Local Testing of Message Sequence Charts is Difficult

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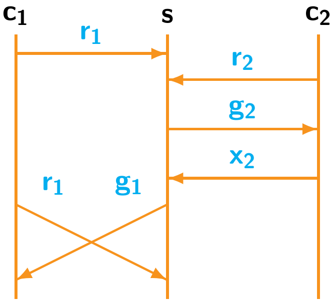
ENS de Cachan
29 May 2007

Message sequence charts (MSC)

- Telecommunications
- Describes a pattern of interaction (a Scenario)
- Attractive visual formalism
- Messages sent between communicating agents
- UML
  - Sequence diagrams
  - Interaction between objects
    e.g., method invocations etc

Message sequence charts: Partial Orders

Two clients and a server



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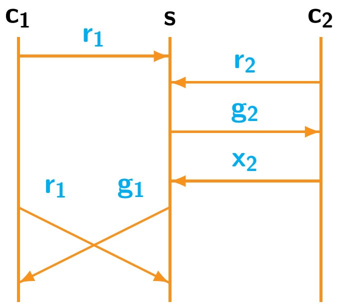

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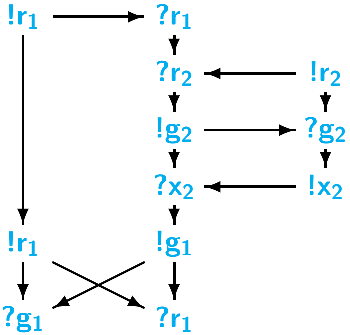

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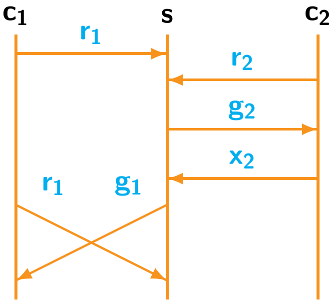

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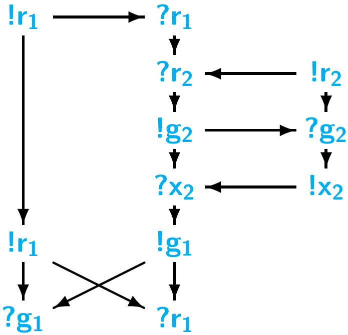

All channels are assumed to be FIFO.

MSC can be regenerated from any one sequentialization.

Collections of MSCs

- Often need to specify a collection of scenarios
- Finite collection can be exhaustively enumerated
- Infinite collection needs a generating mechanism

High level MSCs (HMSCs)

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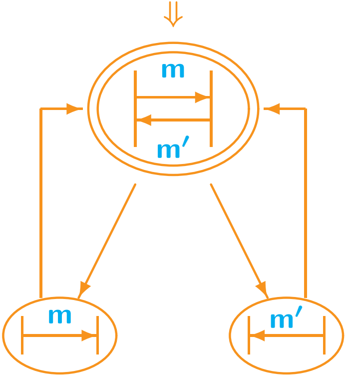

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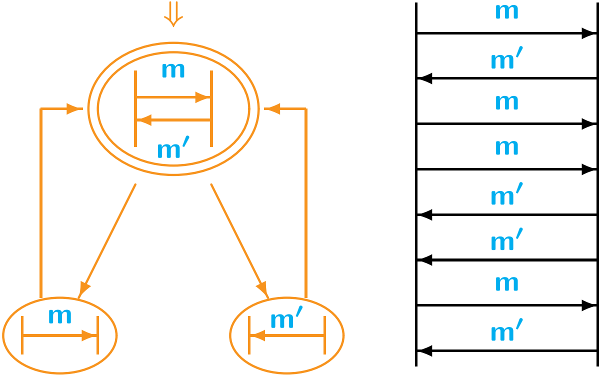

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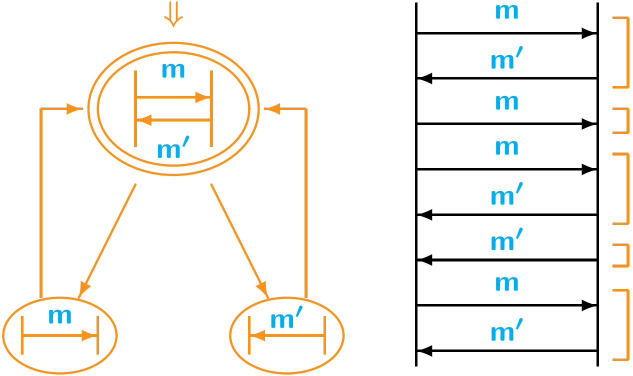

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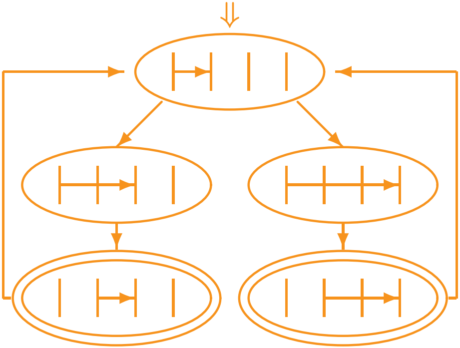

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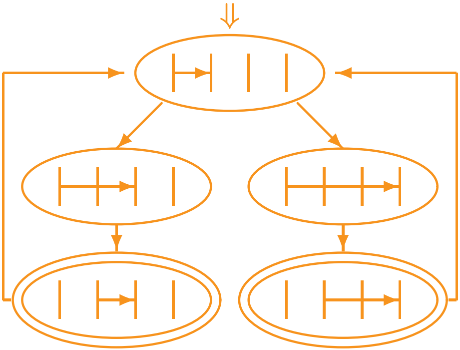

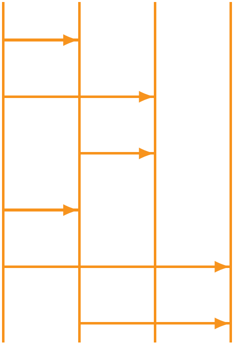

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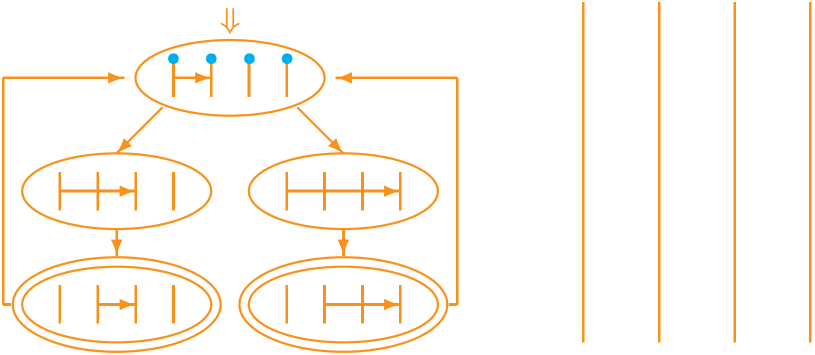

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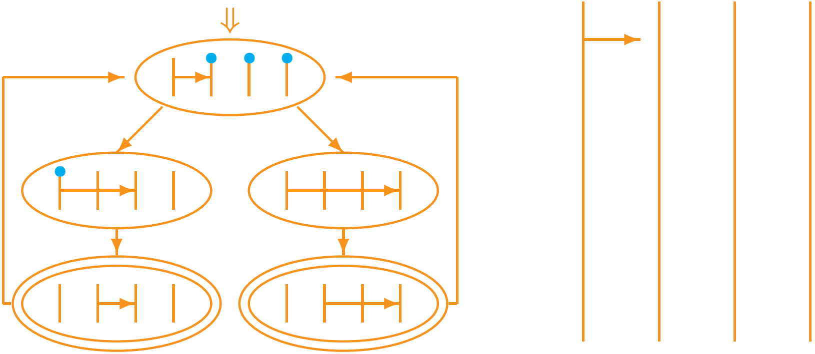

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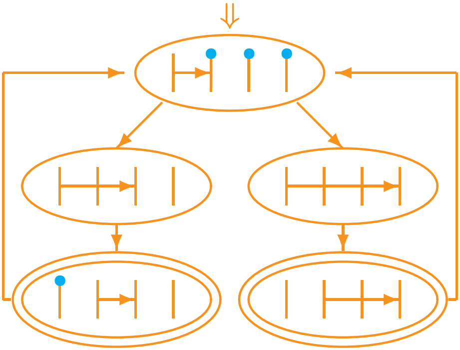

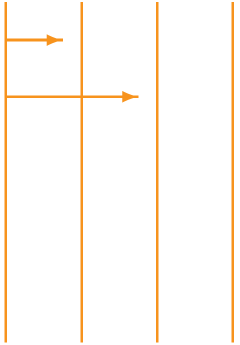

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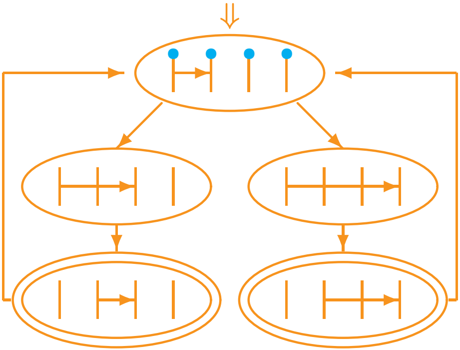

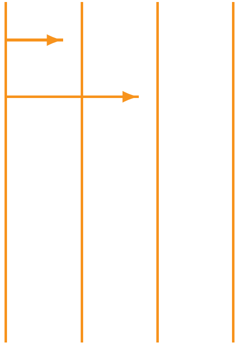

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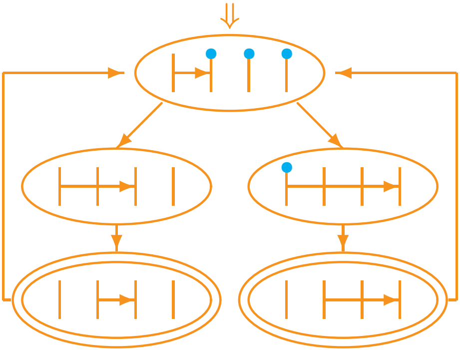

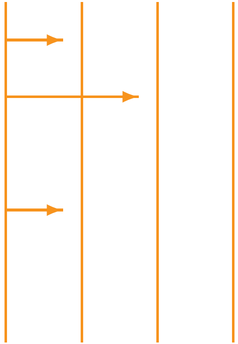

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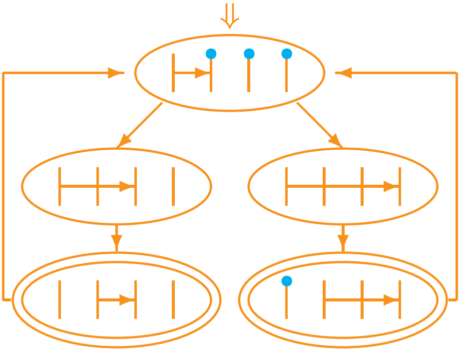

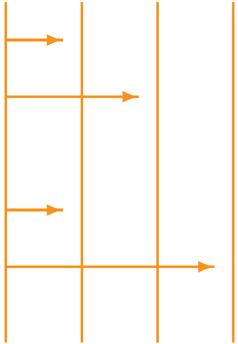

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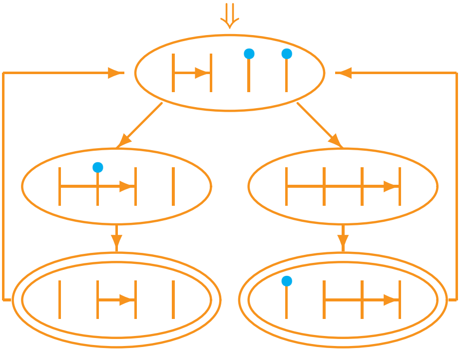

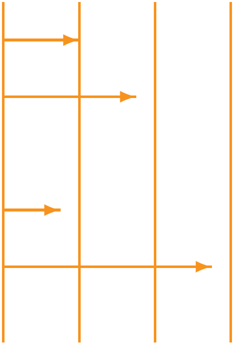

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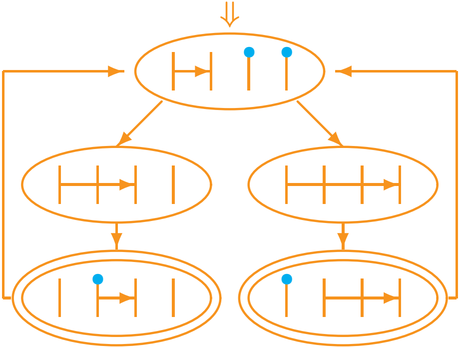

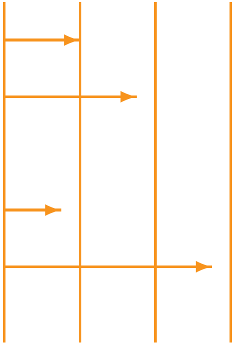

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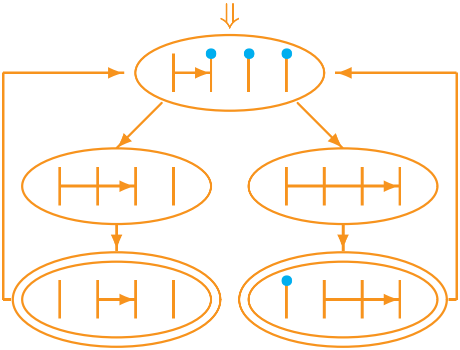

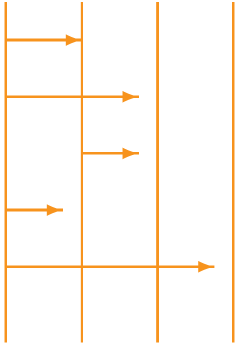

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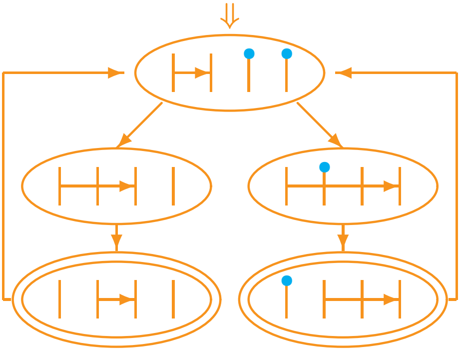

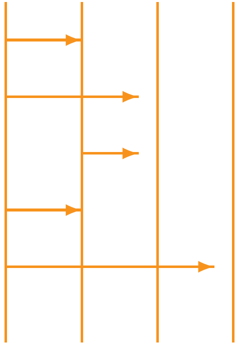

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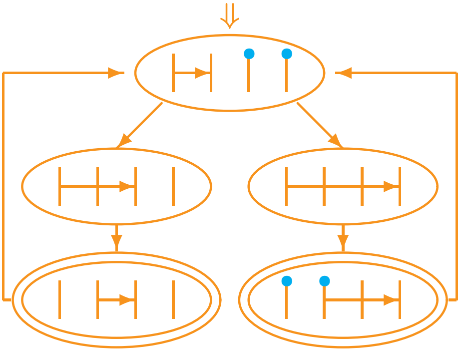

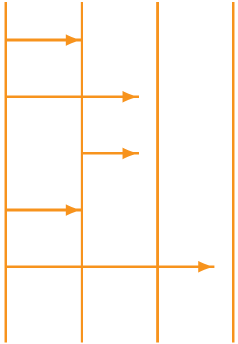

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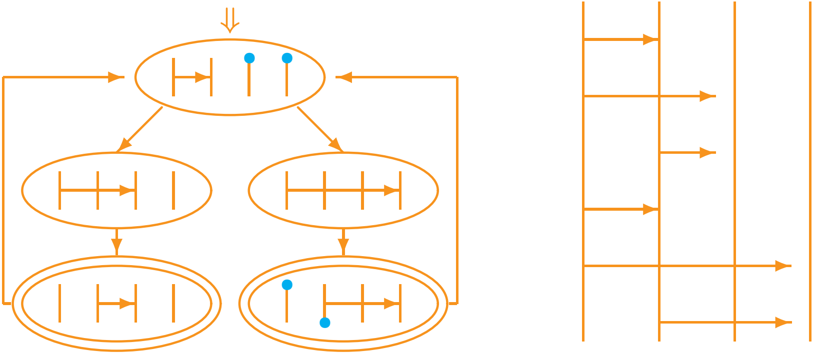

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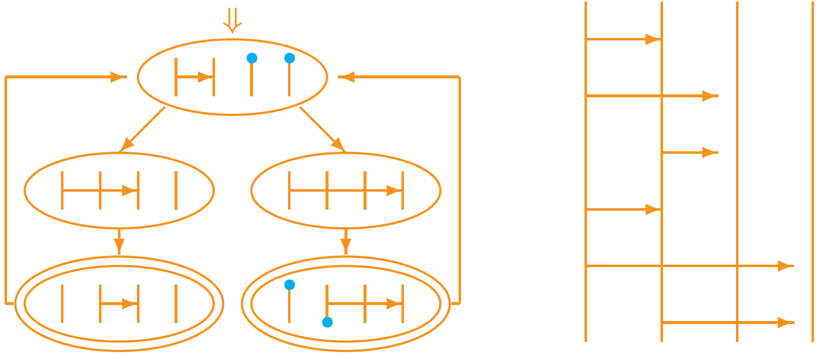
- "Executing" HMSC may require unbounded history



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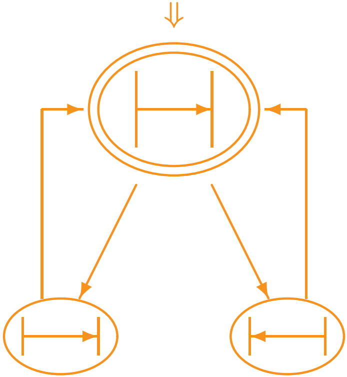
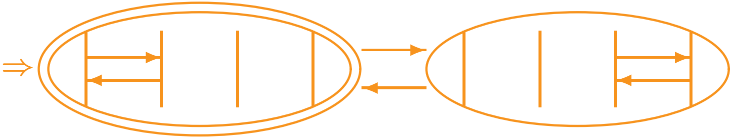


HMSCs and regularity

- HMSC specifications may not be regular
- Problem 1 Unbounded buffers
- Problem 2 Global synchronization yields context-free behaviours



- Sufficient structural conditions on HMSCs to guarantee regularity . . . [AY99,MP99]

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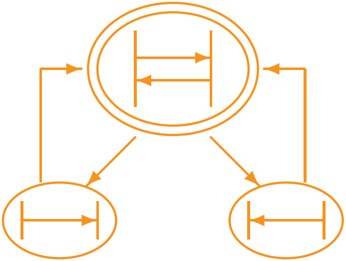

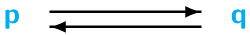

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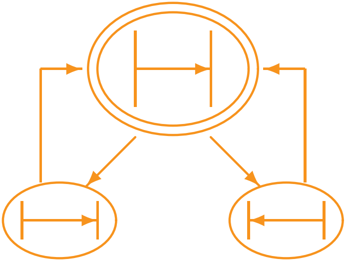

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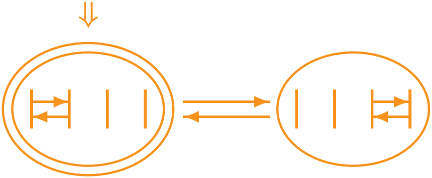

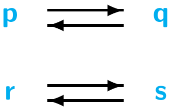

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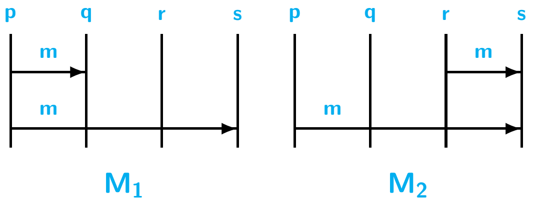
Will local testing suffice to check (regular) HMSC languages?

Implied scenarios [AEY, ICSE '00]



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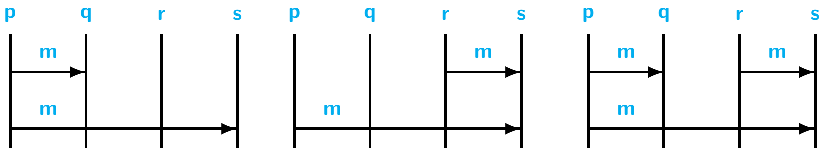

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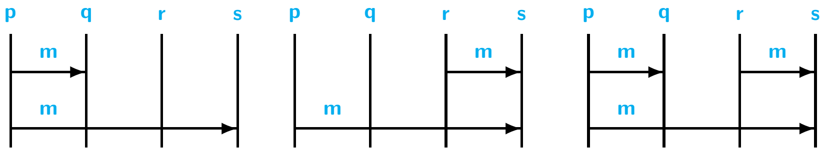

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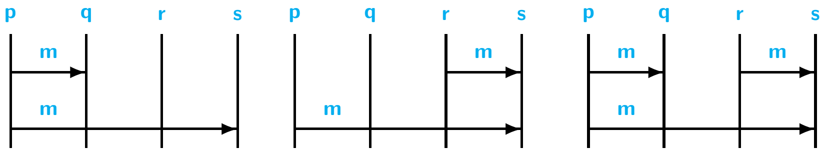

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- An MSC language is weakly realizable if it is closed with respect to implied MSCs

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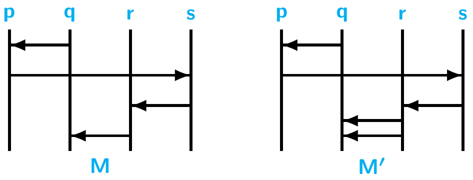

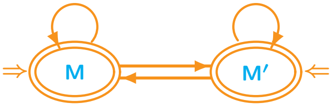

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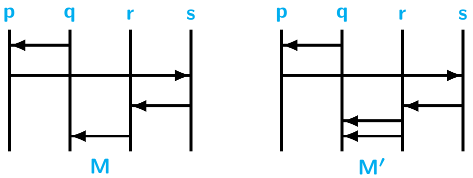

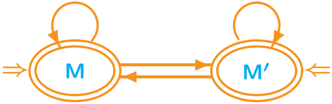

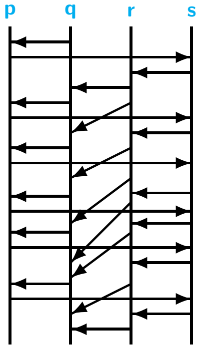

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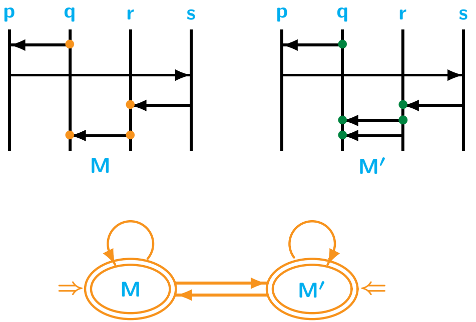
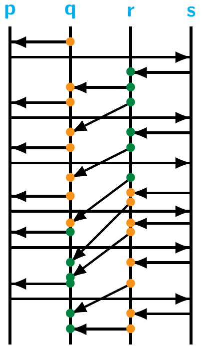
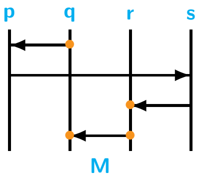
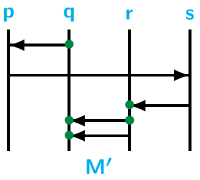
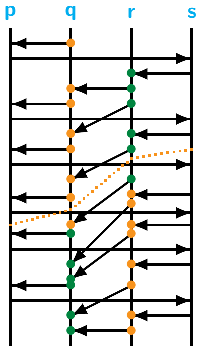




- Even for regular MSC languages, checking weak realizability is undecidable! [AEY, ICALP '01]
- Even if the original language has bounded channels, its weak closure may not







Confusing $M^{2k}M'^{k}$ and $M'^{k}M^{2k}$ generates upto k messages in $p \longrightarrow s$ channel

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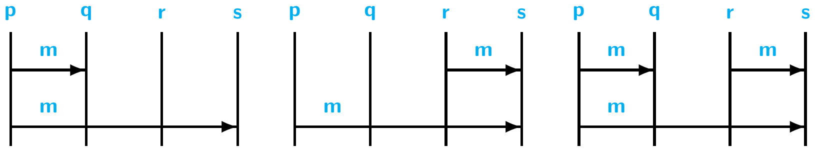

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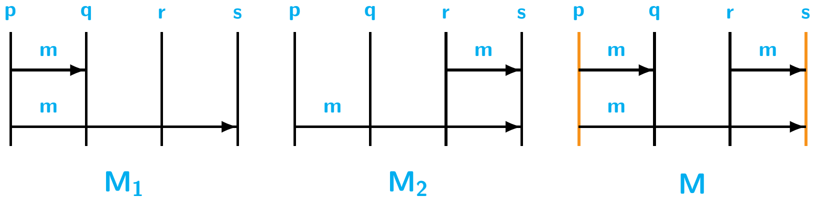

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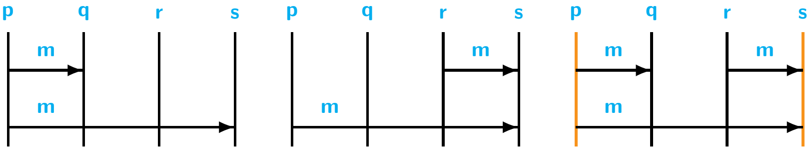

$\mathbf{M_1}$ $\mathbf{M_2}$ $\mathbf{M}$

$\mathbf{M}$ is detected as an illegal MSC.

Observers recording multiple processes can detect more violations.

Combined Observations ...

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- Observer $\mathbf{i}$ records the events on the processes in the set $\mathbf{P_i}$.

Given a HMSC $\mathbf{G}$ can we decide whether its language is testable by the observers $(\mathbf{P_i})_{1 \leq i \leq r}$?

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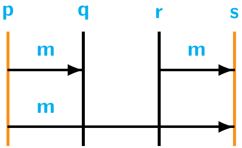

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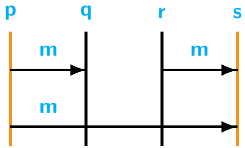

M

The $\{p, s\}$-observation of **M** is (**p!q(m)p!s(m)**,**s?r(m)s?p(m)**).

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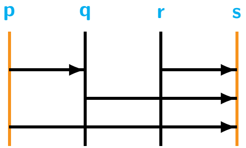

$\mathbf{M'}$

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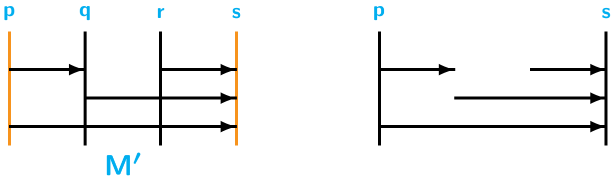

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- For a language $\mathbf{L}$, the $\mathbf{P}$-observation of $\mathbf{L}$ is given by $\mathbf{L}{\restriction}_{\mathbf{P}} \quad = \quad \{\mathbf{M}{\restriction}_{\mathbf{P}} \mid \mathbf{M} \in \mathbf{L}\}$

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  Weak Realizability is 1-testability

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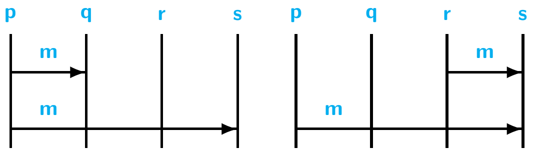

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The set $\{M_1, M_2\}$ is 2-testable but not 1-testable.

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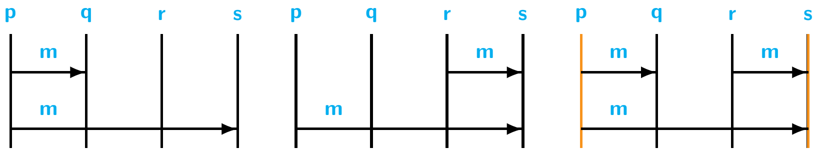

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$\mathbf{k}$-testability ...

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    $\mathbf{k}$-testability is undecidable for all $\mathbf{1} \leq \mathbf{k} < \mathbf{n}$ and all $\mathbf{n} > \mathbf{1}$.
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    $\mathbf{k} = \mathbf{n}$

Modified PCP

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1. the given instance has a solution if and only if, in addition to the above, for each $\mathbf{l} < \mathbf{m}$, $\mathbf{w_1 w_{i_2} \ldots w_{i_l}}$ is a proper prefix of $\mathbf{v_1 v_{i_2} \ldots v_{i_l}}$.
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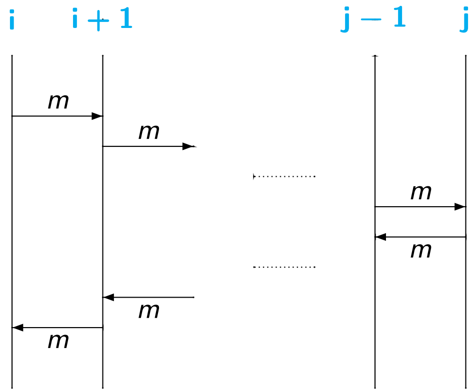
The MPCP problem is undecidable.

Undecidability

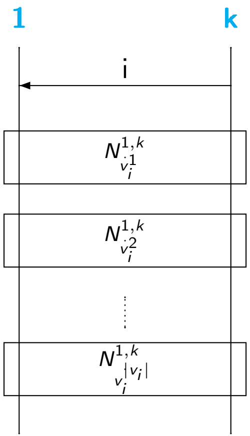
For each pair of processes $\mathbf{i, j}$ with $\mathbf{i < j}$ the MSC $\mathbf{N^m_{ij}}$ is as follows:



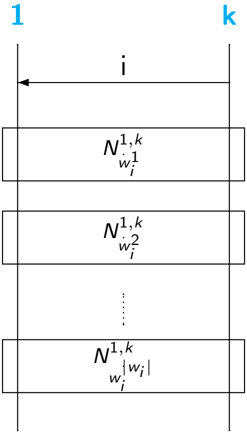
Undecidability ...

For each pair (v_i, w_i) we define three MSCs associated with the pair.

The MSC $\mathbf{M_{v_i}}$

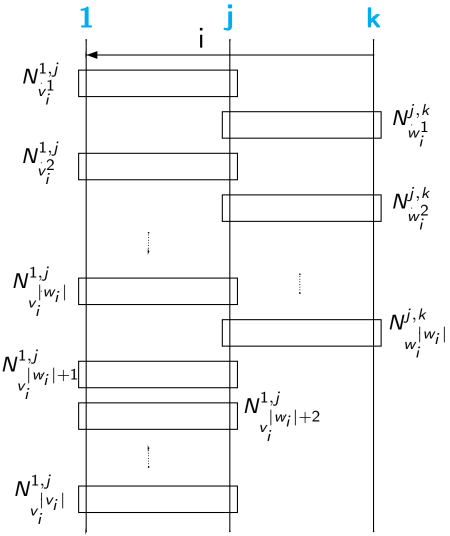


The MSC $\mathbf{M_{w_i}}$



Undecidability ...

The MSC $\mathbf{M^{j}_{v_i w_i}}$



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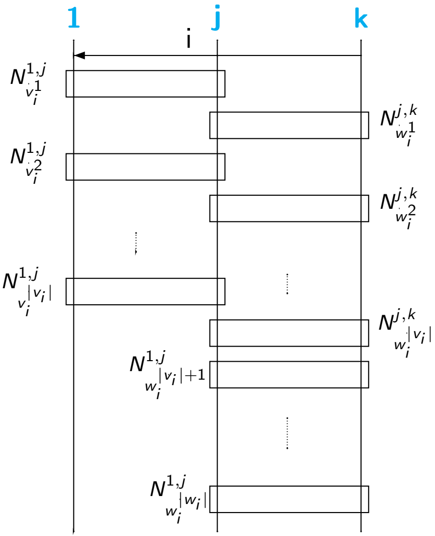


Undecidability ...

Let

$$\begin{array}{rcl} L_v & = & M_{v_1}.\{M_{v_i} \mid 1 \leq i \leq r\}^* \\ L_w & = & M_{w_1}.\{M_{w_i} \mid 1 \leq i \leq r\}^* \\ L^j_{vw} & = & M^j_{v_1 w_1}.\{M^j_{v_i w_i} \mid 1 \leq i \leq r\}^* \end{array}$$

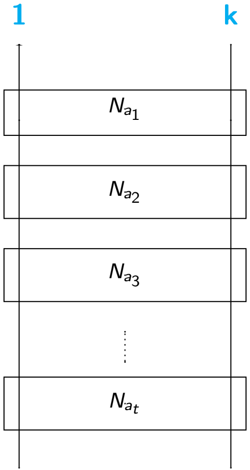
and

$$\mathbf{L_\Delta} \;=\; \mathbf{L_v} \cup \mathbf{L_w} \cup \bigcup_{\mathbf{1<j<k}} \mathbf{L^j_{vw}}$$

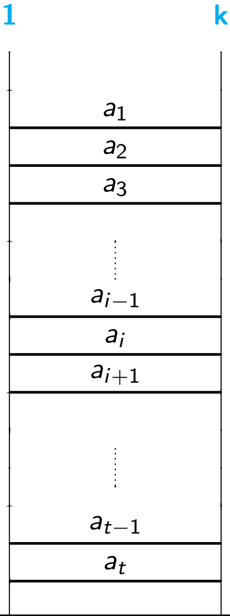
Then, $\mathbf{L_\Delta}$ has a $\mathbf{(k-1)}$-implied scenario if and only if the given instance of MPCP has a solution.

Undecidability ...

Suppose $\mathbf{a_1 a_2 \ldots a_t} \;=\; \mathbf{v_1 v_{i_2} \ldots v_{i_m}} \;=\; \mathbf{w_1 w_{i_2} \ldots w_{i_m}}$



Undecidability ...



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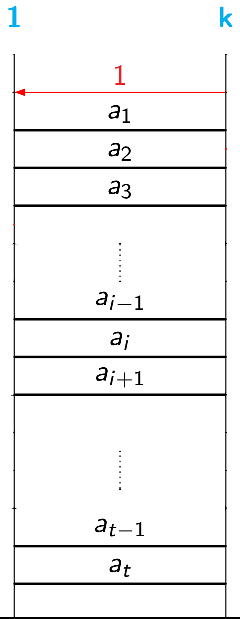

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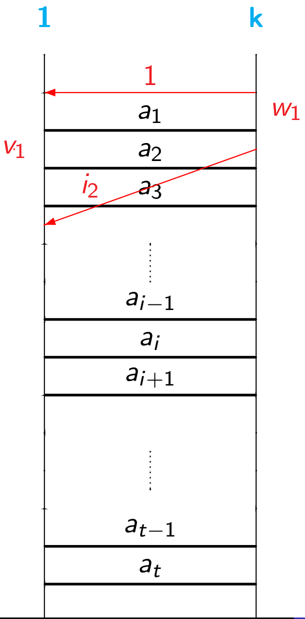

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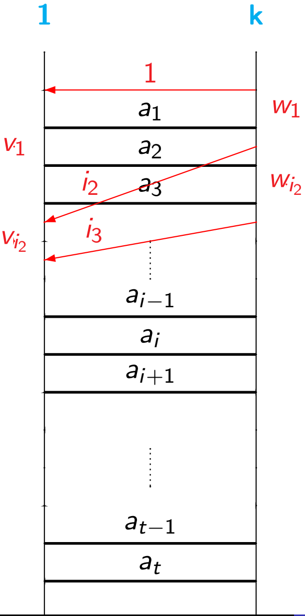

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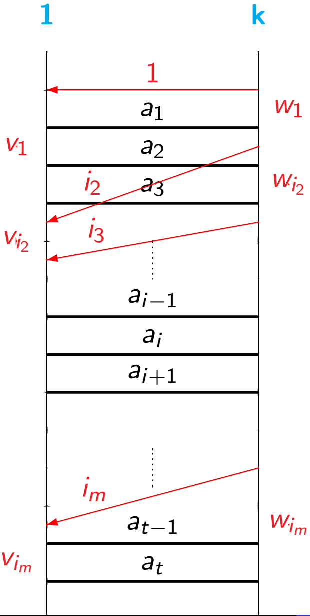
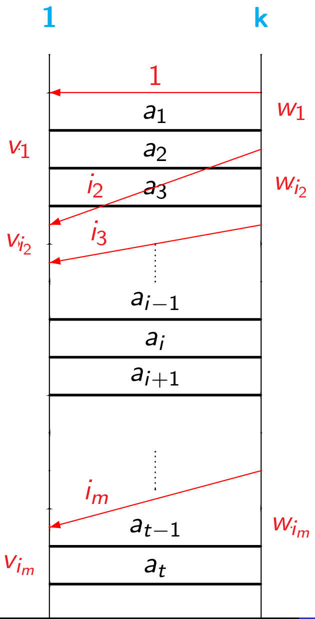


The MSC **M**

Undecidability ...



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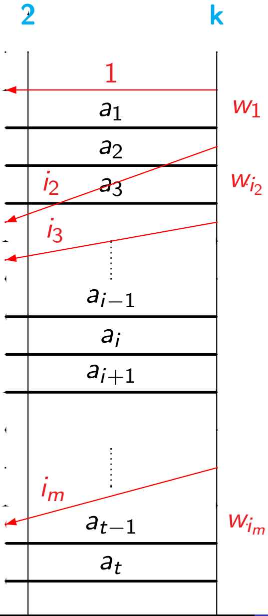
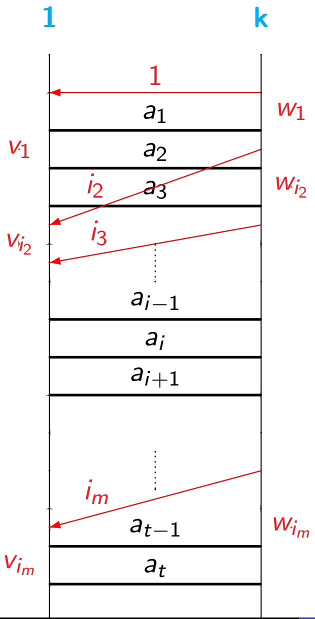


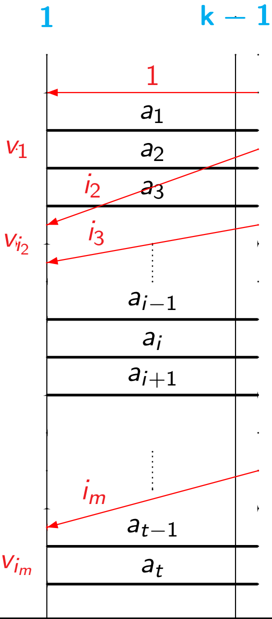
$$\mathbf{M}\restriction_{\{2,3,\ldots,k\}} \in \mathbf{L_w}\restriction_{\{2,3,\ldots,k\}}$$

Undecidability ...



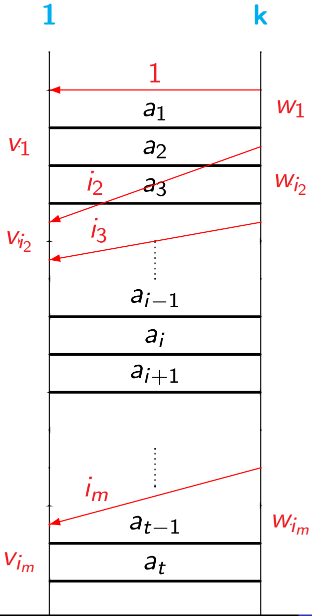
The MSC **M**

Undecidability ...



$$\mathbf{M}\restriction_{\{1,2,\ldots,k-1\}} \in \mathbf{L_v}\restriction_{\{1,2,\ldots,k-1\}}$$

Undecidability ...



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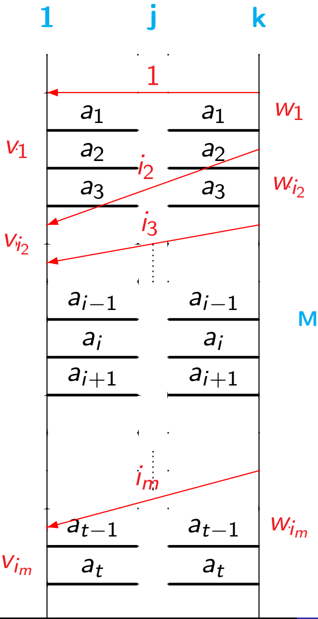
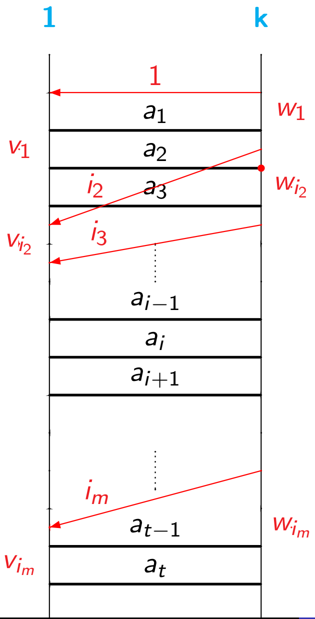


$\mathbf{M}\restriction_{\{1,\ldots,j-1,j+1,\ldots,k\}} \in \mathbf{L}^{j}_{vw}\restriction_{\{1,\ldots,j-1,j+1,\ldots,k\}}$

Undecidability ...



$\mathbf{M} \not\in \mathbf{L}_{\Delta}$

Undecidability ...

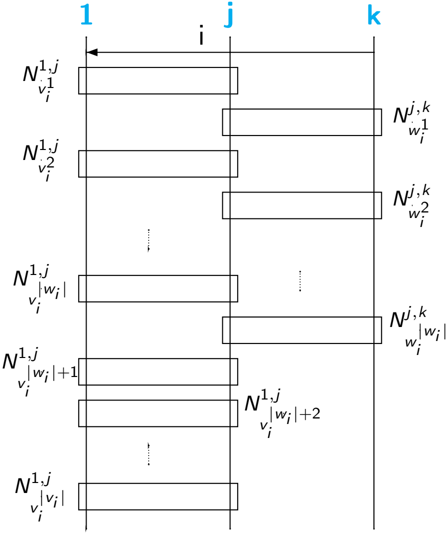
Let $\mathbf{M}$ be a $(\mathbf{k}-\mathbf{1})$-implied scenario.

Undecidability ...

Undecidability ...

Case 1: $\exists \mathbf{j} : \mathbf{1} < \mathbf{j} < \mathbf{k}.\ \mathbf{M}\restriction_{\mathbf{j}} \in \mathbf{L}^{\mathbf{j}}_{\mathbf{vw}}\restriction_{\mathbf{j}}$

Undecidability ...



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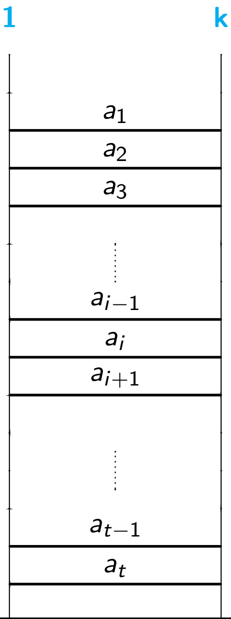

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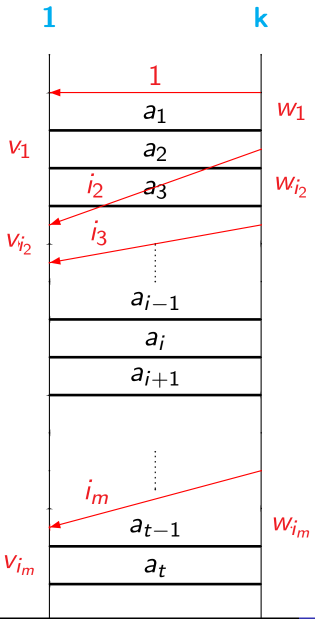

The MSC **M**

Undecidability ...

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$\mathbf{M}$ codes a solution to the MPCP.

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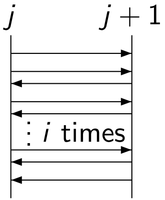
k-testability is undecidable for $1 < k < n - 1$ for all $n > 3$ even when the message alphabet is singleton.

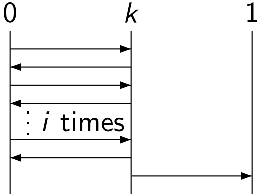
Eliminating Messages

The message **i** from a process to it neighbour can be replaced by the following MSC:



Eliminating Messages ...

The message **i** from **k** to **1** cannot be dealt with similarly. We must permit arbitrary delays in the delivery of this message.



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This also gives an algorithm to check if the **1**-closure of the HMSC is regular, since the infiniteness of the number of intermediate markings of a Net is a decidable problem.

Specification to Implementation

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  Message Passing Automata (MPA)

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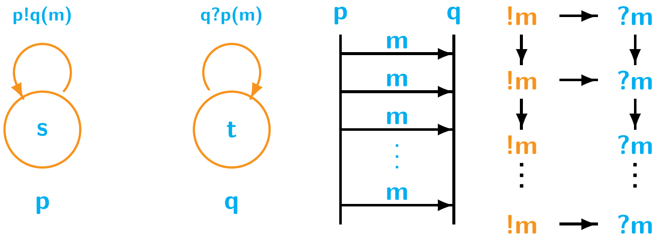

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  - Globally finite state $\Rightarrow$ channels are bounded

MPAs and Regular MSC Languages

- Message Passing Automata with bounded channels generate only regular MSC languages
- What about the converse?

Theorem: [[HMNST, I&C '05],[MNS, Concur'00]] Every regular MSC language is recognized by a deterministic message passing automaton with bounded channels.

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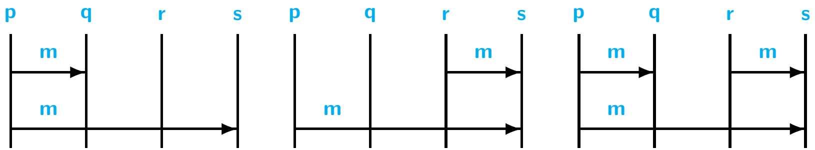

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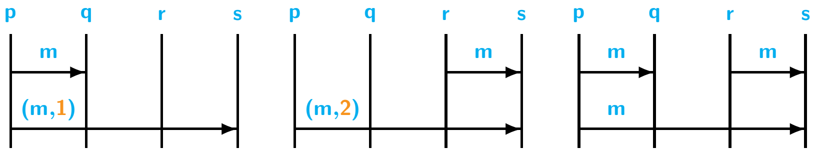

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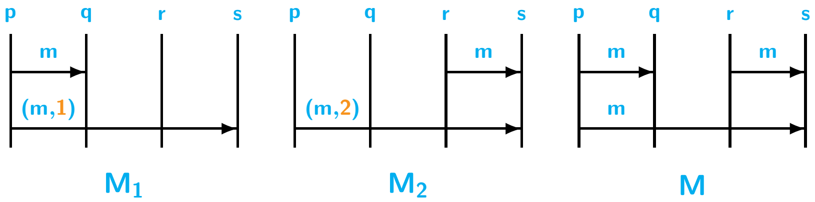

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- This rules out the implied scenario **M**

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- An MSC language is causally closed if it is closed with respect to causal implication

Theorem:[[AMNN FSTTCS'05]] The causal closure of a regular MSC language is always regular and can be effectively constructed.

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- We can effectively construct a bounded message-passing automaton with local accepting states recognizing the causal closure.

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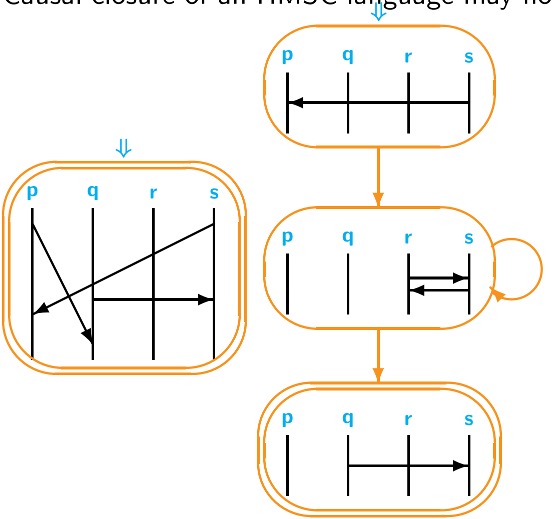

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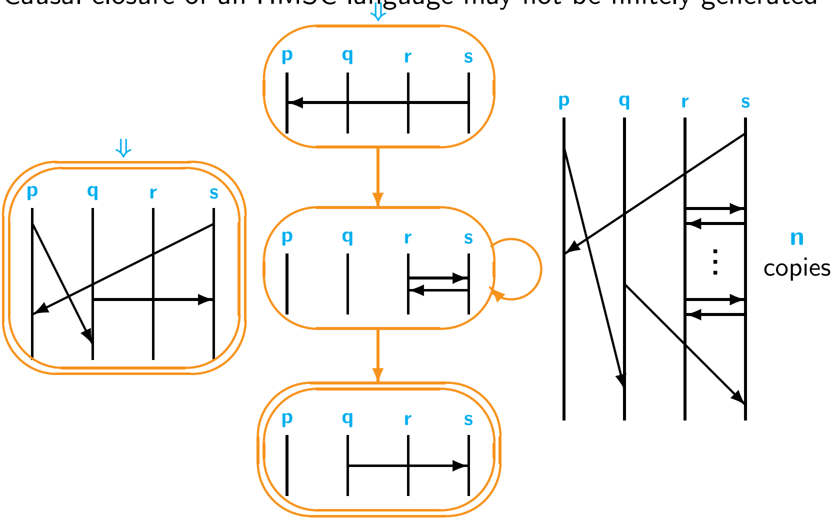

Summary and future work

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- Testability is undecidable in most situations.
- Look for sufficient conditions that indicate violation of testability.