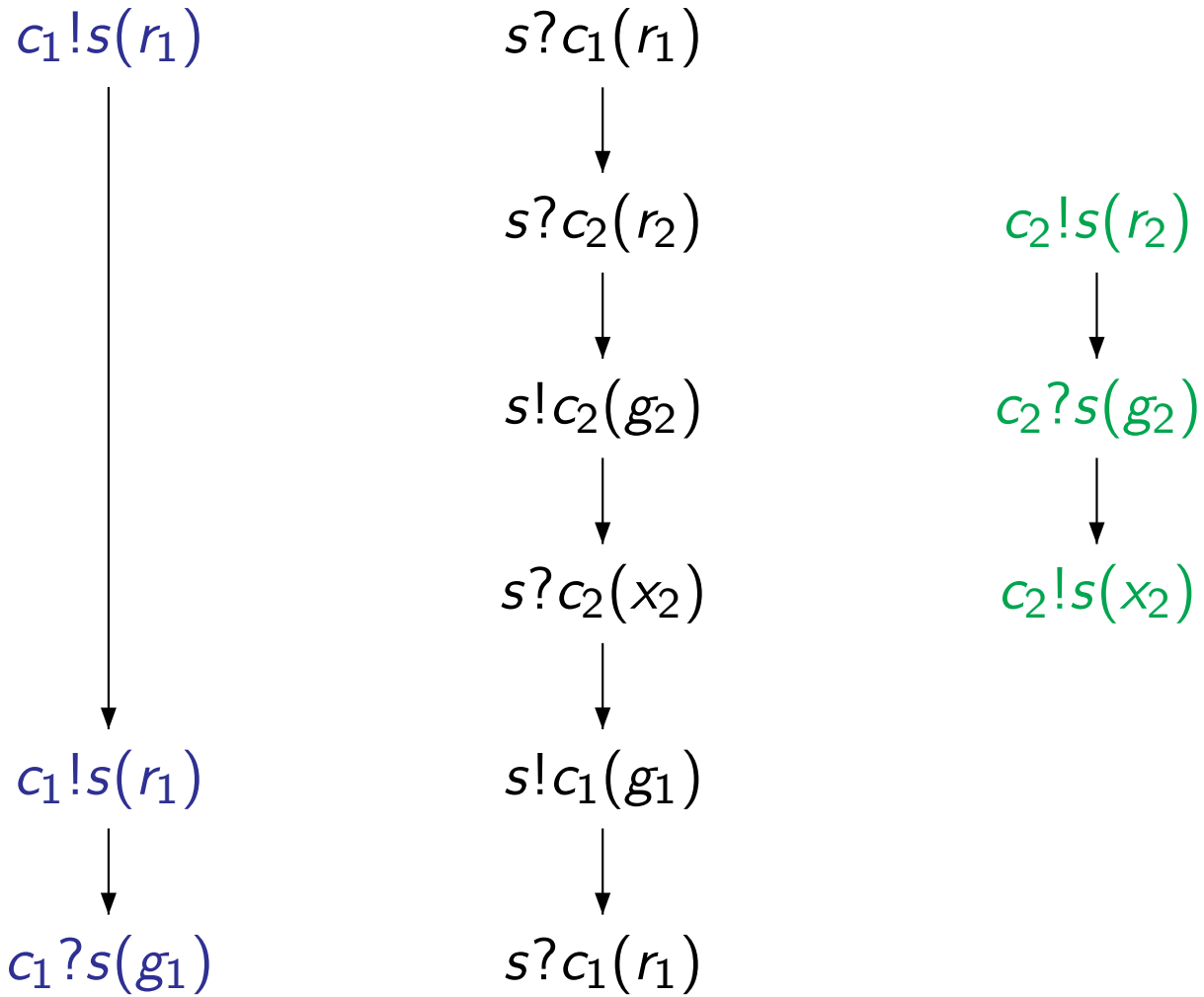

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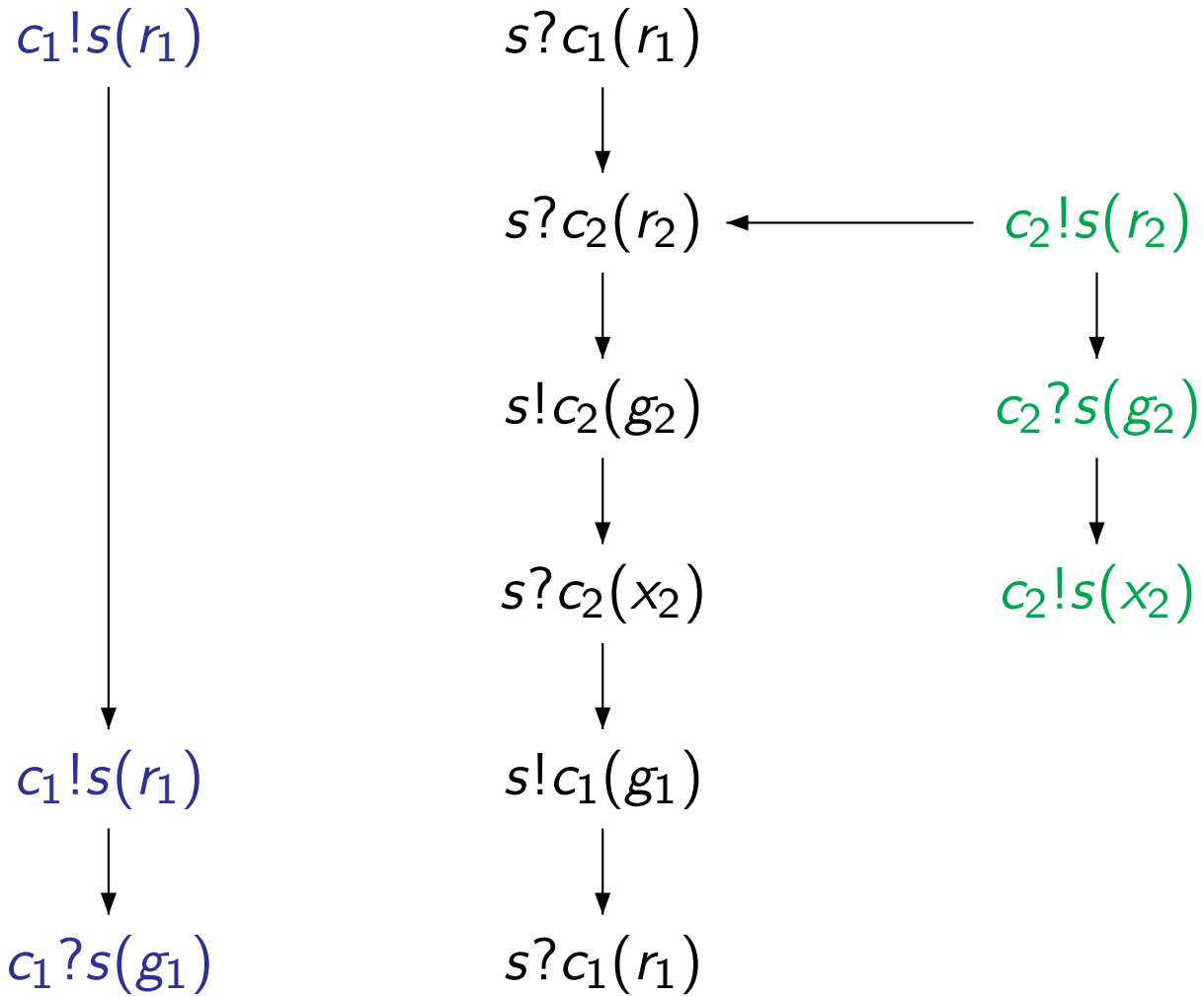

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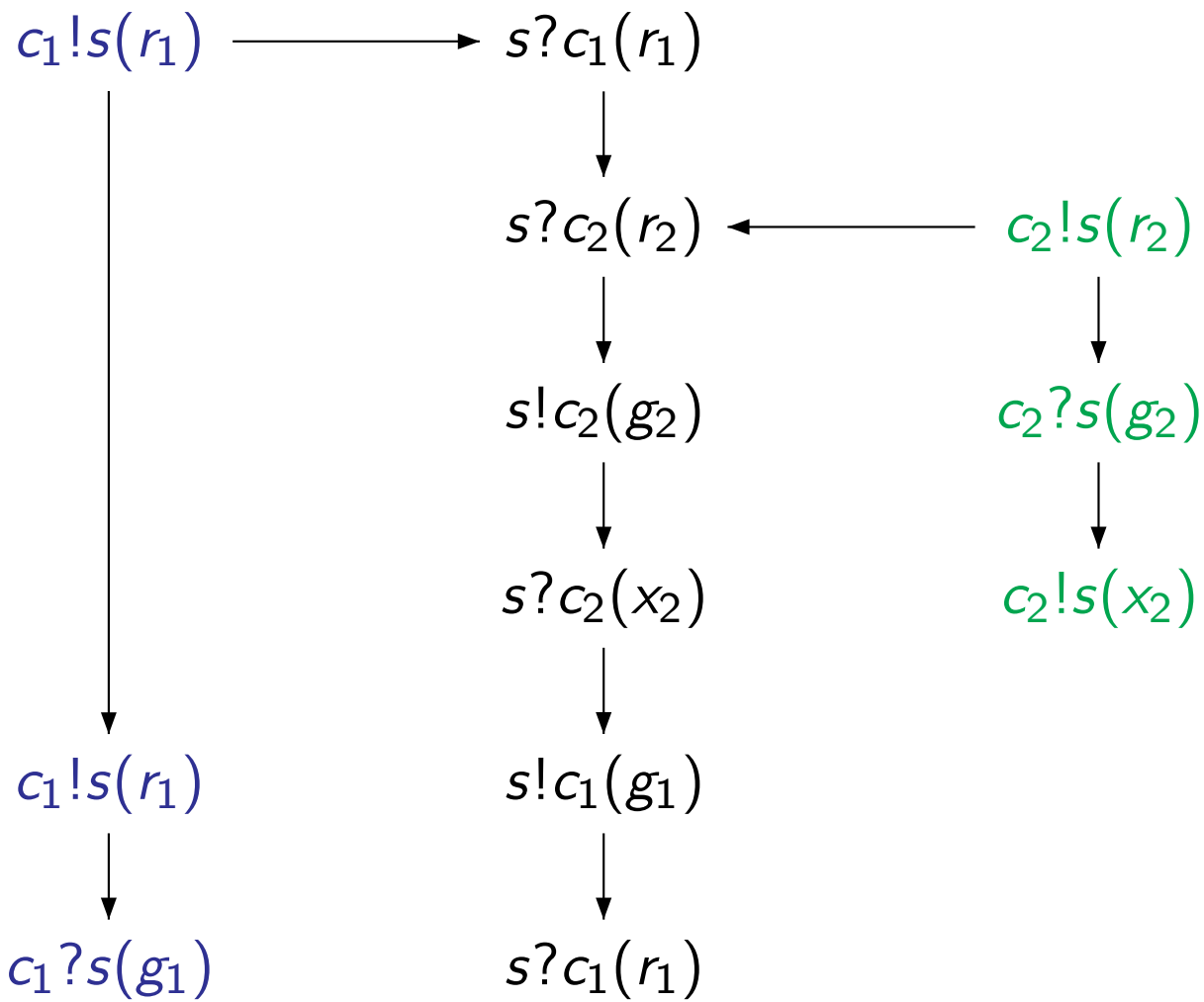

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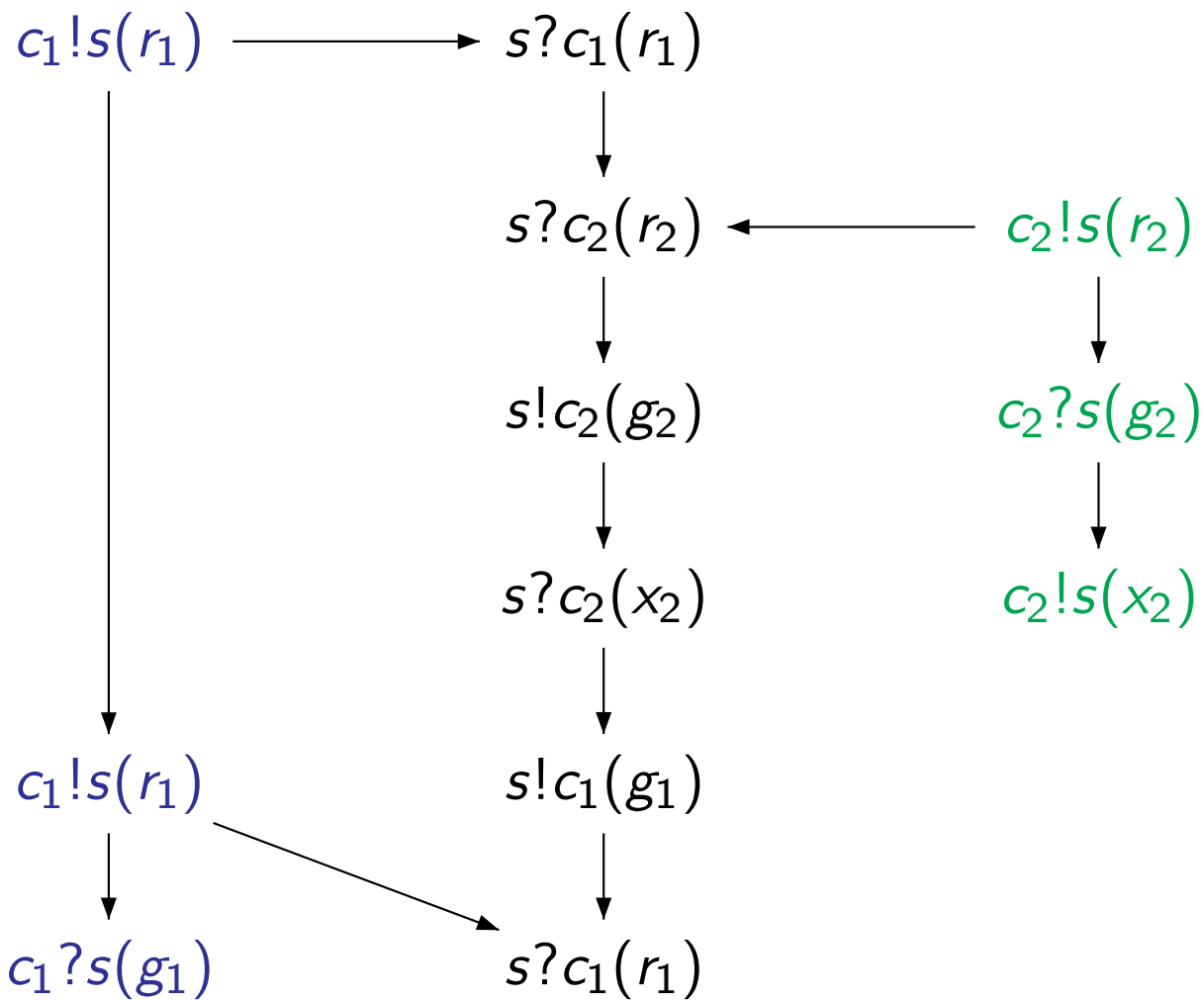

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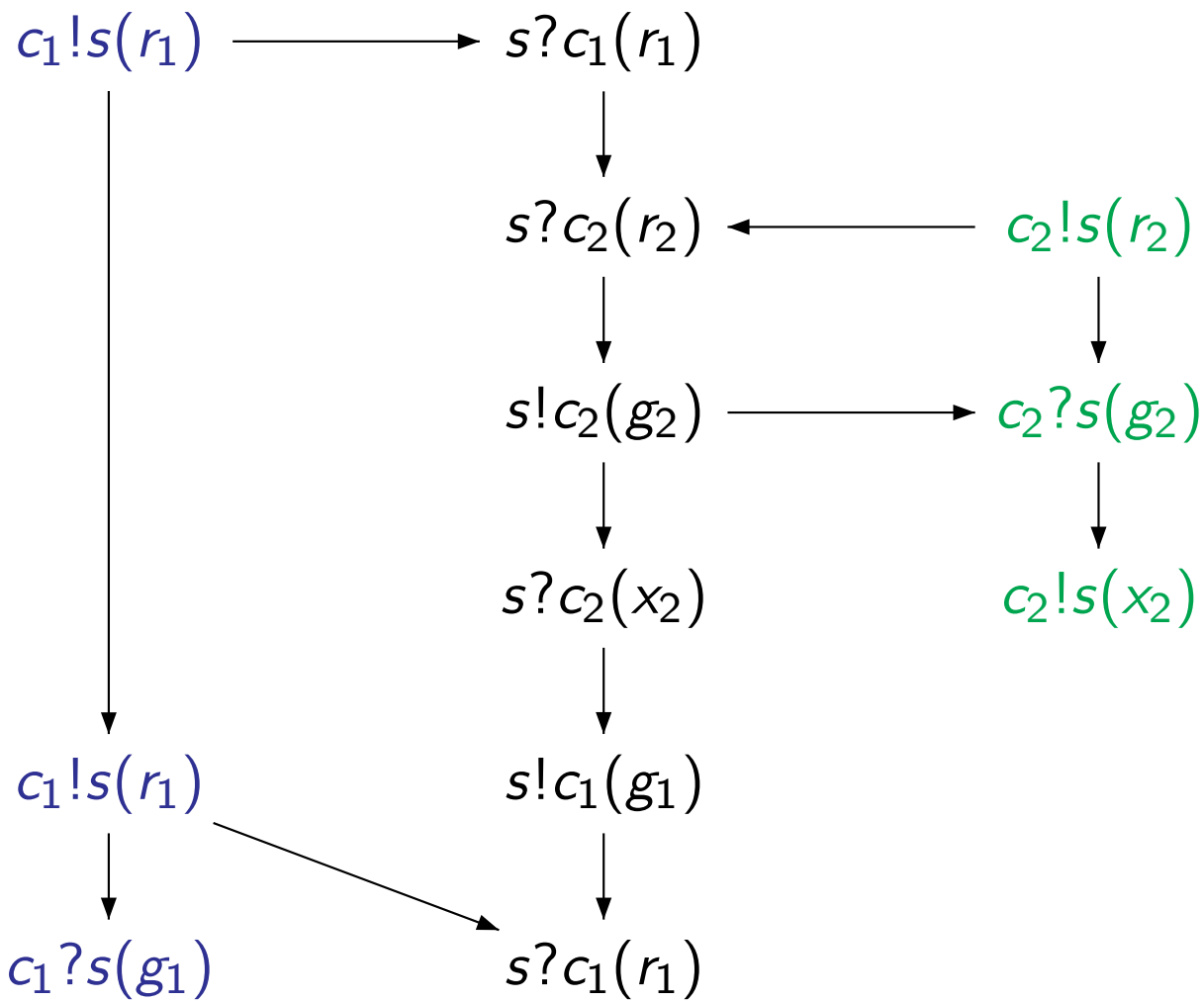

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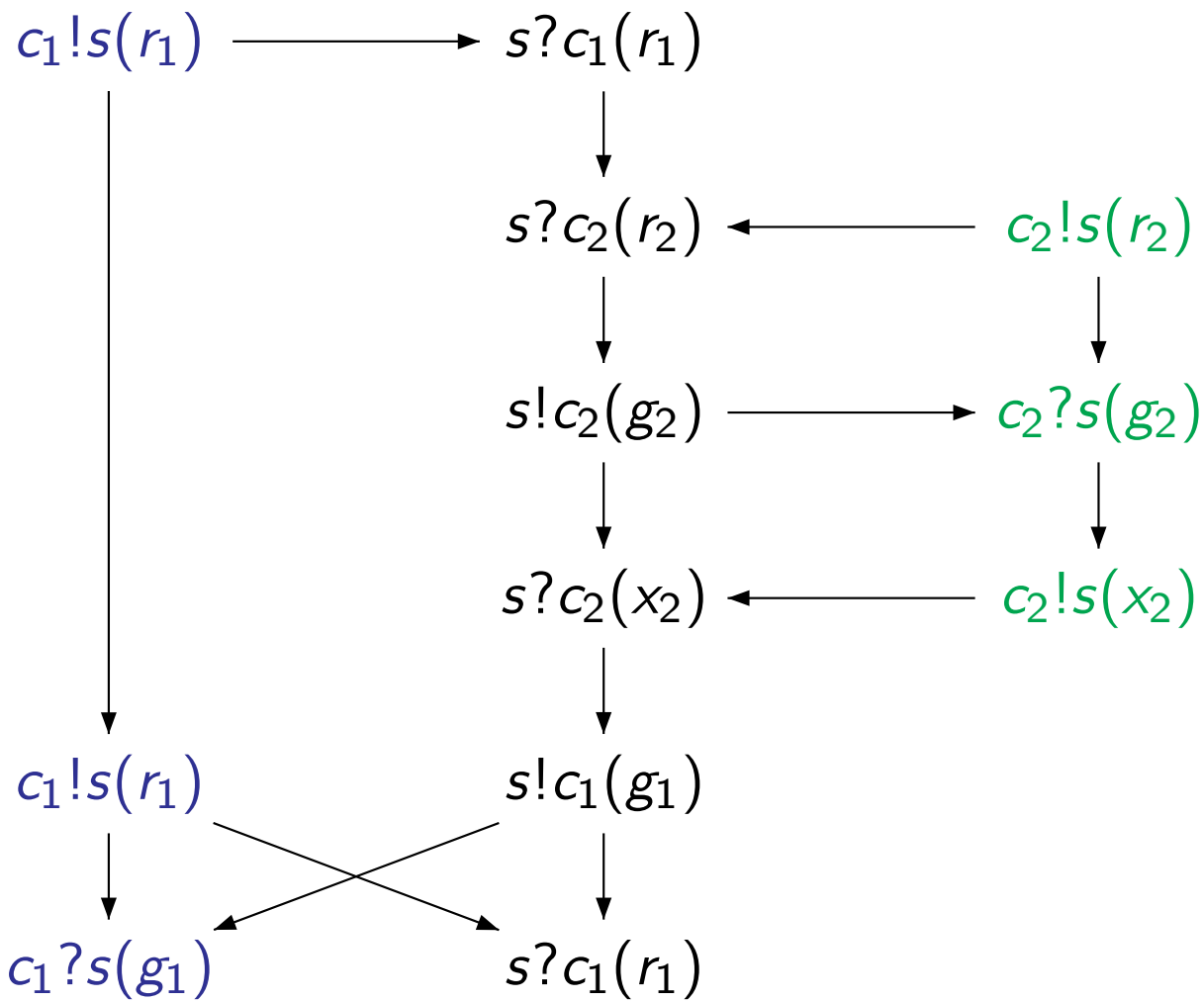

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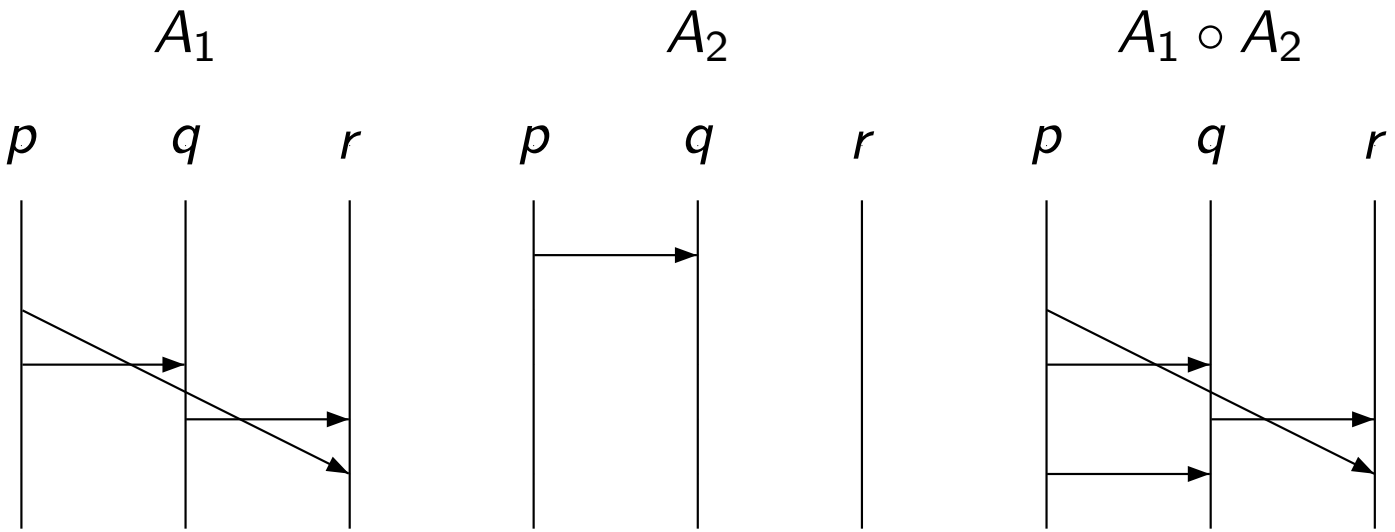
How do we describe MSC languages?

Concatenation of MSCs



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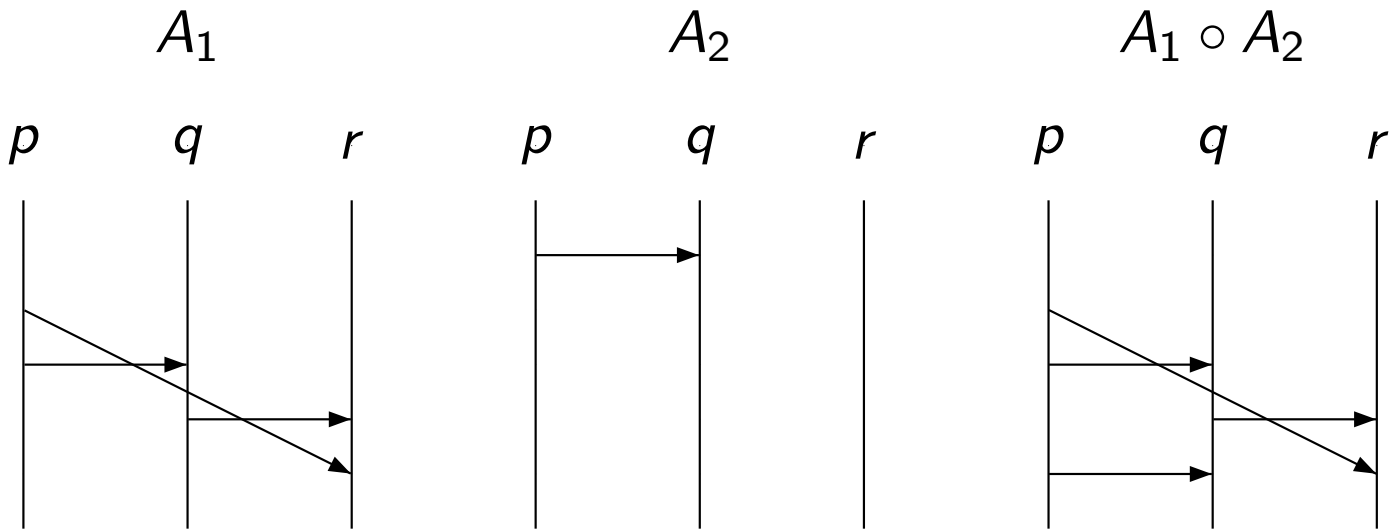

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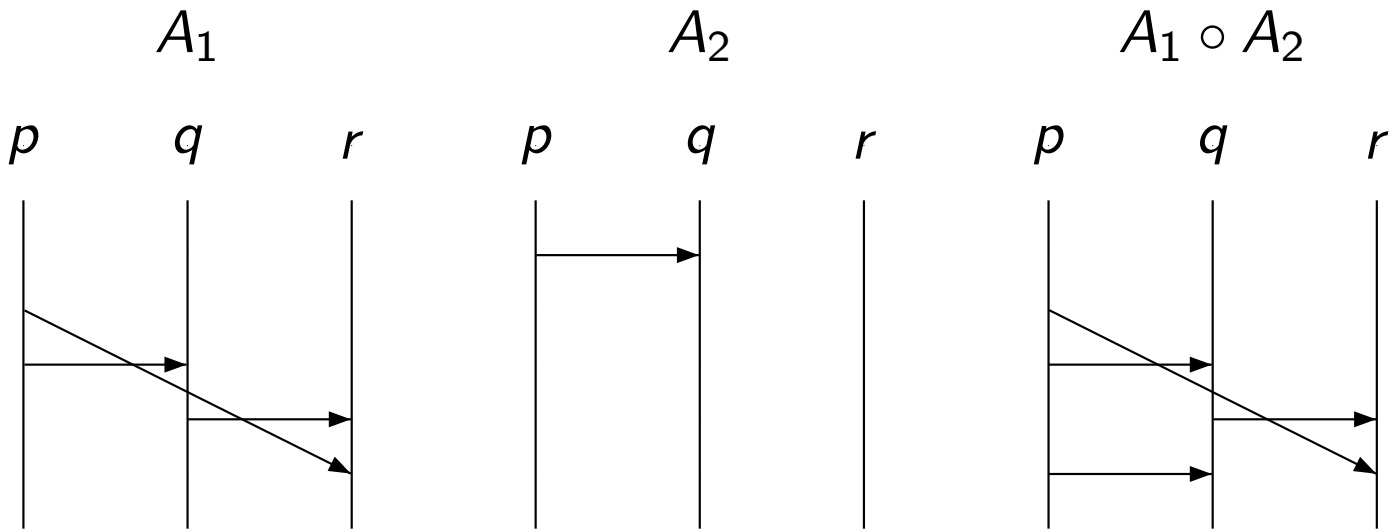

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p!r p!q q?p q!r r?q p!q q?p r?p

is a linearization of $A_1 \circ A_2$.

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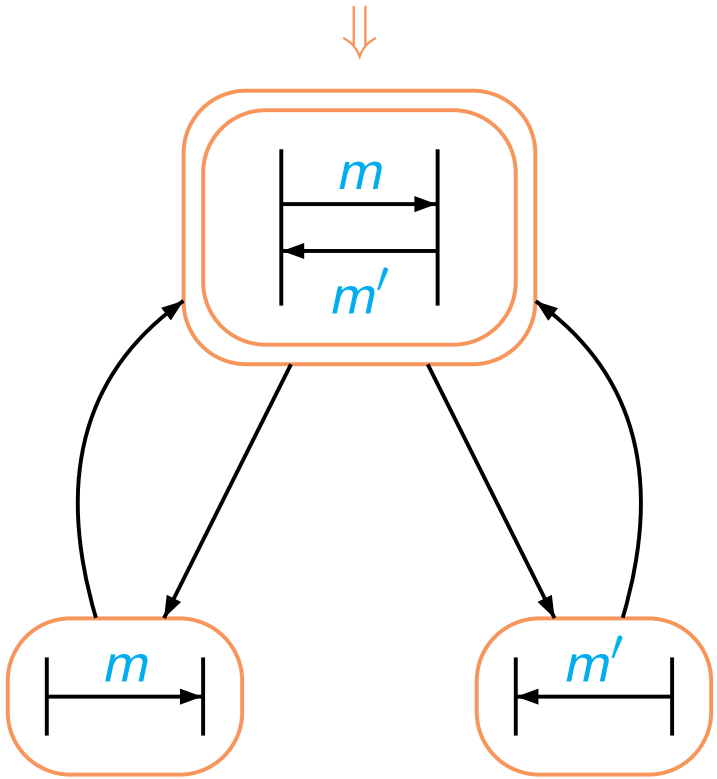

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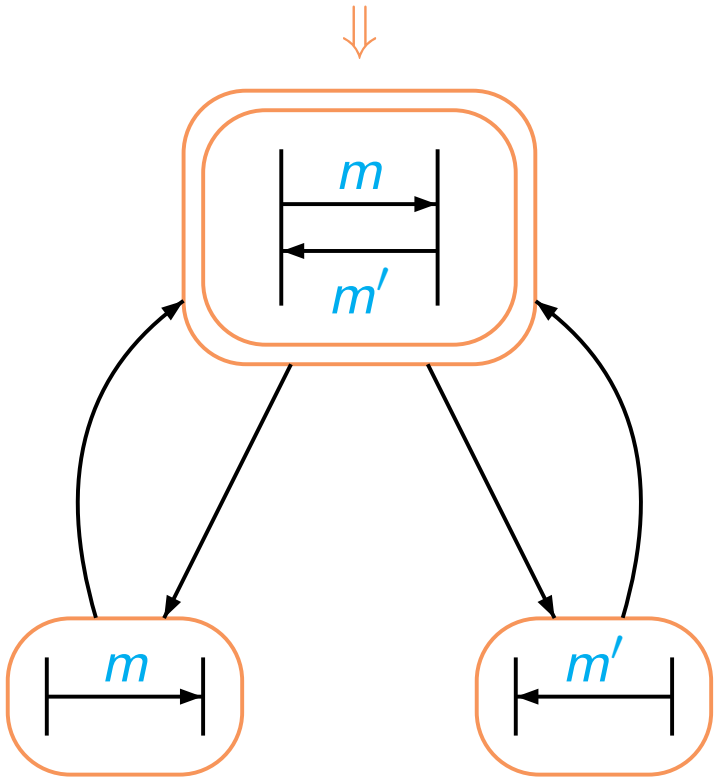

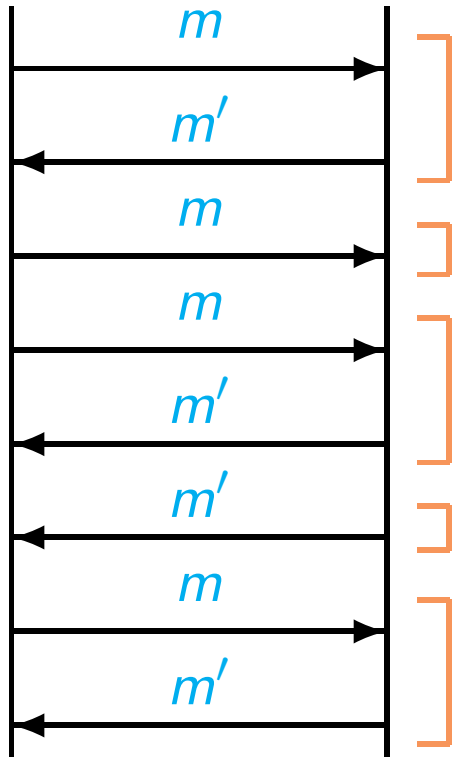

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- They are global descriptions and do not give any guarantee about implementability.

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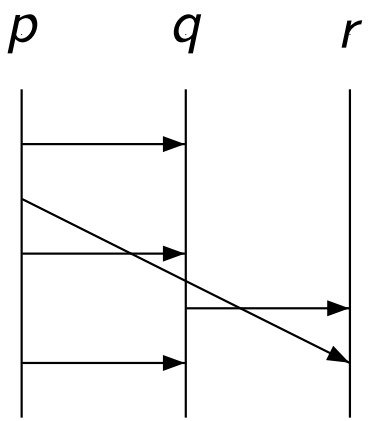

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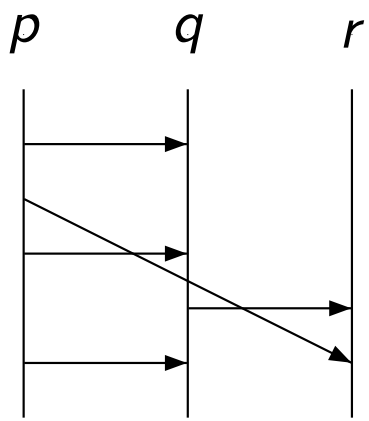

The linearization

p!q q?p p!r p!q q?p q!r r?q p!q q?p r?p

is 1-bounded while the linearization

p!q p!r p!q p!q q?p q?p p!r q?p r?q r?p

is 3-bounded.

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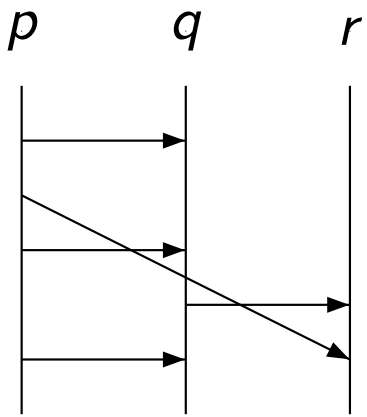

An existentially 1-bounded and universally 3-bounded MSC.

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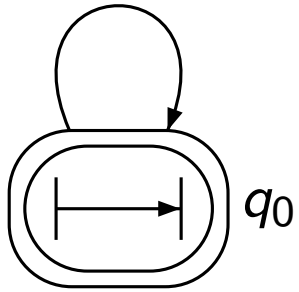

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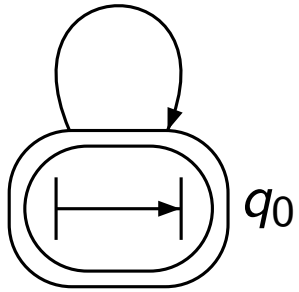

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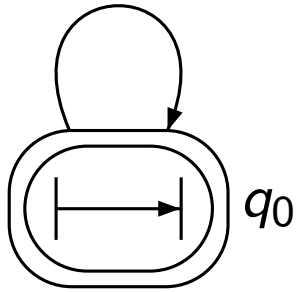

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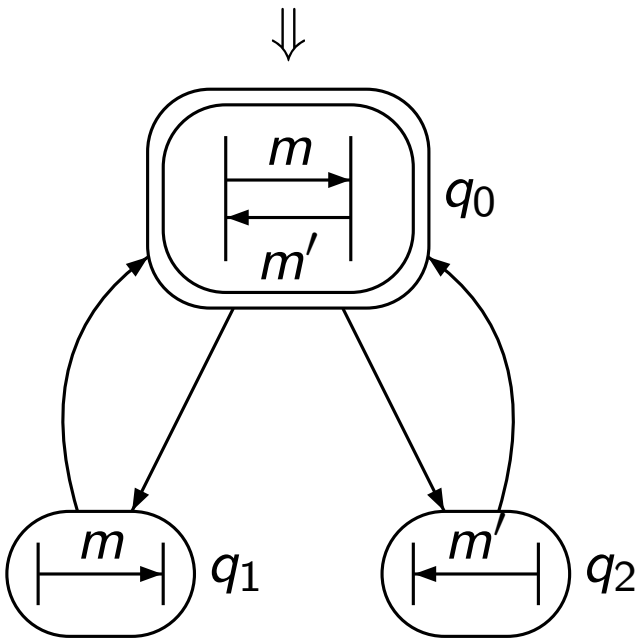
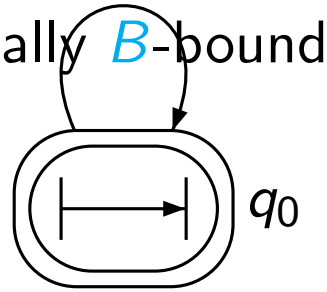


Boundedness ...

An MSG is existentially bounded if there exists a B such that every MSC it generates is existentially B-bounded.



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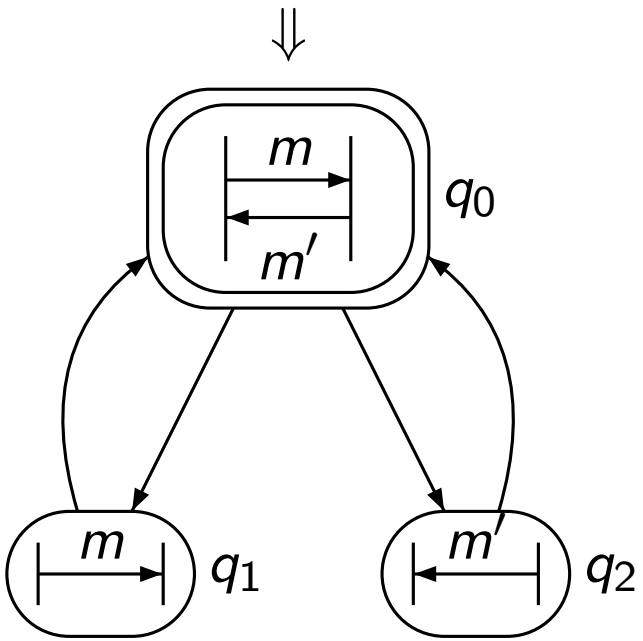

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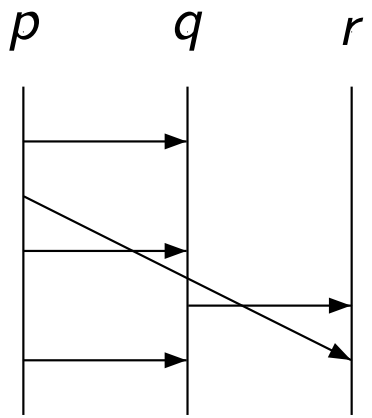
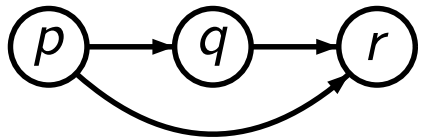
- Checking whether an MSG is universally bounded (B-bounded for a given B) is decidable.

Communication graph of an MSC

Nodes are the processes. An edge from p to q if there is a message from p to q.





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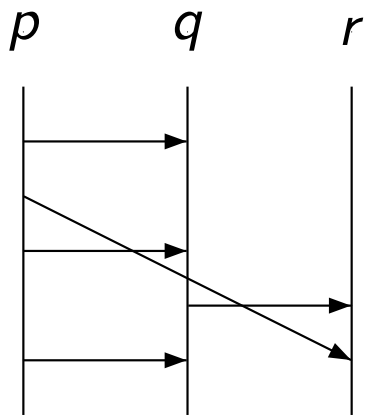

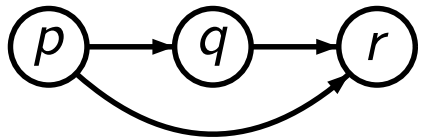

An MSG is locally strongly connected if and only if the MSC generated by any loop has a communication graph that is the disjoint union of SCCs.

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- The channel from p to q is bounded by the size of M^{k+1}.

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Other definitions are possible ...

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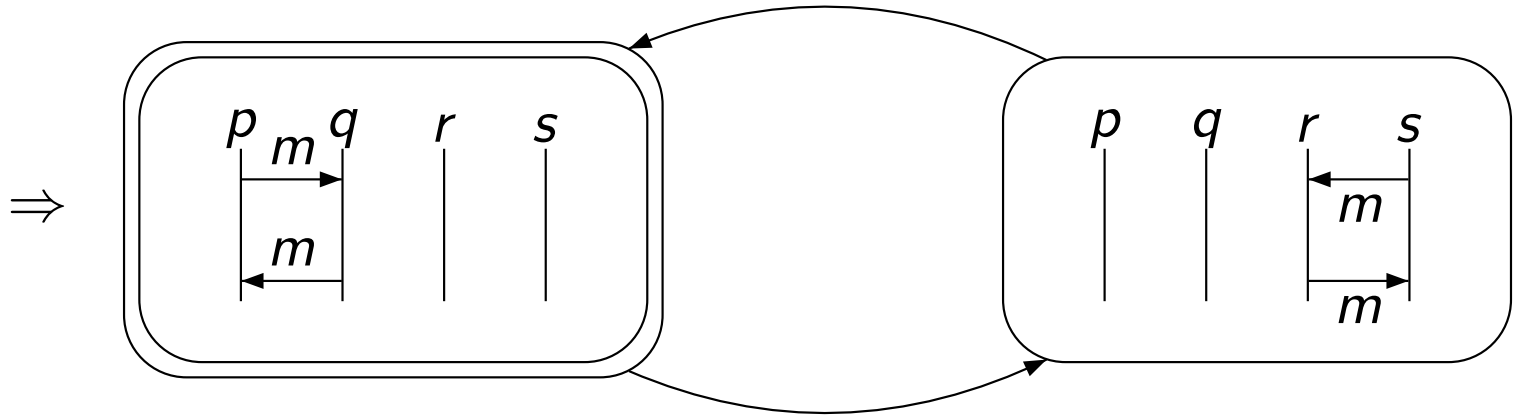
What about the converse?

Iteration of concurrent events



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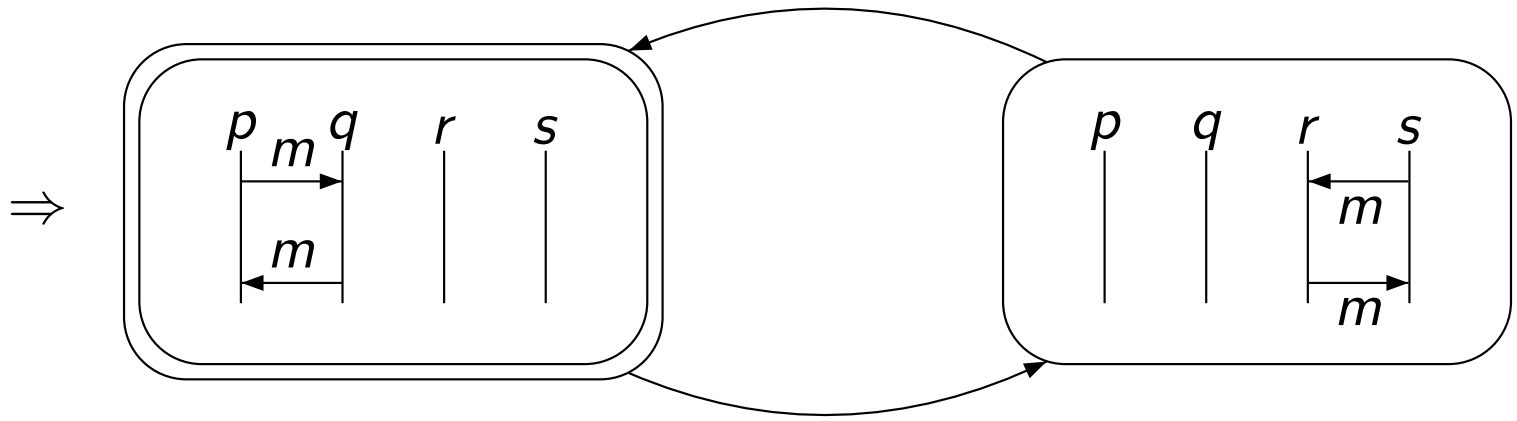

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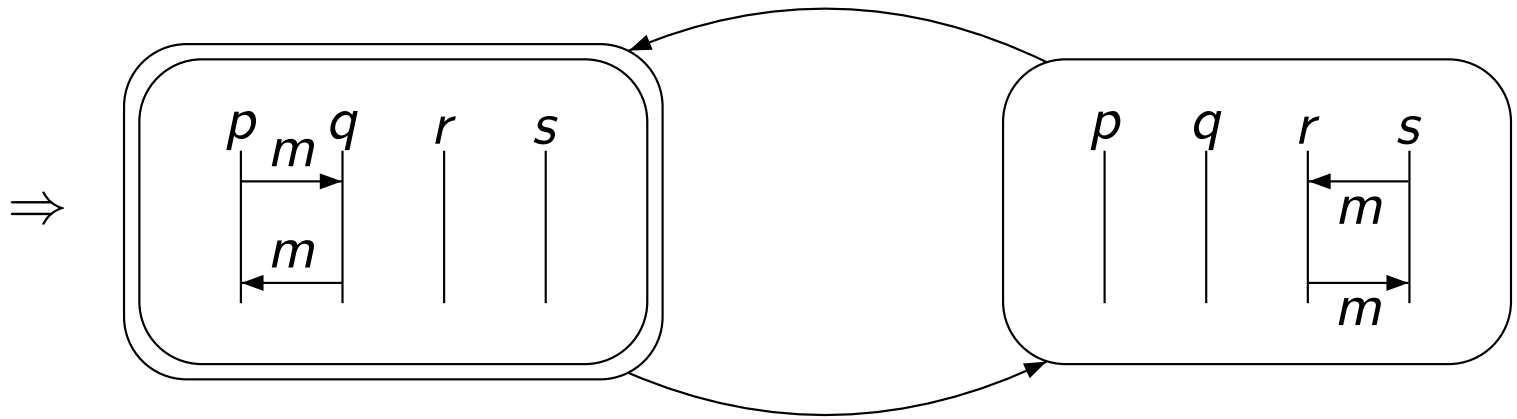

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Thus even bounded MSGs can exhibit nonregular behaviour.

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What about sufficient conditions for regularity?

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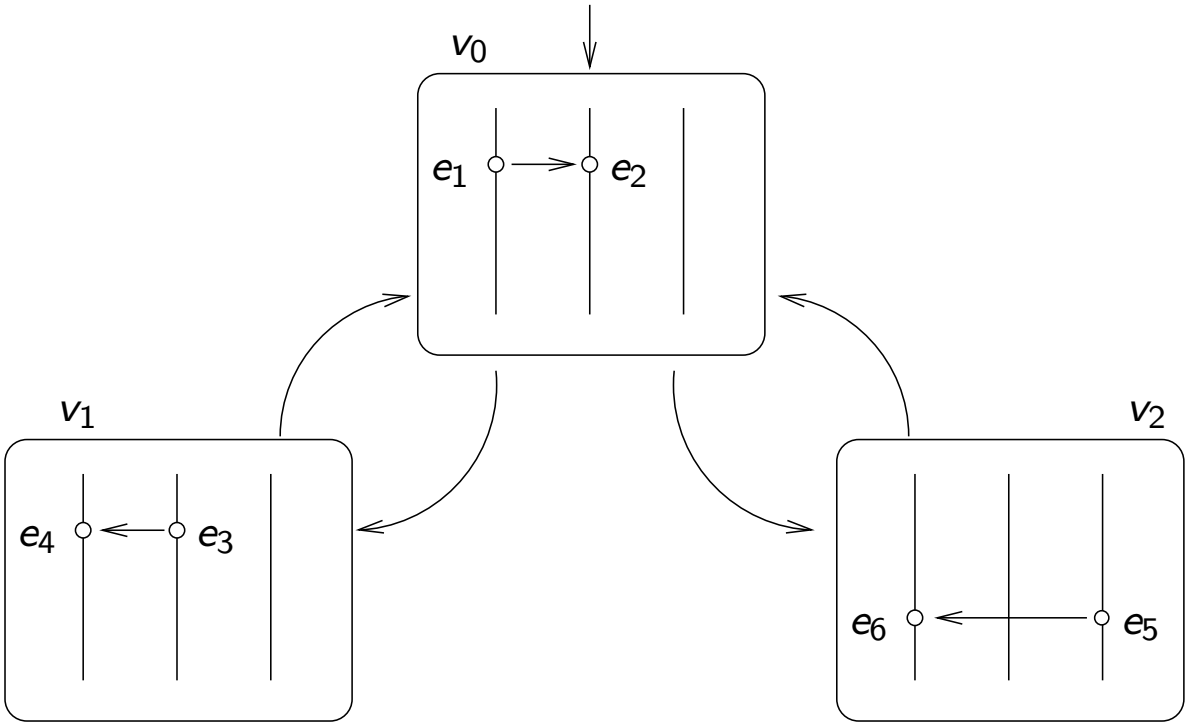
Theorem: Every locally synchronized MSG generates a regular MSC language.

Locally synchronized MSGs ...

- Every linearization of the MSCs generated must be exhibited.



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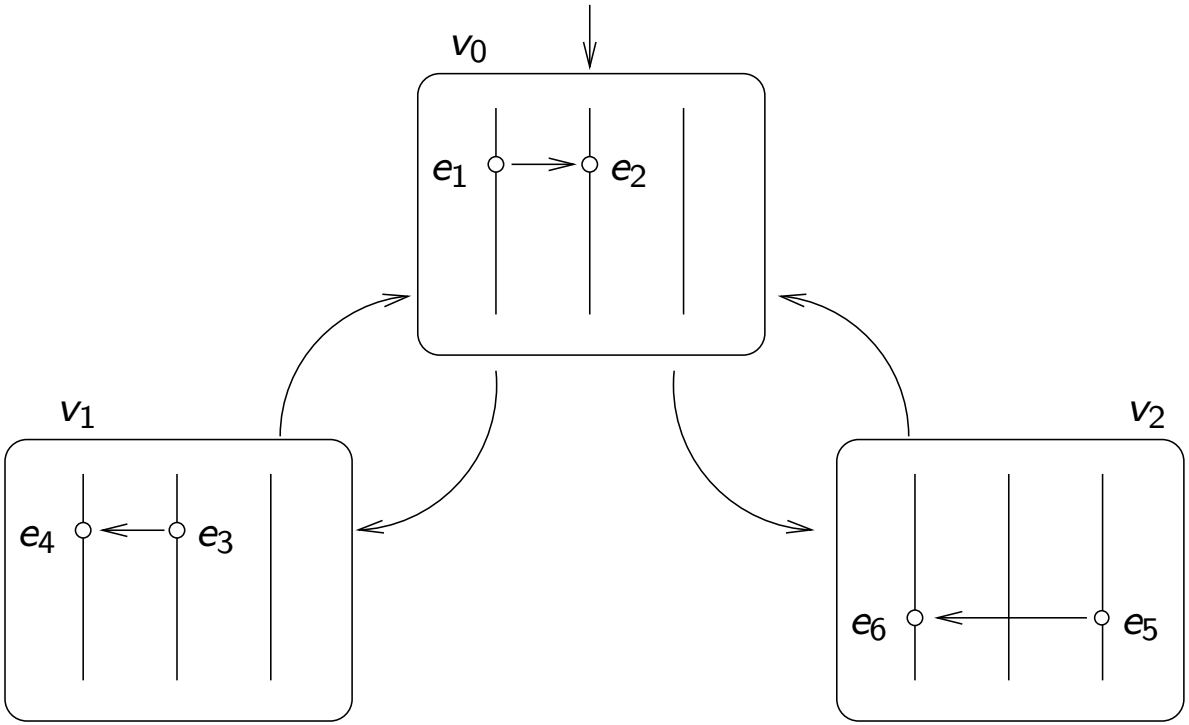

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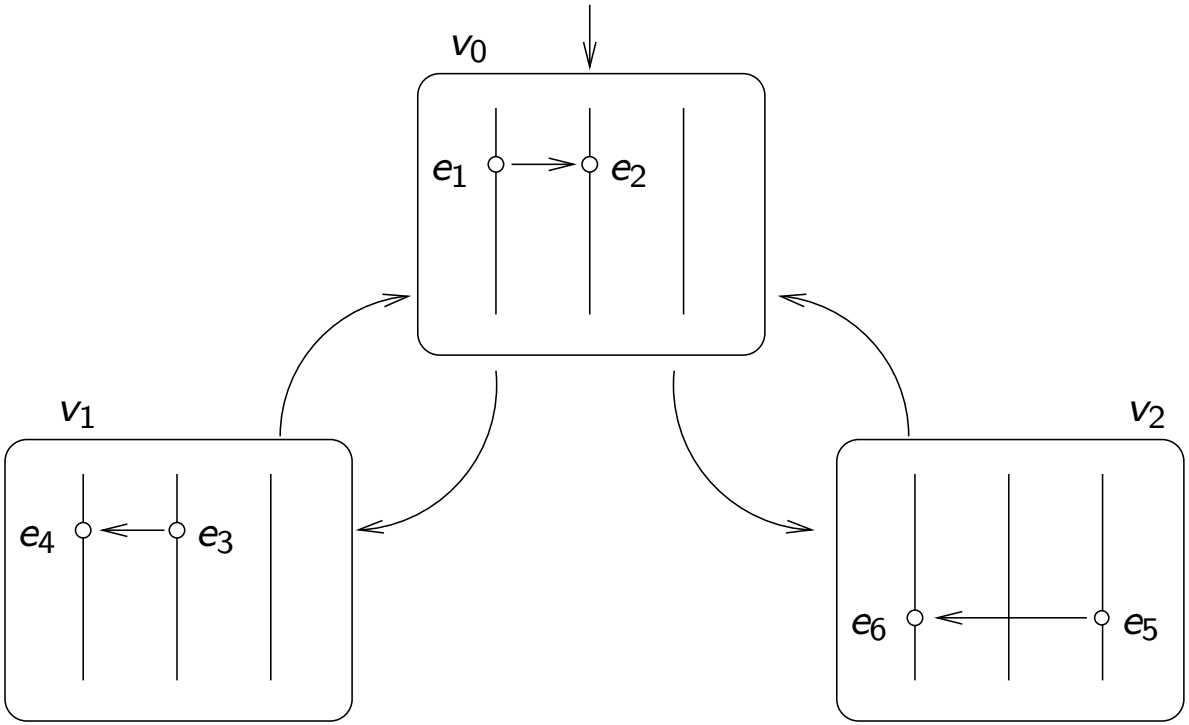

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- How many and which nodes of the path in the MSG do we need to carry with us to generate these sequentializations ?
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Theorem: Every finitely generated MSC regular language is the language of a locally synchronized MSG.

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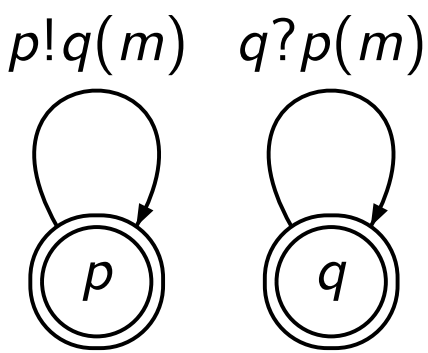
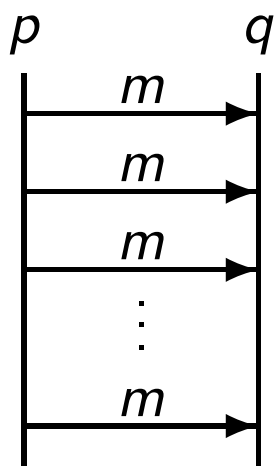
An Example: The producer–consumer system.





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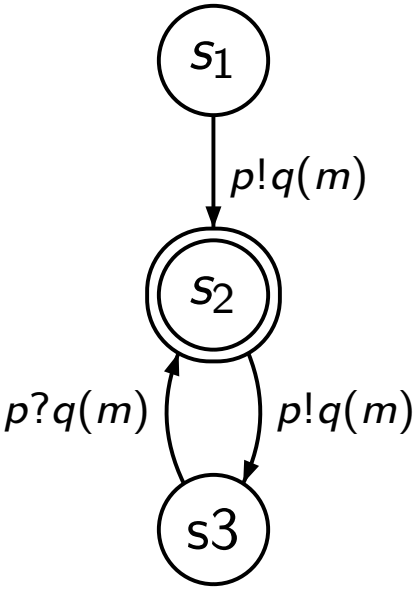

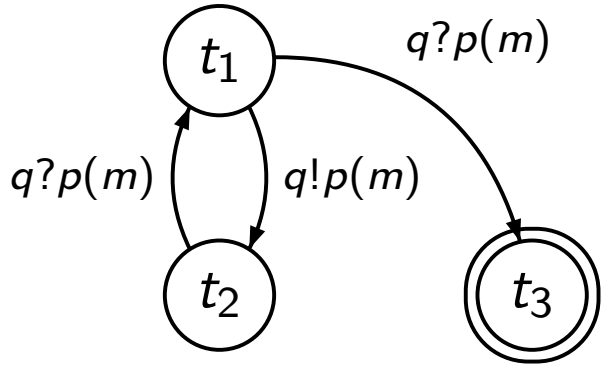

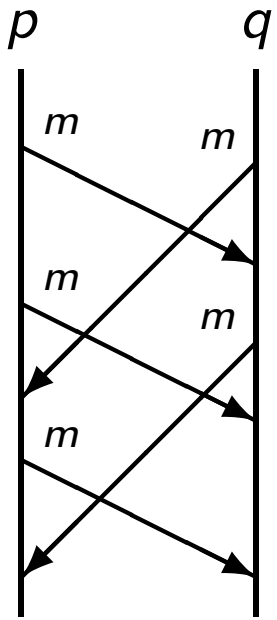

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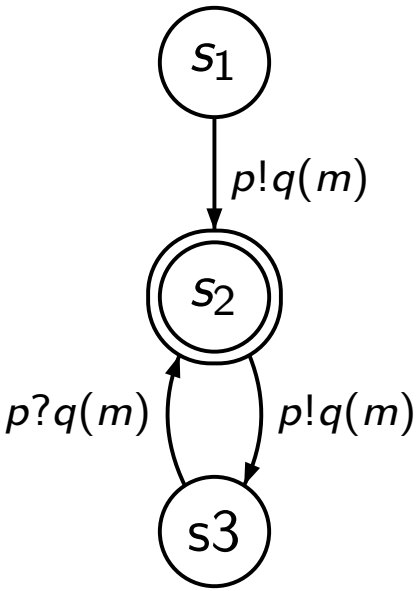

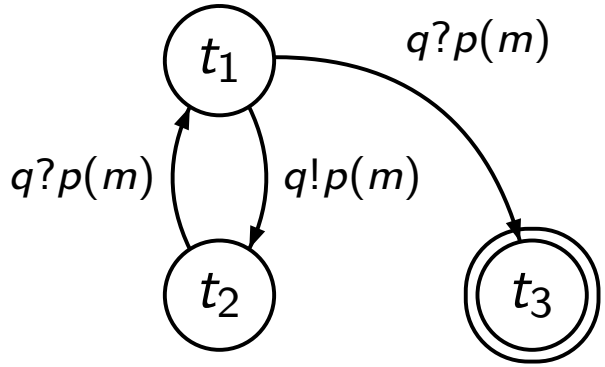

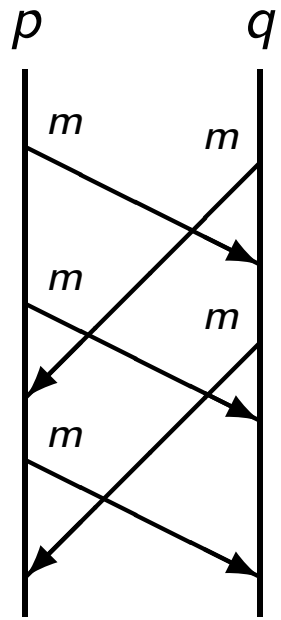

MPAs can accept languages that cannot be generated by MSGs.

MSGs to MPAs

When are MSG languages implementable using MPAs?

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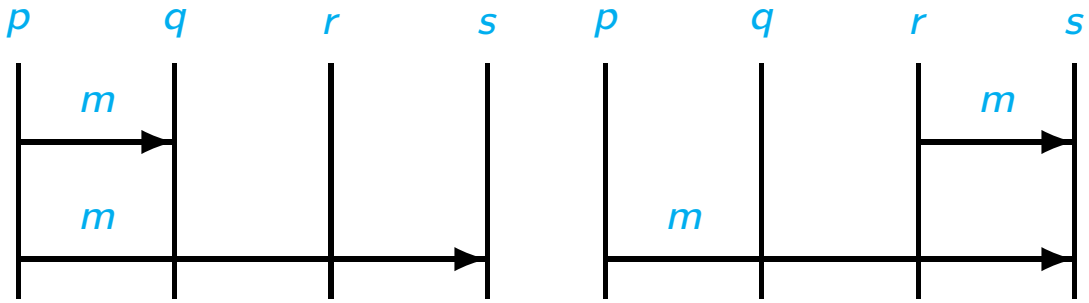
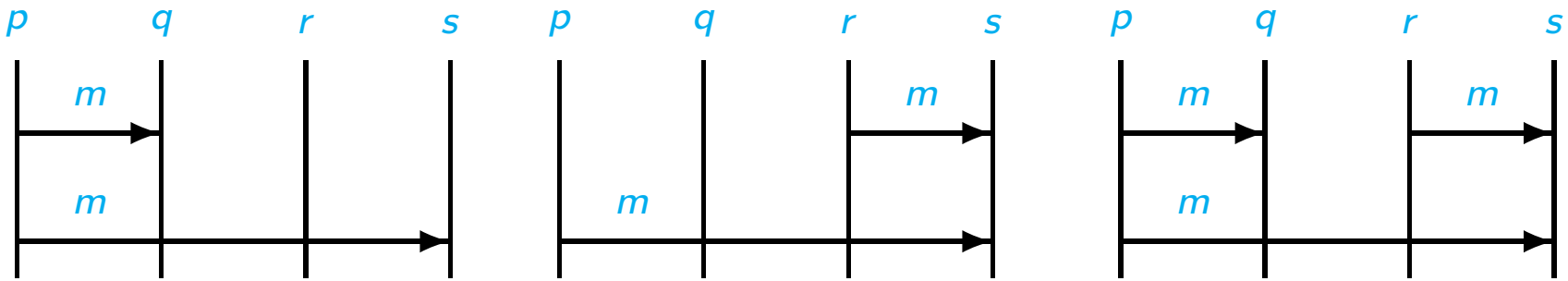
- An MSG language is an MPA language if and only if every MSC implied by its language also belongs to its language (it is implied closed).

Implied closure



M_1 M_2

Implied closure



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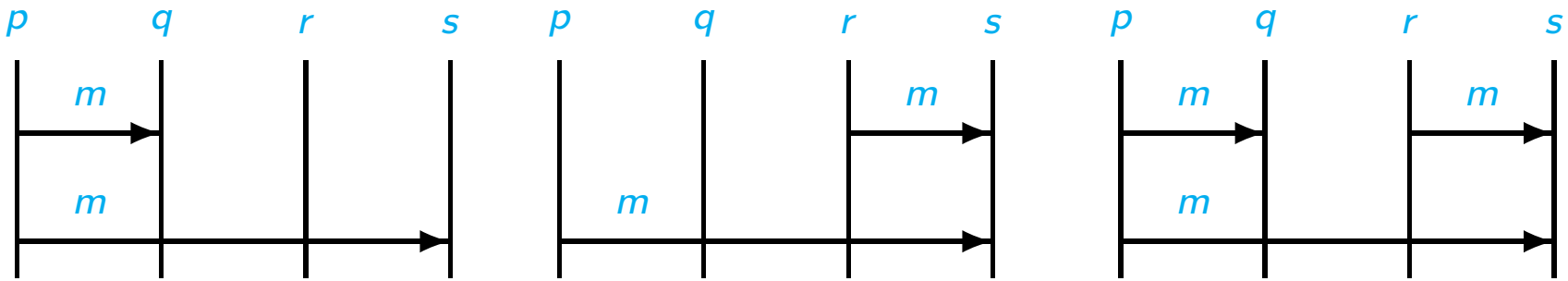

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Implied closure

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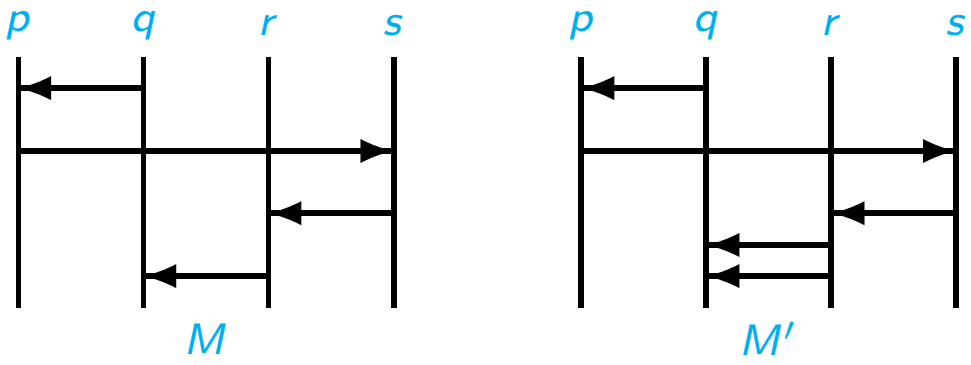

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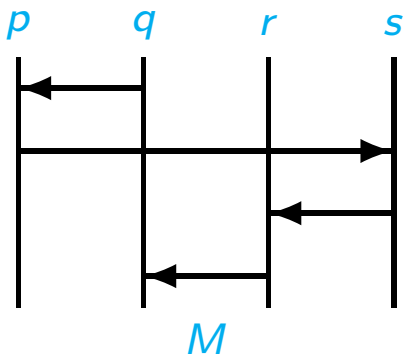

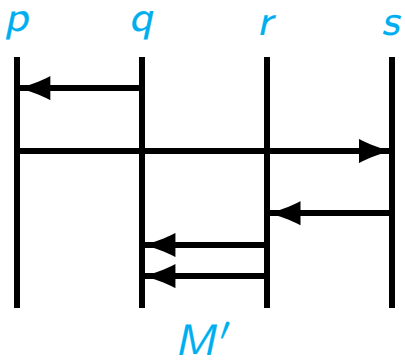

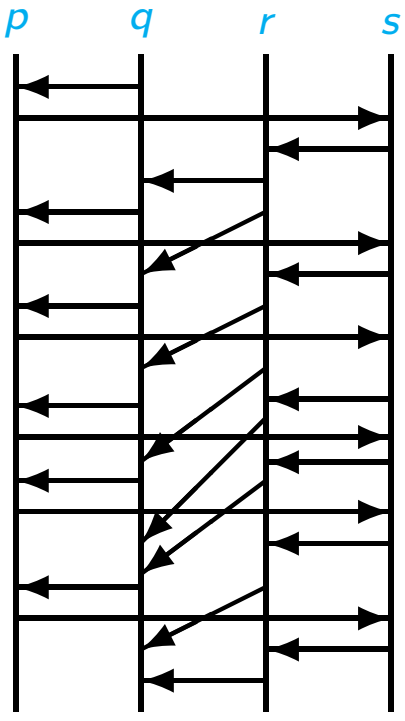

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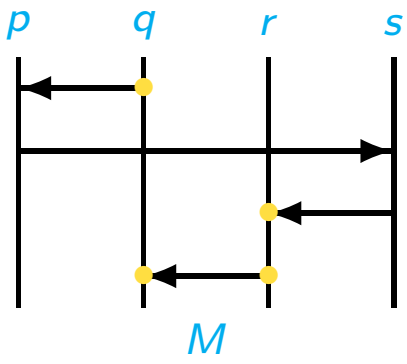

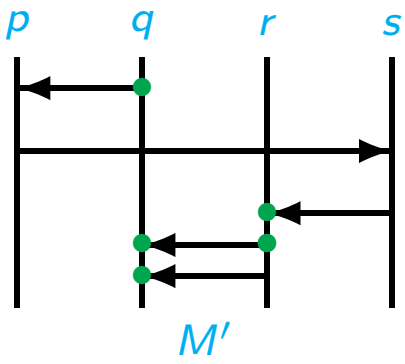

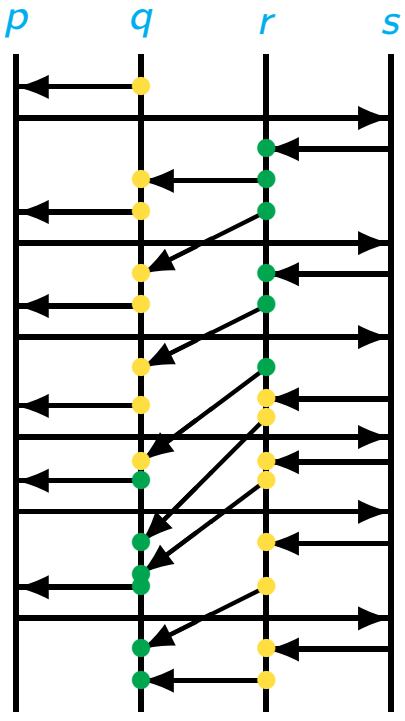

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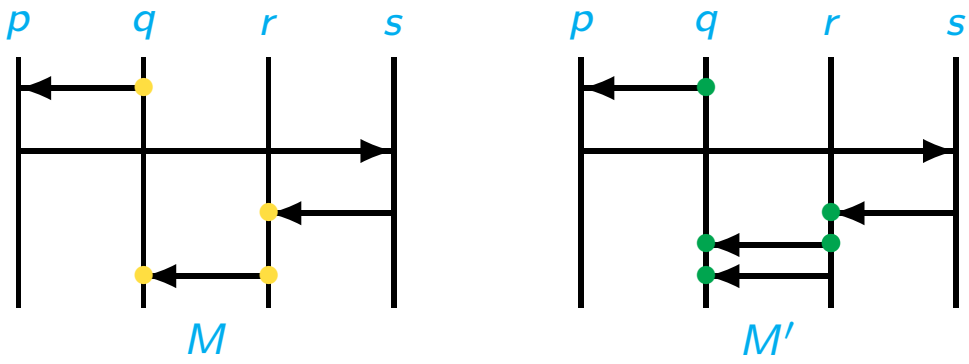
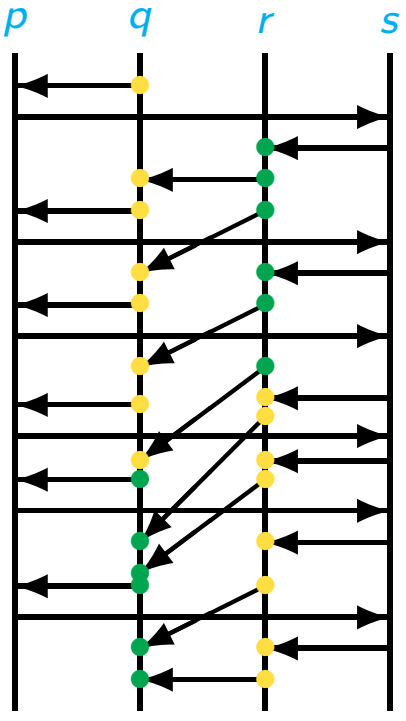


Confusing $M^{2k}M'^{k}$ and $M'^{k}M^{2k}$ generates upto k messages in $p \longrightarrow s$ channel



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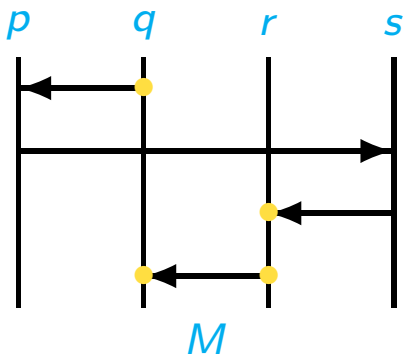

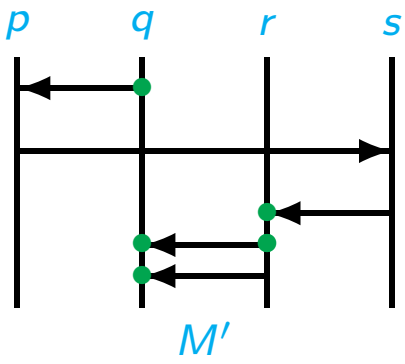

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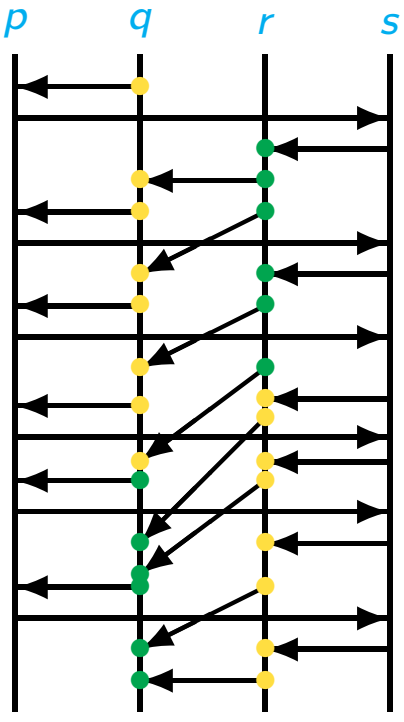
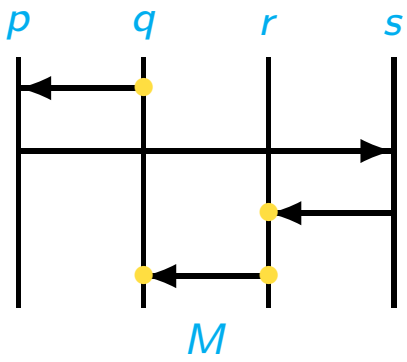
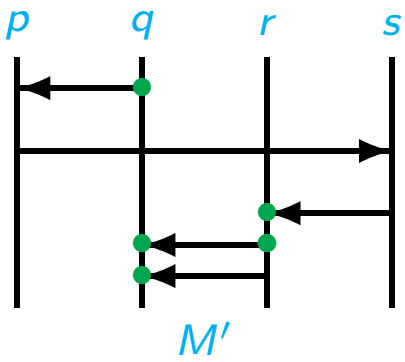


Theorem: Checking whether the language of an MSG is implied closed is undecidable. It is undecidable even for locally synchronized MSGs.

Implied closure





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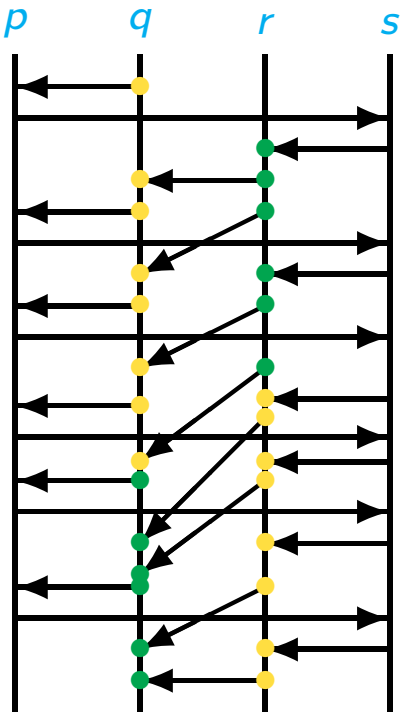

Theorem: Checking whether the language of an MSG is implied closed is undecidable. It is undecidable even for locally synchronized MSGs.

A weaker notion of implementability yields interesting positive results.

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- Checking whether the language of an MPA is regular is undecidable.

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  Are all the negative instances avoided?

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- Given B, we can effectively construct $Lin^B(L)$ consisiting of all the B bounded linearizations of MSCs in L.

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[One of the many results best proved via a translation to Mazurkiewicz traces.]

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In the next lecture...

- A distributed synthesis theorem w.r.t. a weaker notion of implementability.
- Generalizing the decidability of model-checking beyond GC-MSGs.
- Generalizing the distributed synthesis theorem.
- Monadic Second order Logic (MSO) over MSCs and its relationship to regularity and MPAs.
- Extending MSCs with time.