

# Analyzing time constrained MSGs

K Narayan Kumar

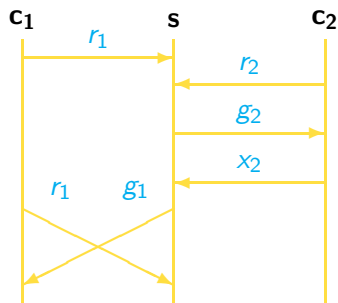
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Joint work with P Gastin, Madhavan Mukund

Chennai, 30 January 2009

# Message Sequence Charts

Two clients and a server



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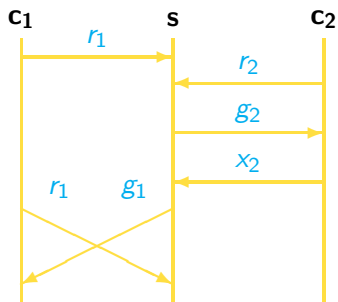
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Unfortunately, most of the results are negative.

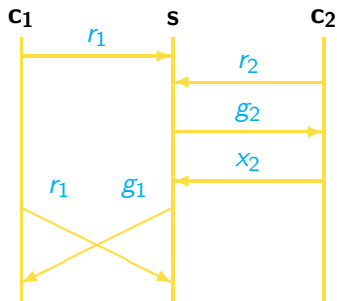
# MSCs

Two clients and a server

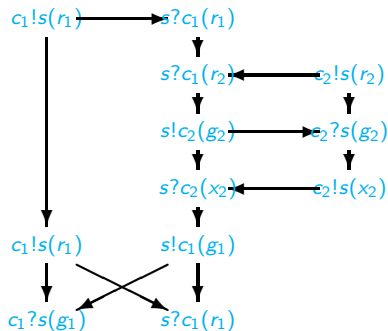


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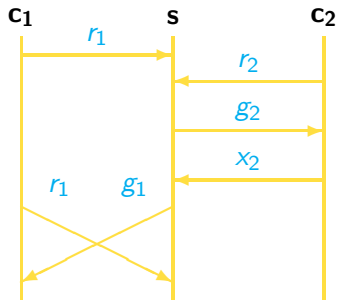


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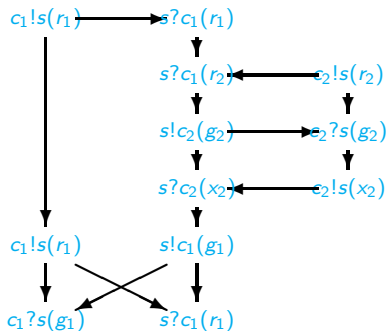


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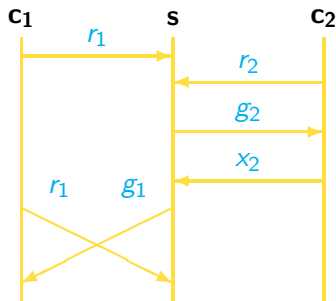
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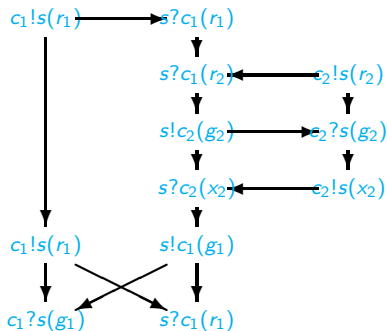
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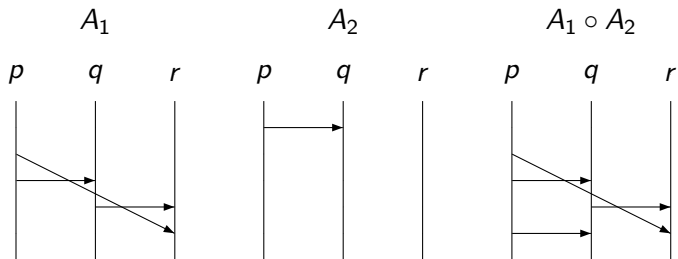


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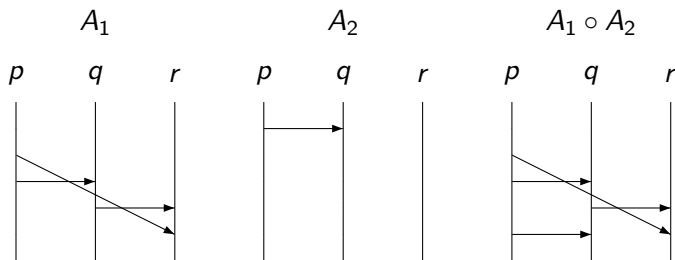


- ▶ All channels are assumed to be FIFO.
- ▶ An MSC can be regenerated from any one sequentialization.

# Concatenation of MSCs



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**$p!r \ p!q \ q?p \ q!r \ r?q \ p!q \ q?p \ r?p$**

is a sequentialization of of  $A_1 \circ A_2$ .



# Message Sequence Graphs

- ▶ A finite state automaton

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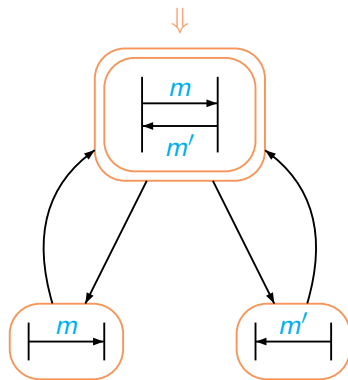
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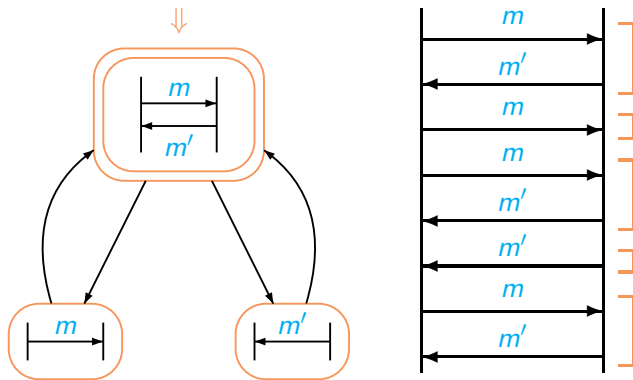
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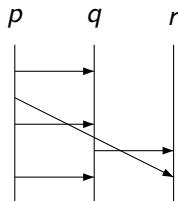


# Boundedness

A sequentialization of an MSC is  $B$ -bounded if no channel has more than  $B$  messages at any point.

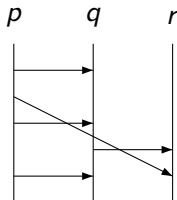
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The linearization

**$p!q \ q?p \ p!r \ p!q \ q?p \ q!r \ r?q \ p!q \ q?p \ r?p$**

is 1-bounded while the linearization

**$p!q \ p!r \ p!q \ p!q \ q?p \ q?p \ p!r \ q?p \ r?q \ r?p$**

is 3-bounded.



## Boundedness ...

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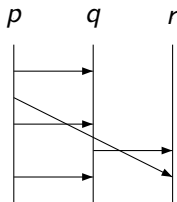
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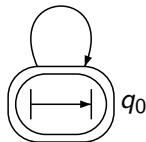
An existentially 1-bounded and universally 3-bounded MSC.

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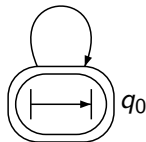
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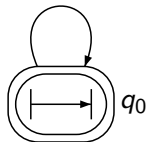
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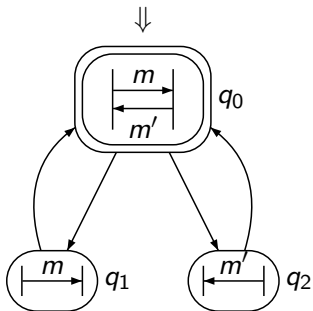
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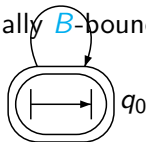


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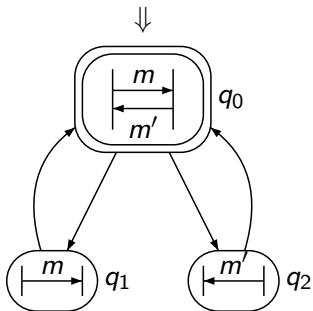


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An MSG is **existentially bounded** if there exists a  $B$  such that every MSC it generates is existentially  $B$ -bounded.



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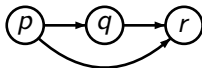
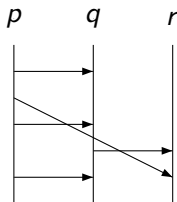
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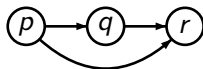
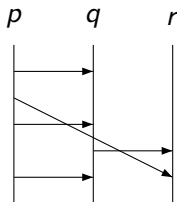
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Nodes are the processes. An edge from  $p$  to  $q$  if there is a message from  $p$  to  $q$ .



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An MSG is bounded if and only if every the MSC generated by every loop has a communication graph that is a disjoint union of SCCs.

# Adding time to scenarios



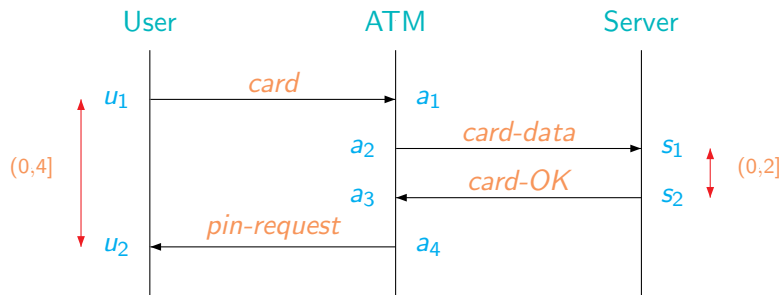
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- ▶ Time constrained Message Sequence Graphs
  - ▶ Generate infinite families of time constrained MSCs

# MSCs with time constraints



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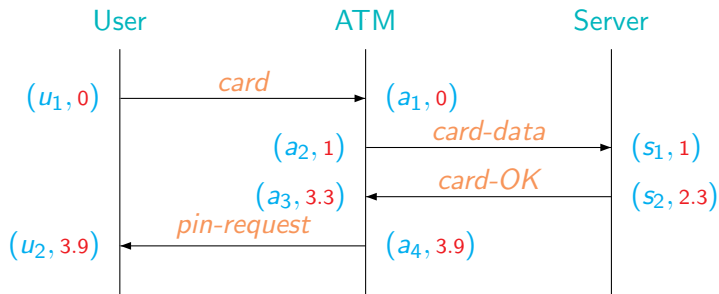
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    - ▶  $e$  is  $p!q(m)$  and  $e'$  is corresponding receive  $q?p(m)$

# A timed behaviour



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- ▶ TC-MSC  $T \Rightarrow L(T)$ , set of timed MSCs that cover  $T$

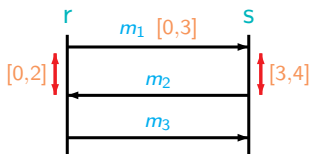
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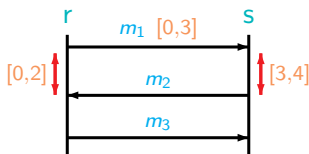
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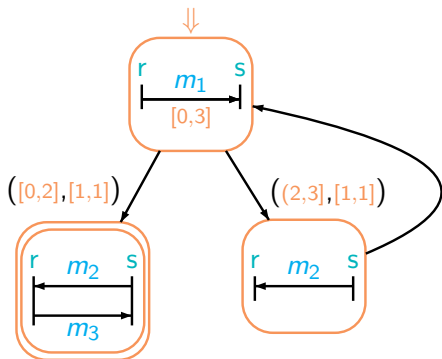
- ▶ The set of timed MSCs covering a TC-MSC may be empty.
- ▶ A TC-MSC is said to be **realizable** if it is covered by at least one timed MSC.





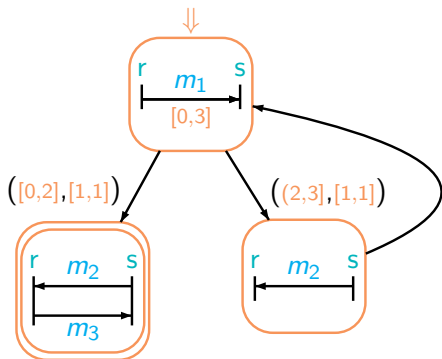
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- ▶ States labelled by time constrained MSCs
- ▶ Local constraints for each process along edges
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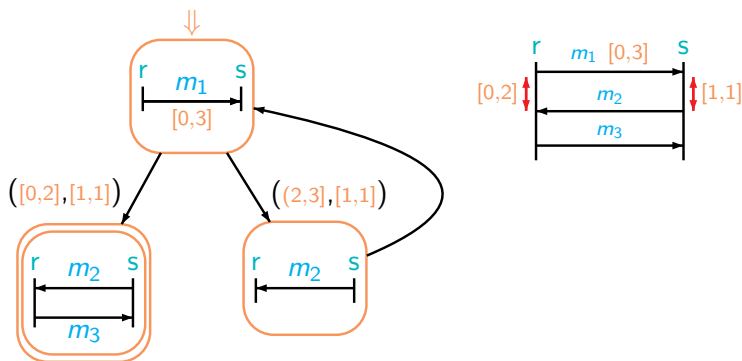
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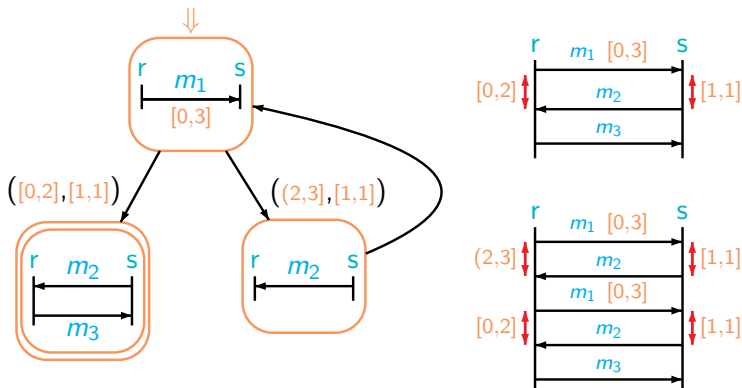
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Given a TC-MSG  $G$  and a state  $q$  in  $G$ , does there exist a path  $q_0 q_1 \dots q_k = q$  from an initial state  $q_0$  such that the TC-MSG generated by this path is realizable ?

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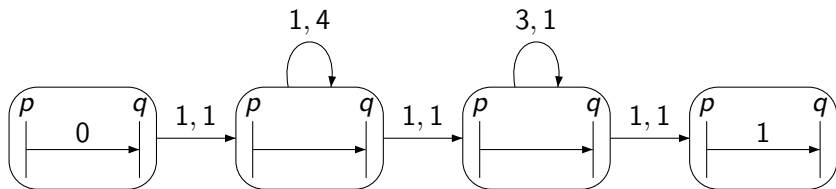
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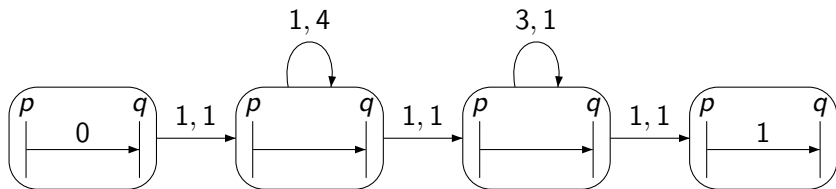
This problem is trivial for ordinary MSGs.



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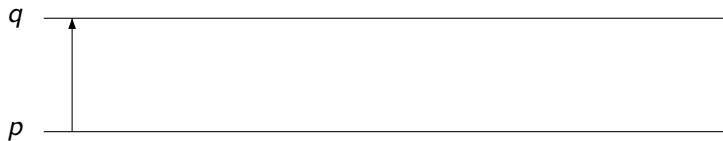
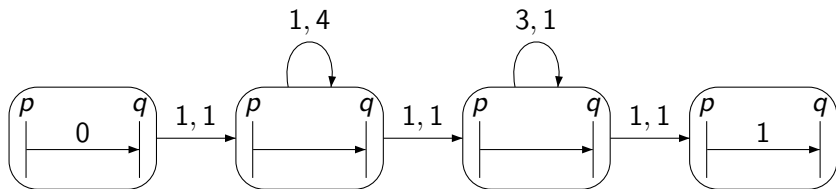
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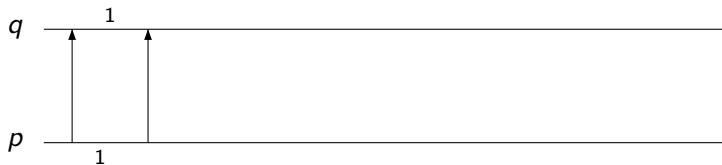
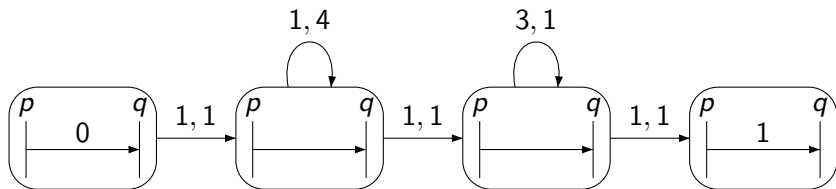
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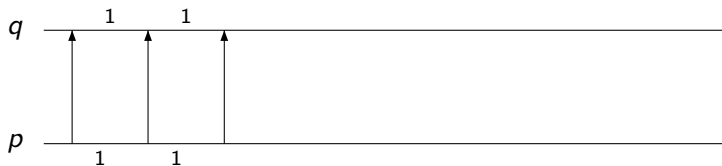
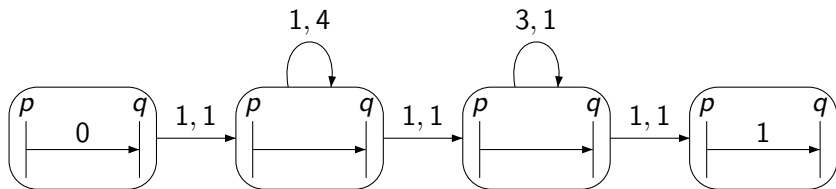
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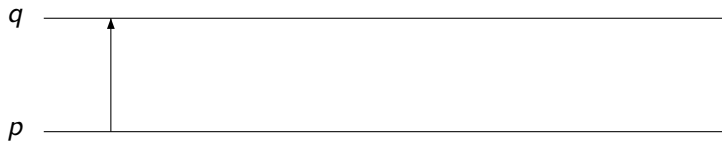
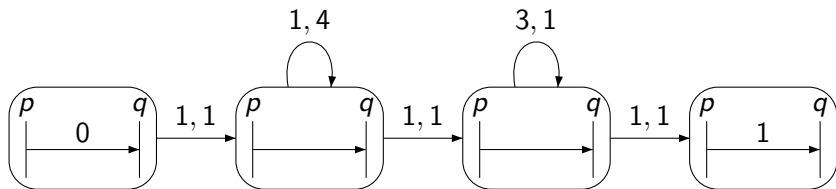
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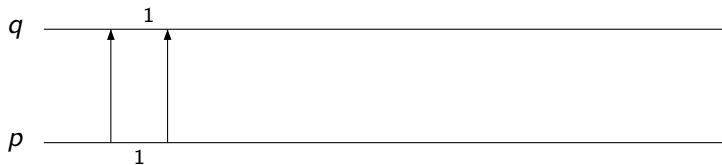
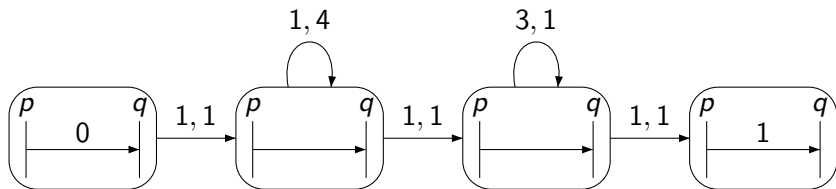
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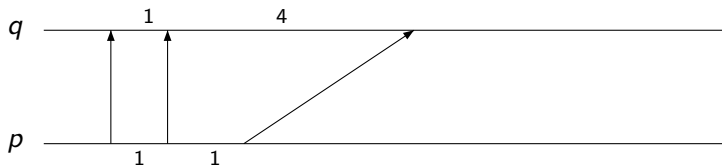
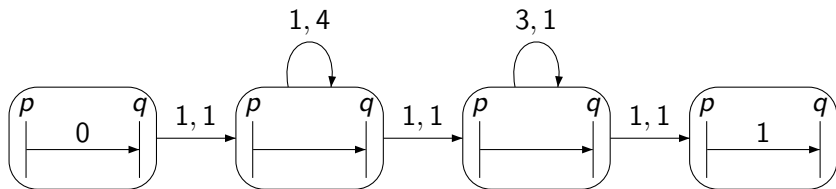
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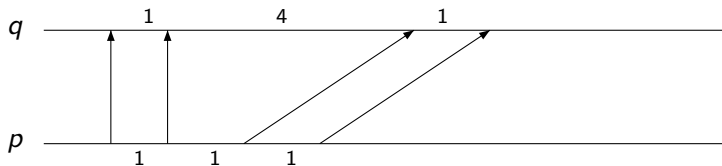
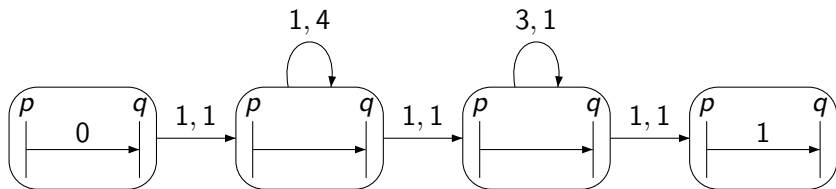


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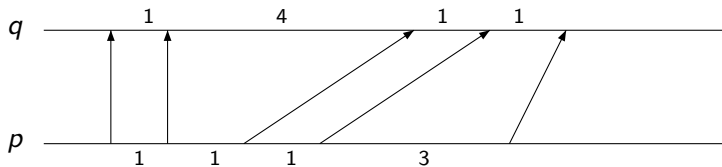
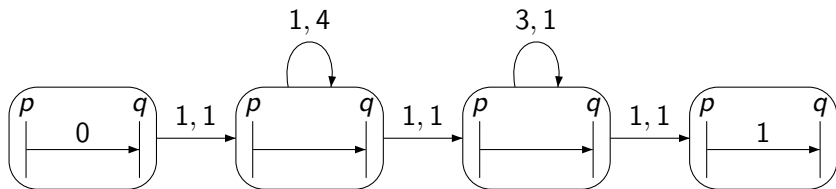




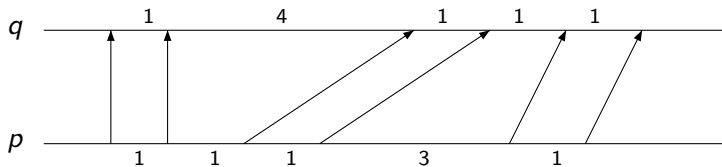
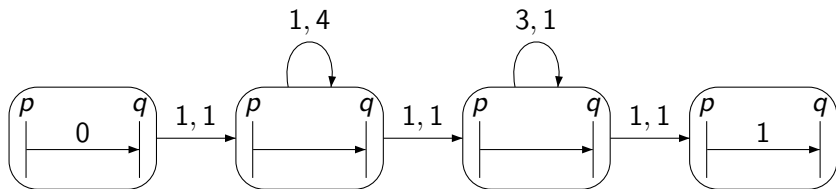
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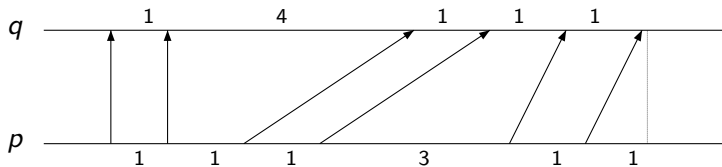
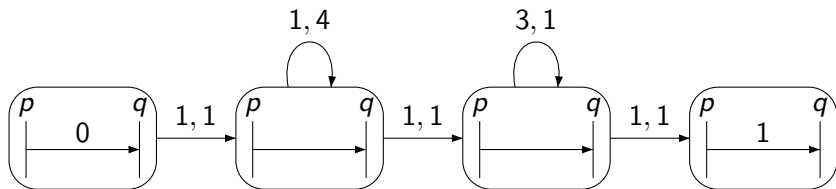
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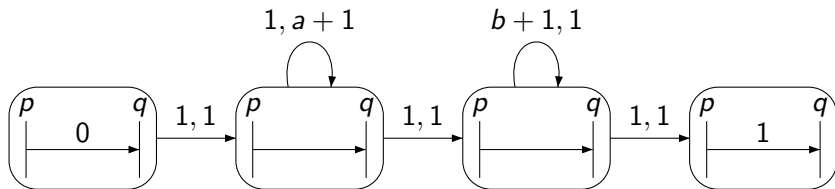
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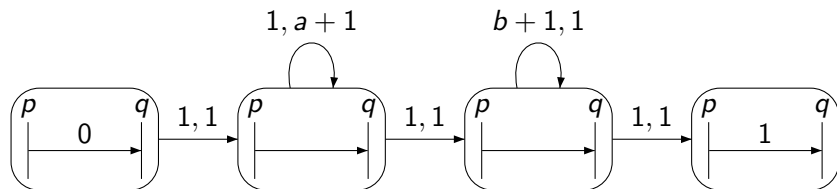
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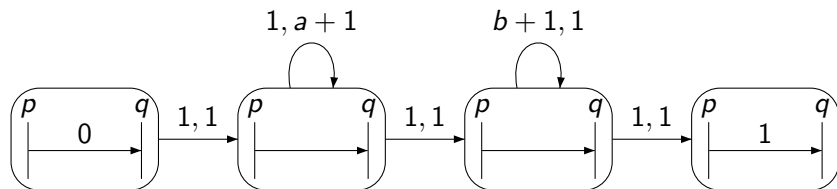


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- ▶ Simple paths may not be realizable while those with loops may be.

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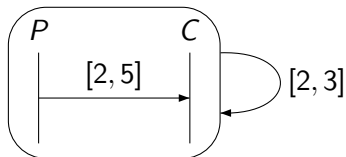
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Time constraints may ensure boundedness.

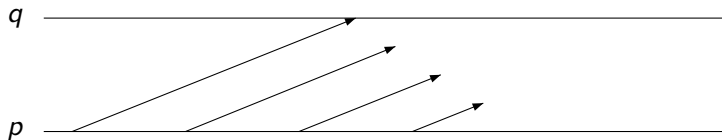
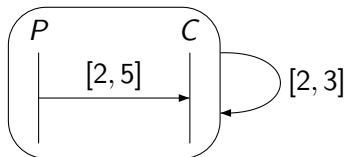
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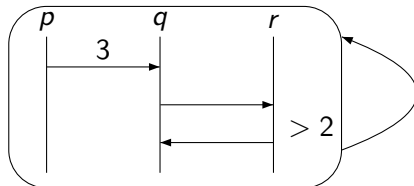


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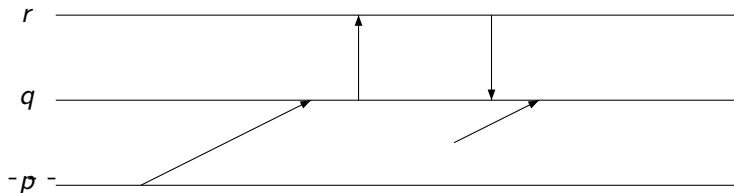
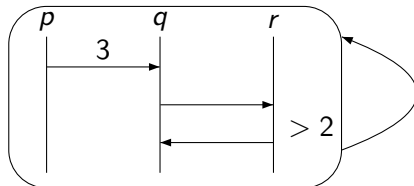
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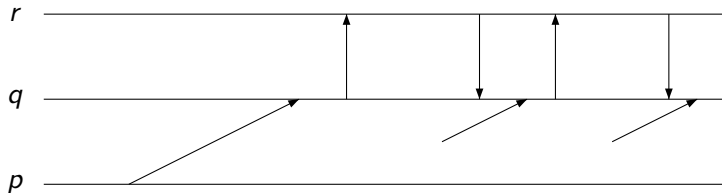
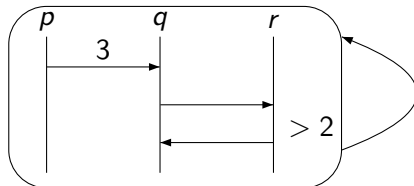
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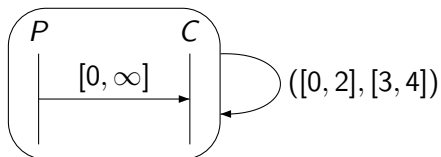
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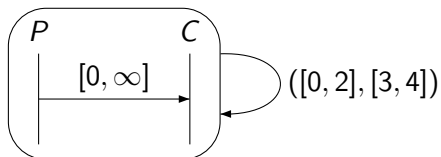
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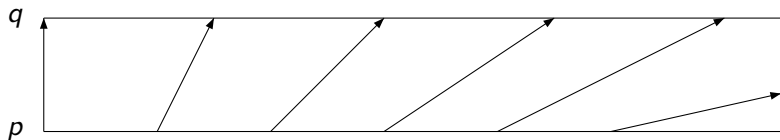
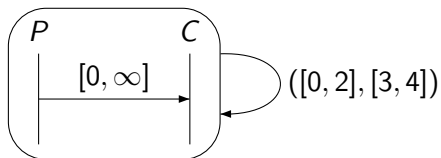


$q$  \_\_\_\_\_

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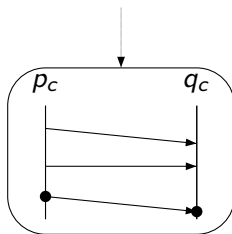
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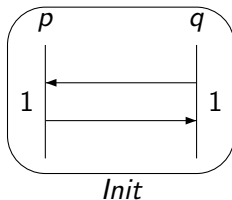
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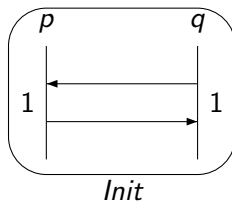
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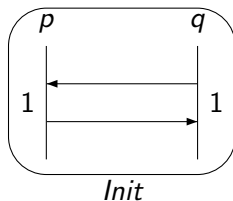
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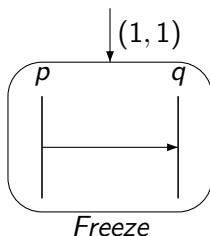
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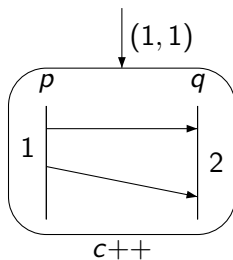


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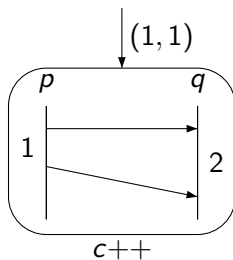
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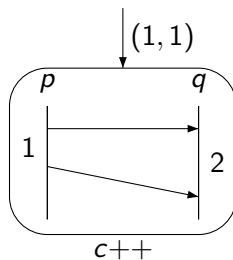
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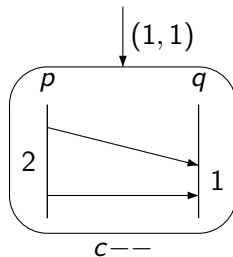
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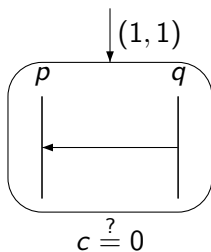


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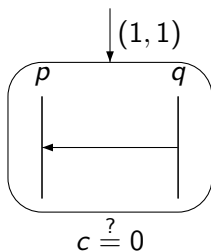
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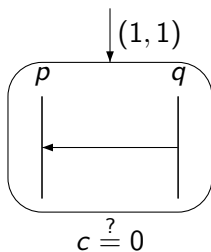
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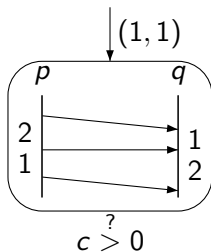
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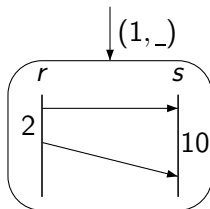
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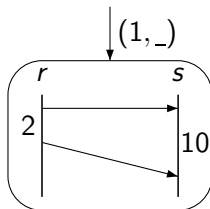
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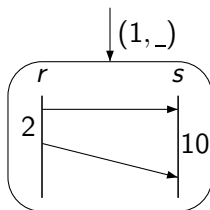
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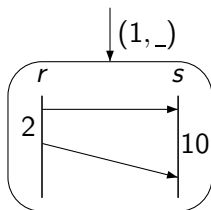
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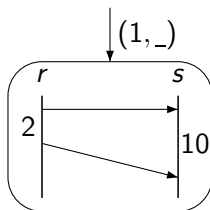
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Checking boundedness for TC-MSGs is undecidable



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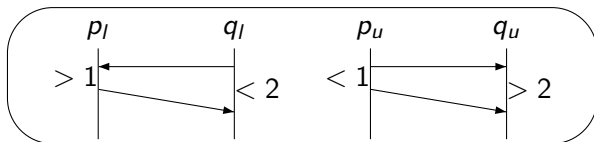
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- ▶ The value of  $p_u - q_u$  is used to check for 0.

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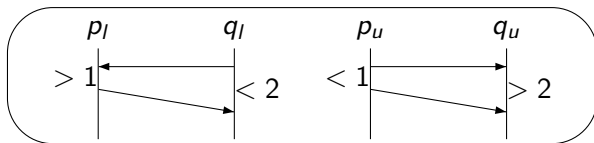
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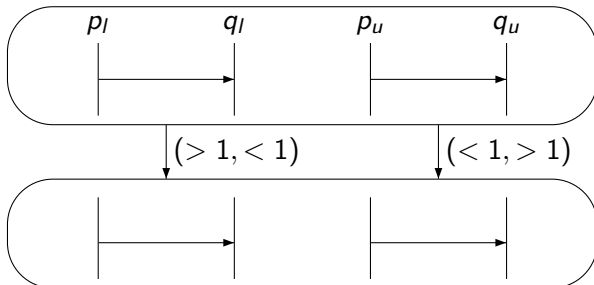


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Composition between Nodes

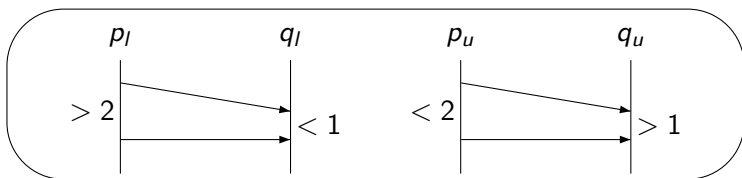


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The decrement instruction

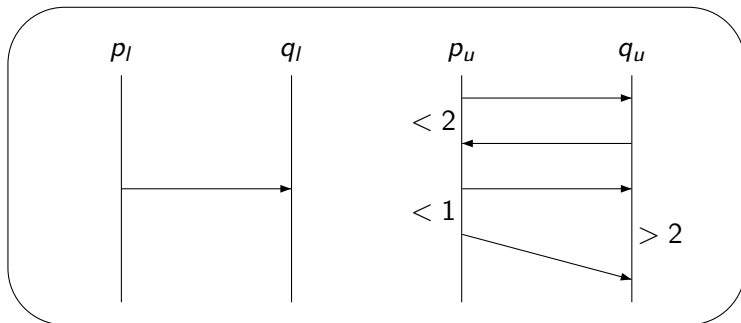
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## More Undecidability – 2

What about the reachability problem for channel bounded TC-MSGs?

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The reachability problem for channel bounded TC-MSGs is also undecidable.

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Even with the restriction that constraints across nodes are permitted only on a fixed process, the reachability and boundedness problems for TC-MSGs remain undecidable.

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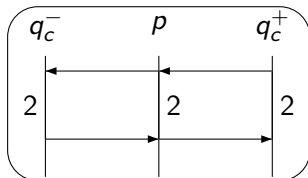
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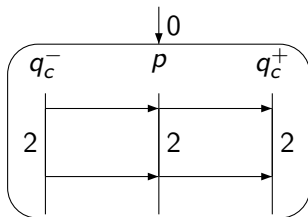
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# More Undecidability - 3

Initialize

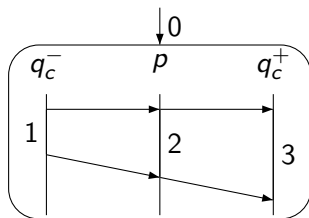


Freeze

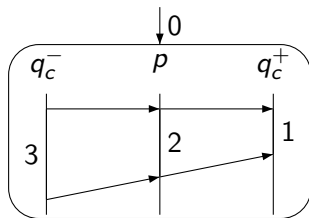


# More Undecidability – 3

## Increment



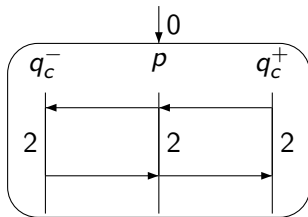
## Decrement





# More Undecidability – 3

## Check for Zero



# Locally synchronized MSGs

- ▶ Construct communication graph for an MSC  
One node per process, edge  $p \rightarrow q$  iff  $p$  sends a message to  $q$

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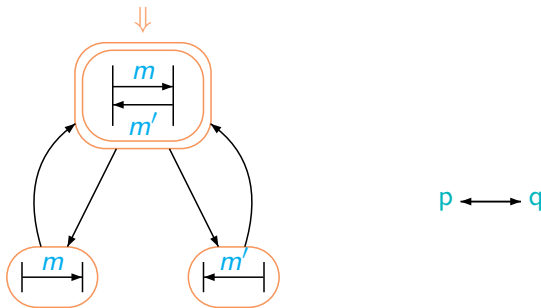
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# Locally synchronized MSGs

- ▶ Construct communication graph for an MSC  
One node per process, edge  $p \rightarrow q$  iff  $p$  sends a message to  $q$
- ▶ For each loop, communication graph is one strongly connected component plus isolated vertices
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  - ▶ The number of **active** clocks is bounded.
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  - ▶ Works like an event-clock automaton (upto some extra labelling).

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Thank you.

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- ▶ The control state reachability problem is decidable. A path is realizable if and only if each node in the path is realizable.
- ▶ The boundedness problem is still open. Time constraints can enforce boundedness.