Tutorial

Graph Decompositions and the Verification of Concurrent Recursive Programs I

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IMS, Singapore, August 2016

Outline

* Models
* Behaviours
* Specifications
* Verification via graph decompositions
* Conclusion

Concurrent Recursive Programs

Concurrent Recursive Programs

Variables range over finite domains

Concurrent Recursive Programs

Variables range over finite domains

Functions can be recursive

Concurrent Recursive Programs

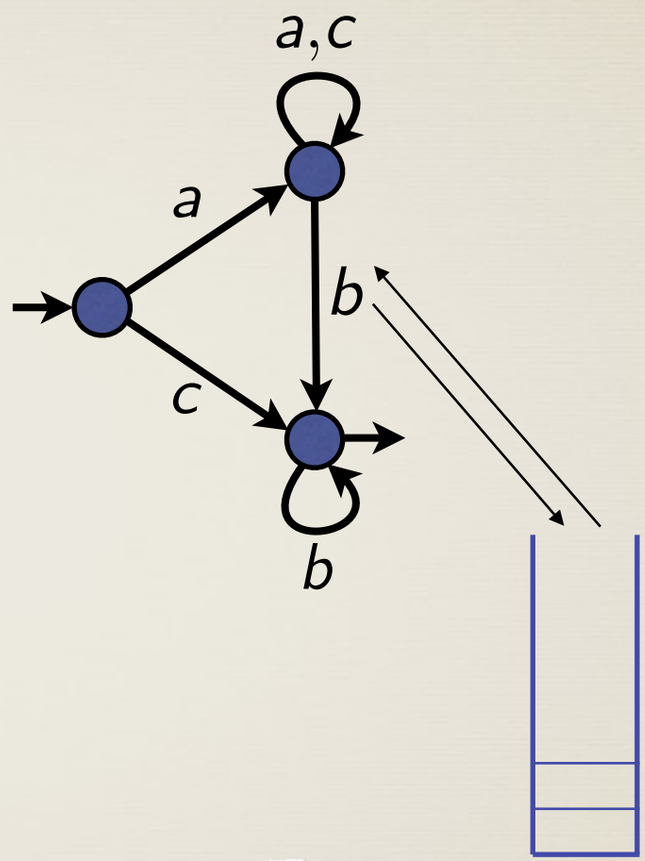
Multi-threaded or Distributed

Variables range over finite domains

Functions can be recursive

Modeling Recursion

```
func f1
{while <true>
 {call f1 OR
    a OR
    exit;}
 return;}
```



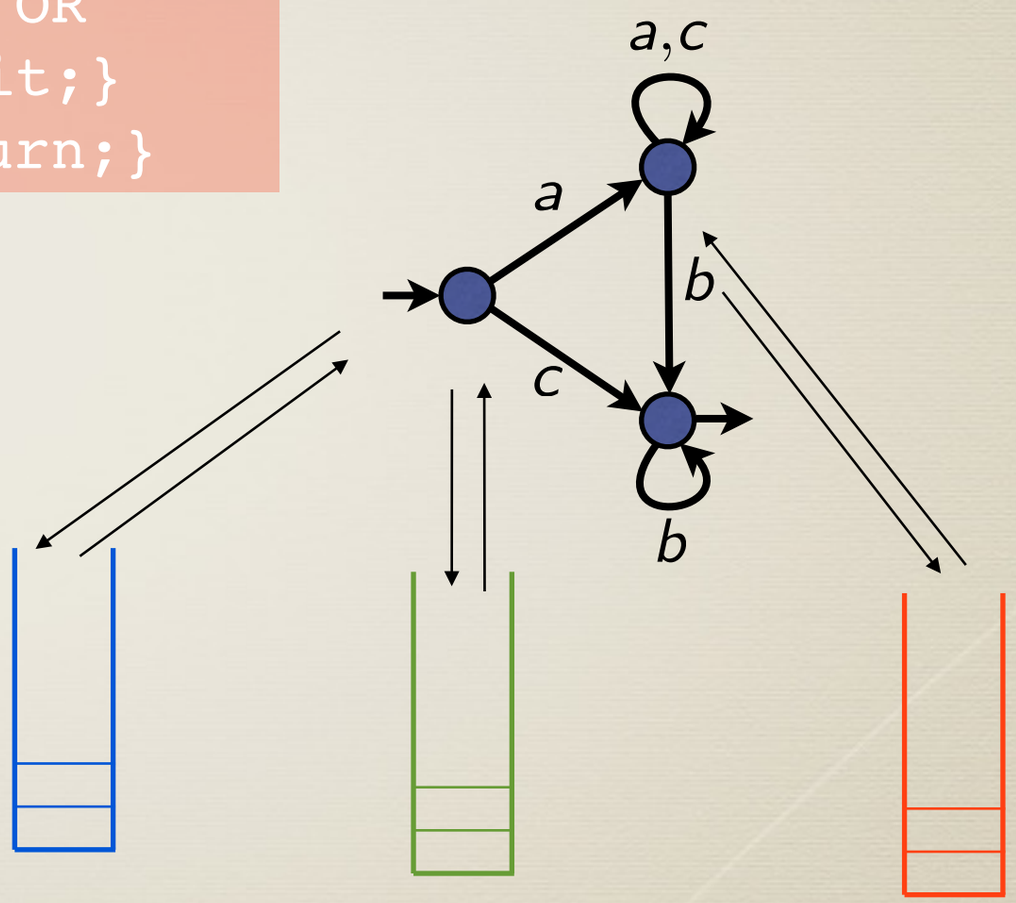
Recursive Programs are Pushdown Systems

Modeling Recursion …

```
func f1
{while <true>
 {call f1 OR
  a OR
  exit;}
 return;}
```

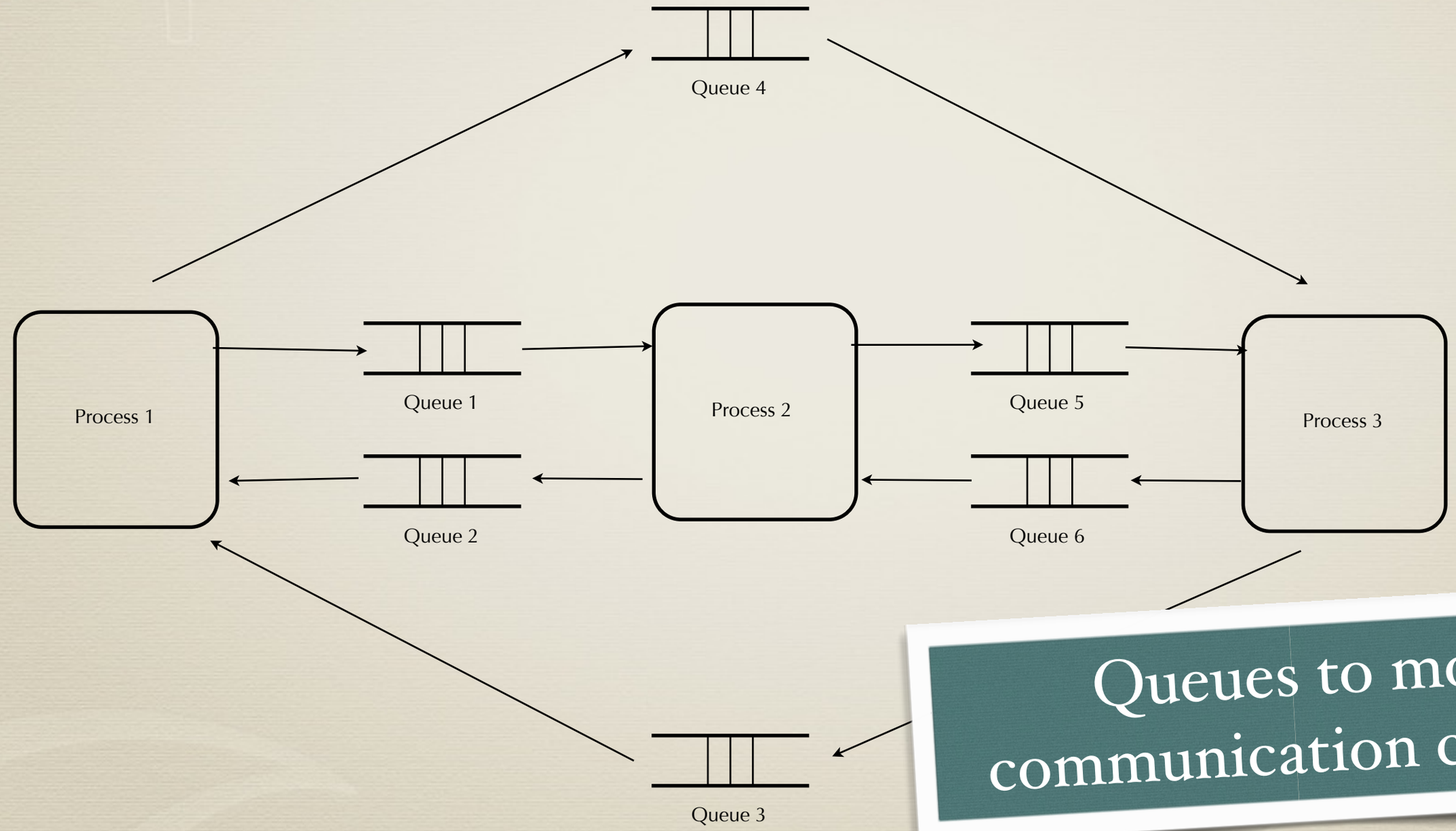
```
func f2
{while <true>
 {call f2 OR
  a OR
  exit;}
 return;}
```

```
func f3
{while <true>
 {call f3 OR
  a OR
  exit;}
 return;}
```



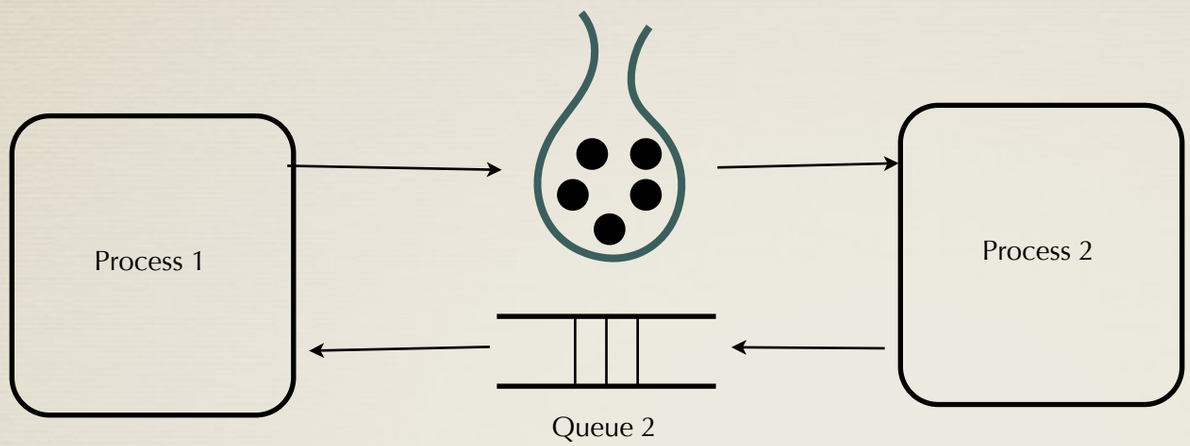
Multi-threaded Programs are Multi-Pushdown Systems

Concurrent Communicating Programs



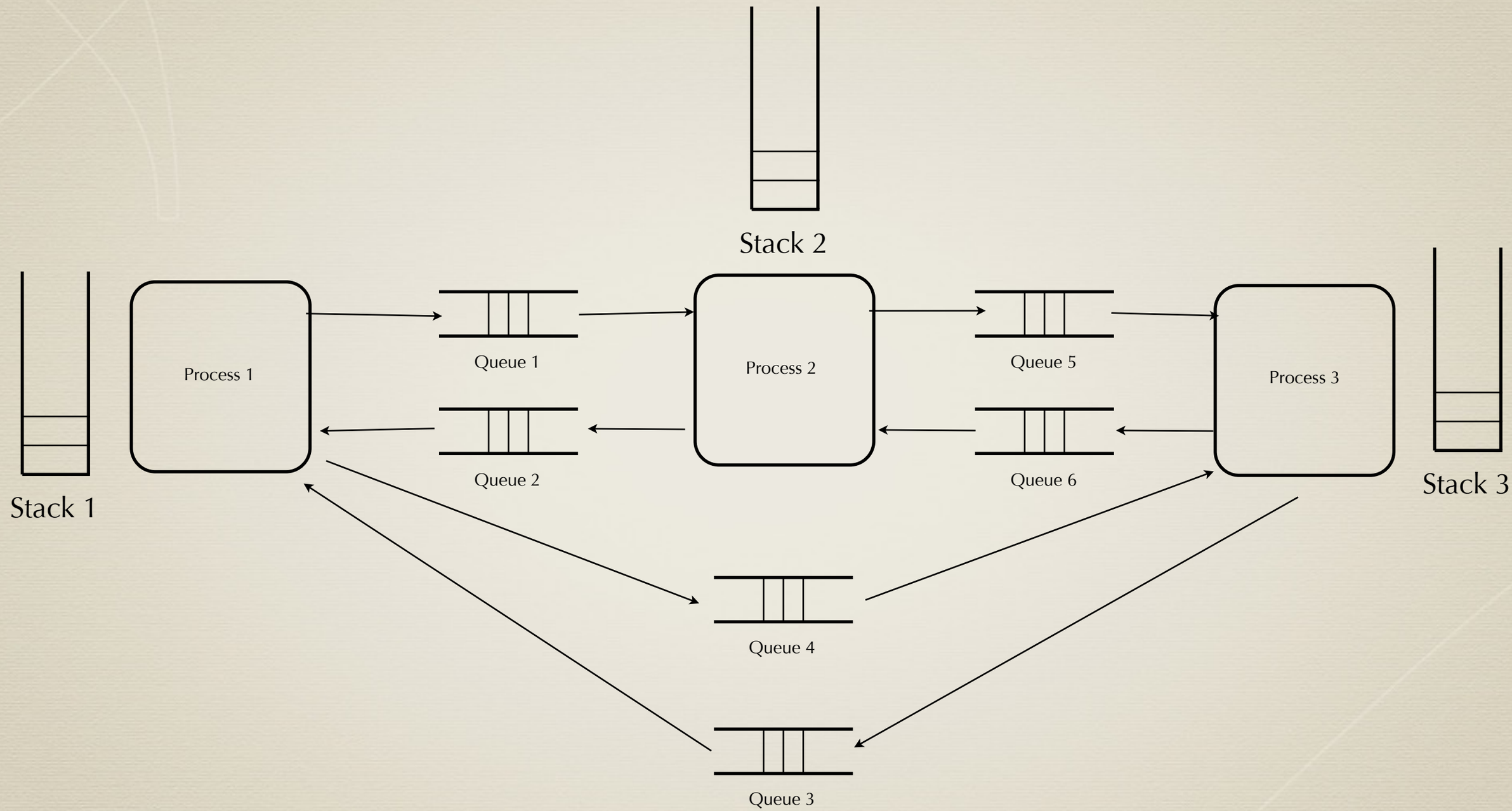
Queues to model communication channels

Unordered Channels



Bags to model unordered communication channels

Communicating Recursive Processes



System: Concurrent Processes with Data-Structures

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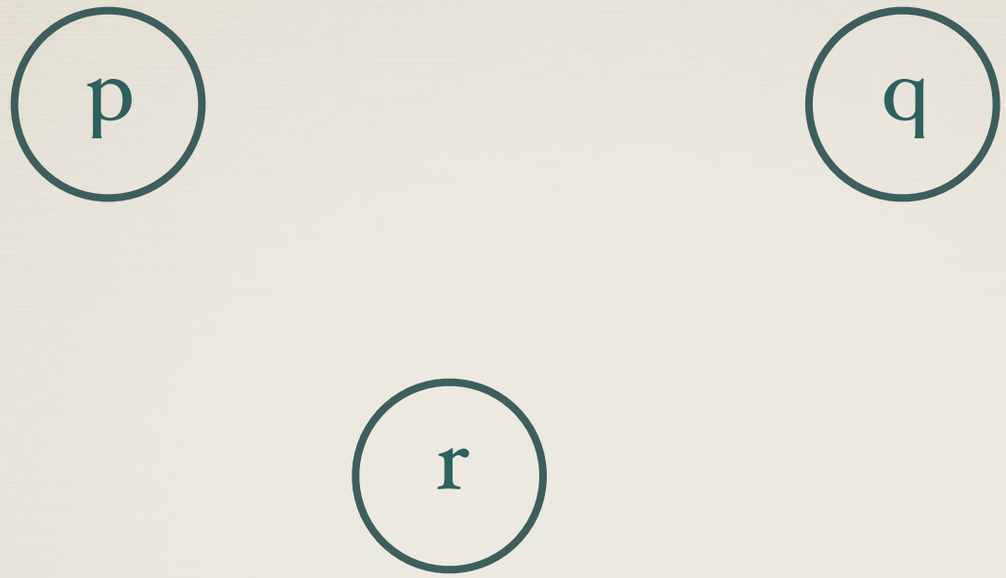
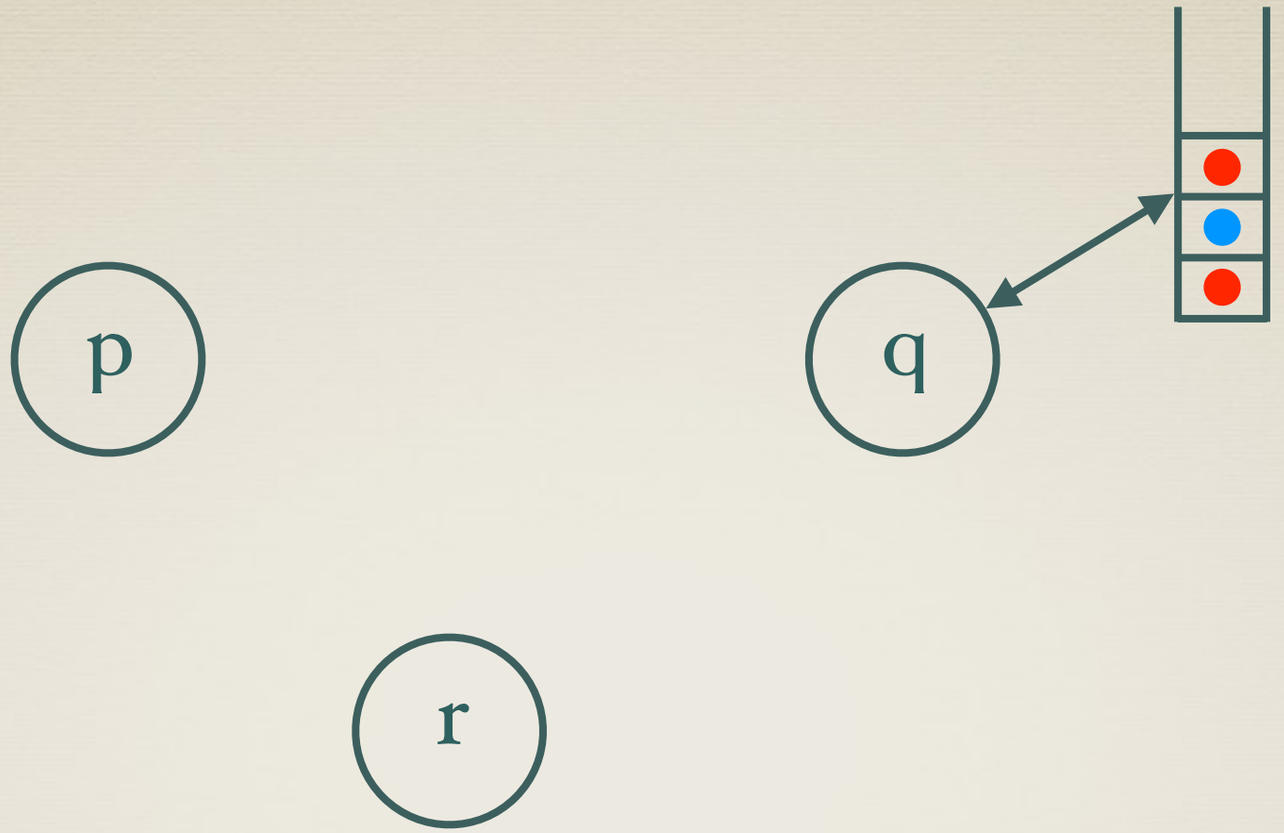


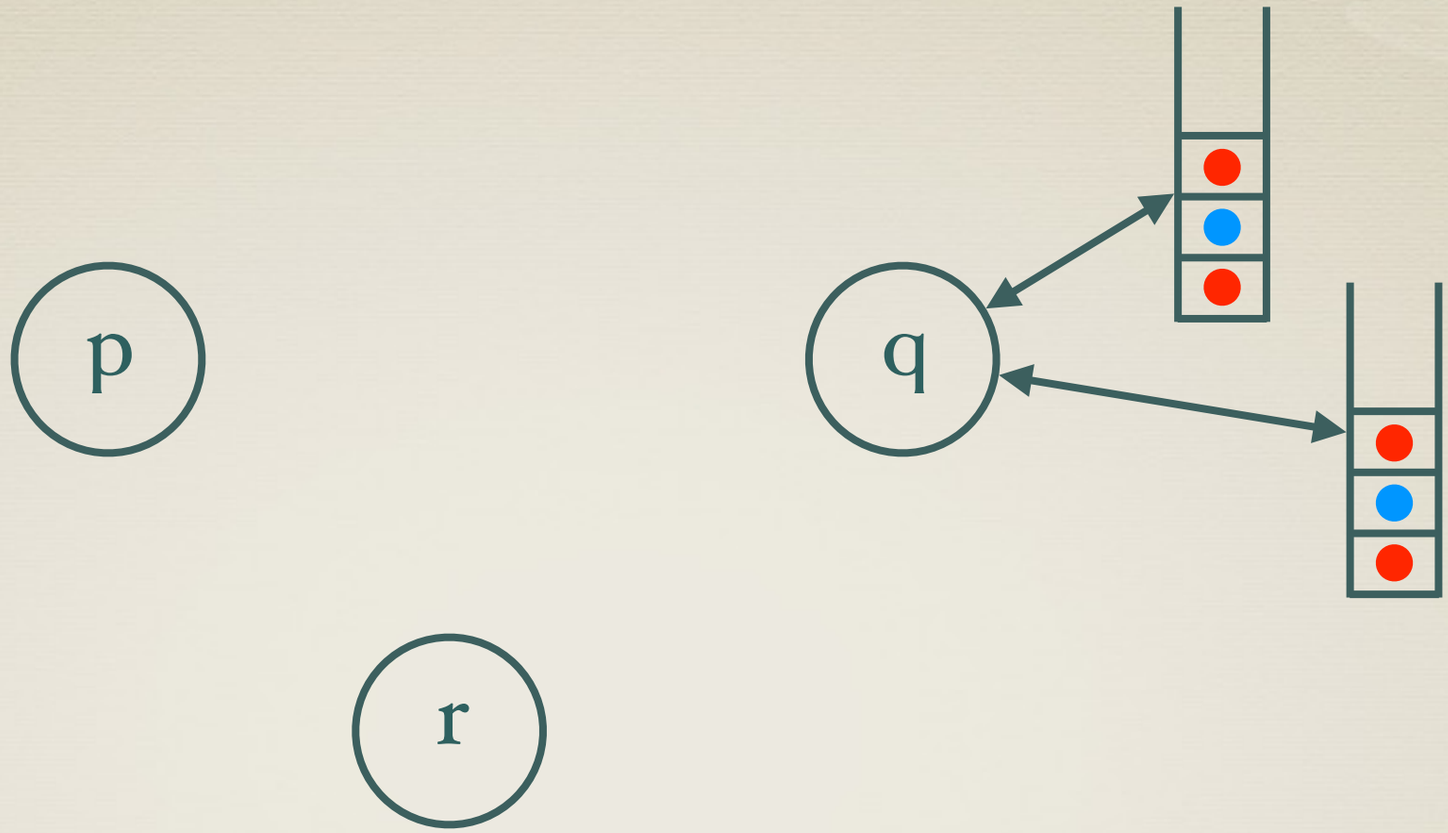
- Processes

System: Concurrent Processes with Data-Structures



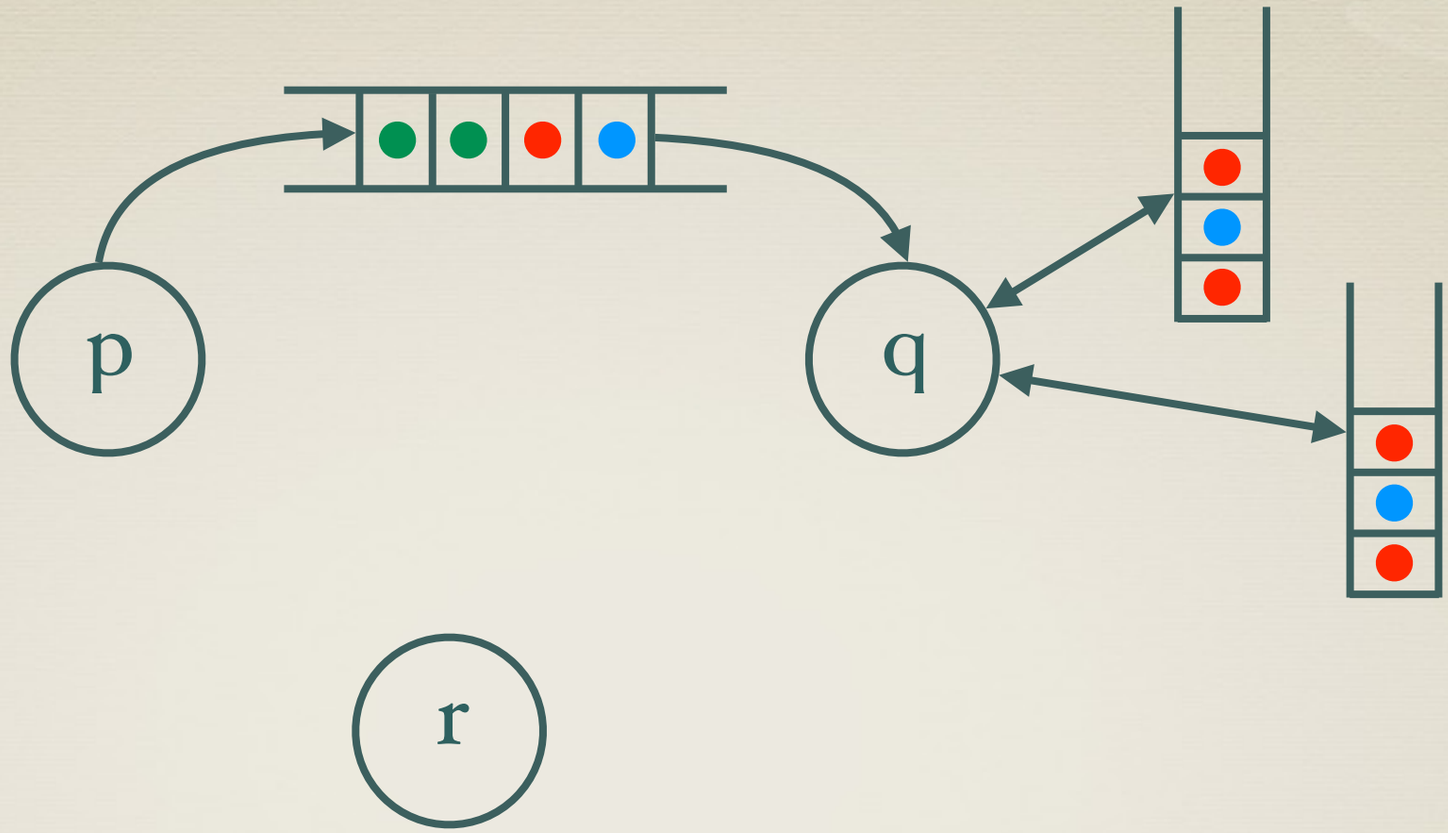
- Processes
- Data structures
  - Stacks: recursive programs

System: Concurrent Processes with Data-Structures



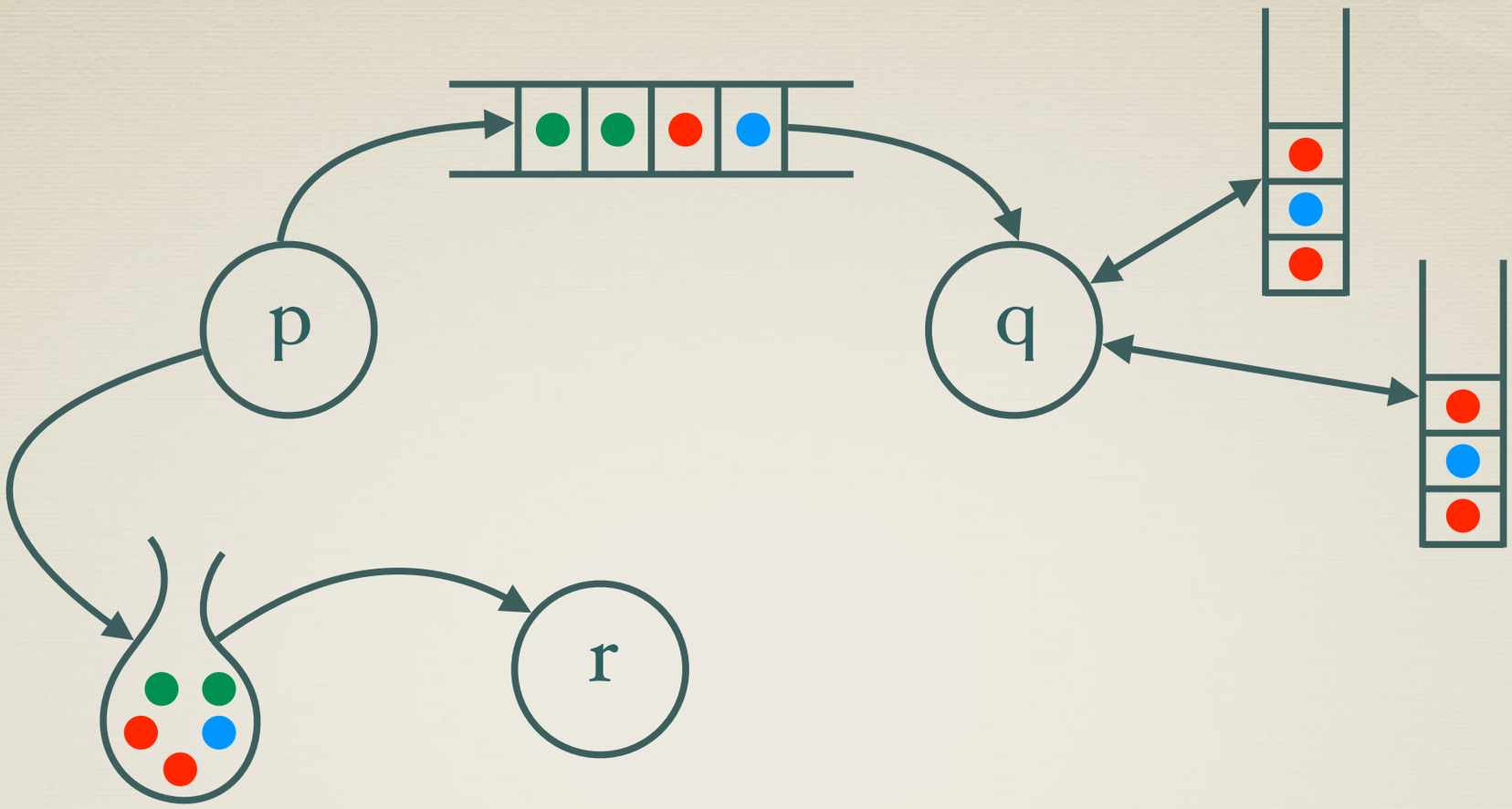
- Processes
- Data structures
  - Stacks: recursive programs, multithreaded

System: Concurrent Processes with Data-Structures



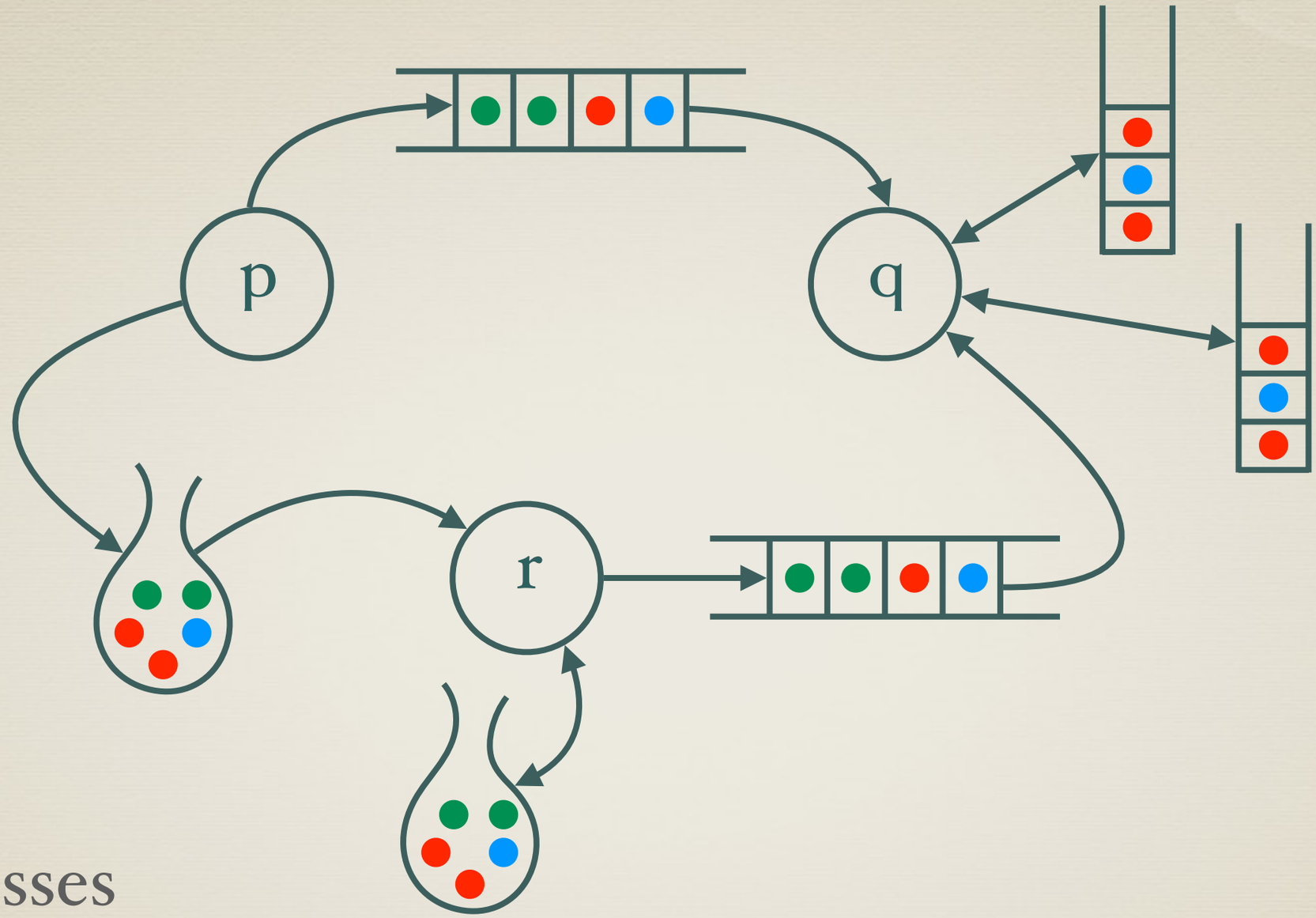
- Processes
- Data structures
  - Stacks: recursive programs, multithreaded
  - Queues: communication (FIFO)

System: Concurrent Processes with Data-Structures



- Processes
- Data structures
  - Stacks: recursive programs, multithreaded
  - Queues: communication (FIFO)
  - Bags: communication (unordered)

System: Concurrent Processes with Data-Structures



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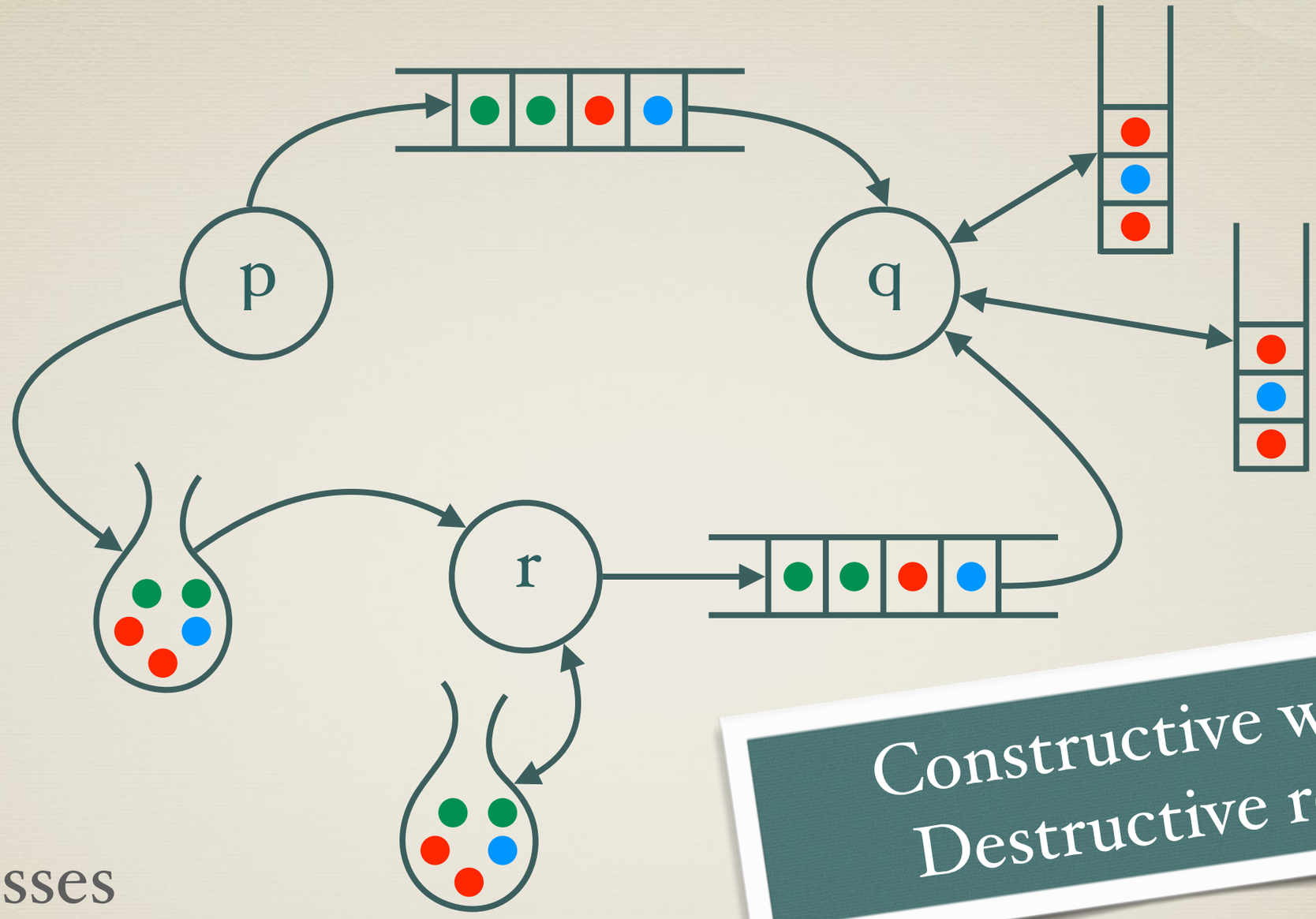
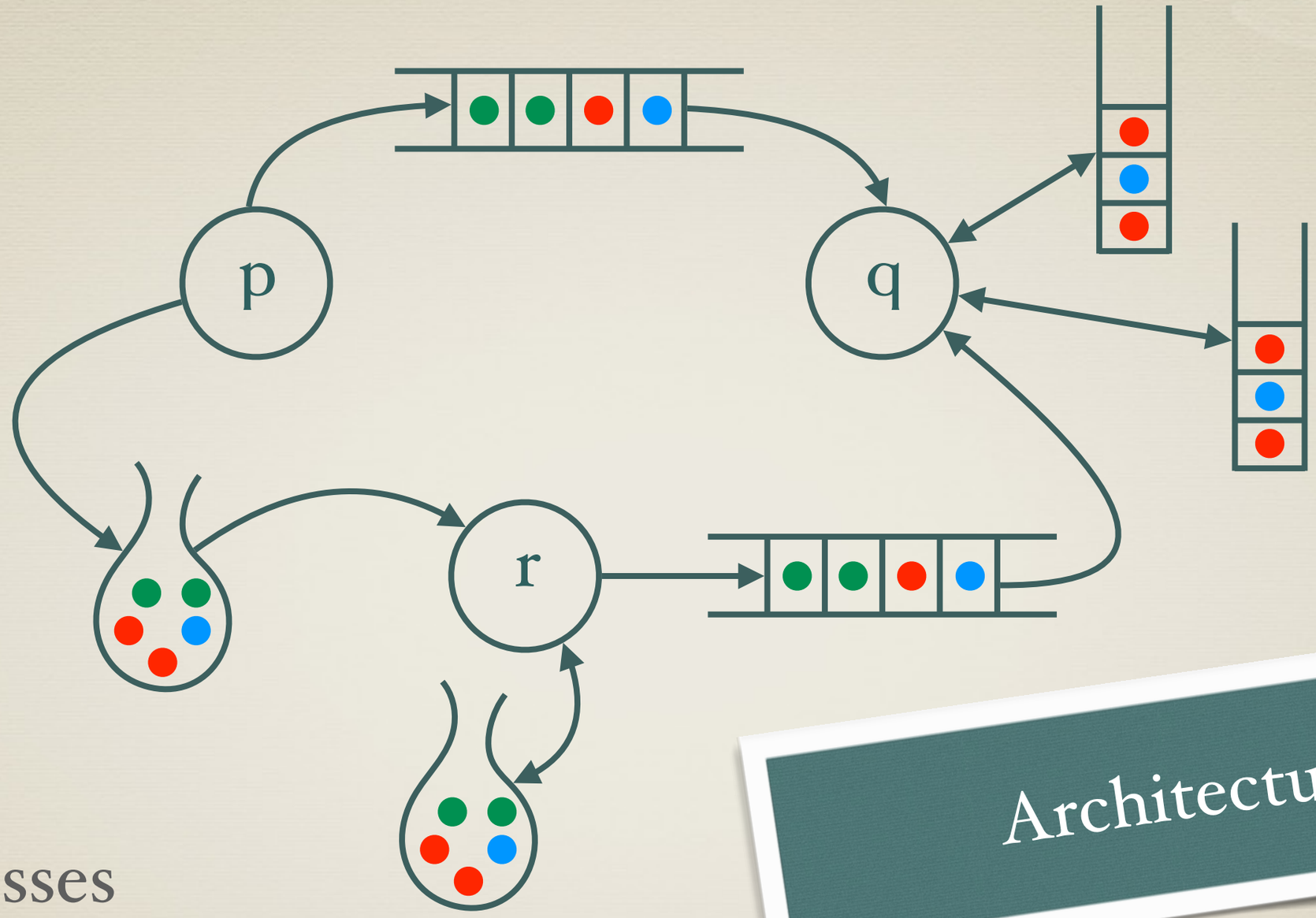


Constructive writes
Destructive reads

- Processes
- Data structures
  - Stacks: recursive programs, multithreaded
  - Queues: communication (FIFO)
  - Bags: communication (unordered)

System: Concurrent Processes with Data-Structures

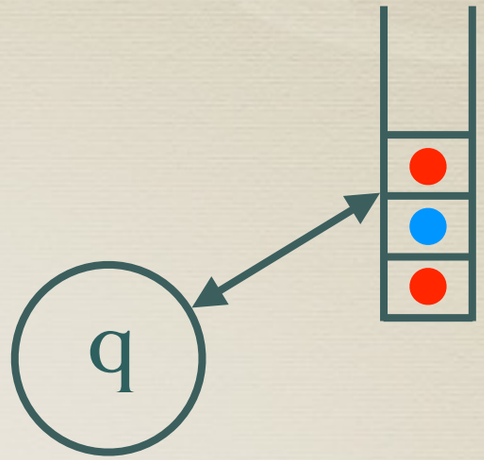


Architecture

- Processes
- Data structures
  - Stacks: recursive programs, multithreaded
  - Queues: communication (FIFO)
  - Bags: communication (unordered)

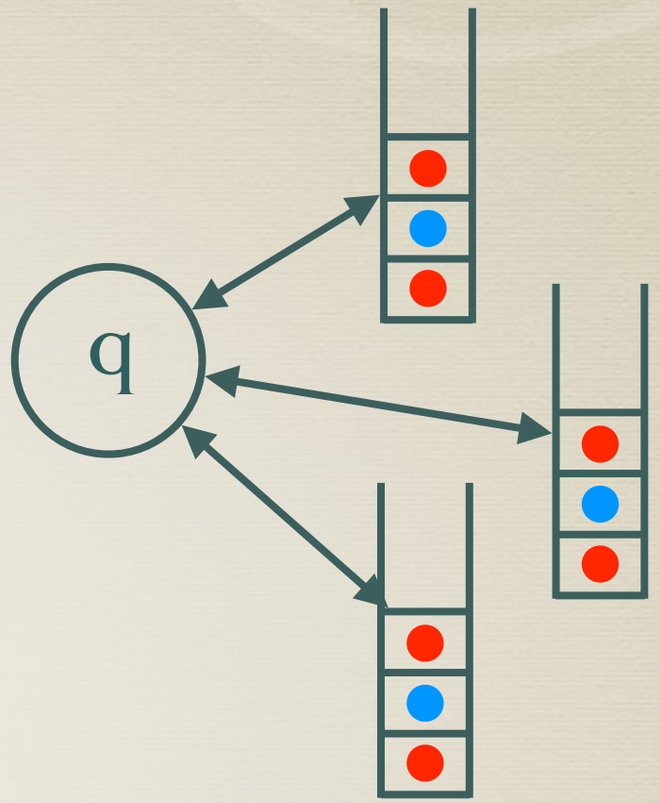
Special cases

- PDA: Pushdown automata
  Recursive programs



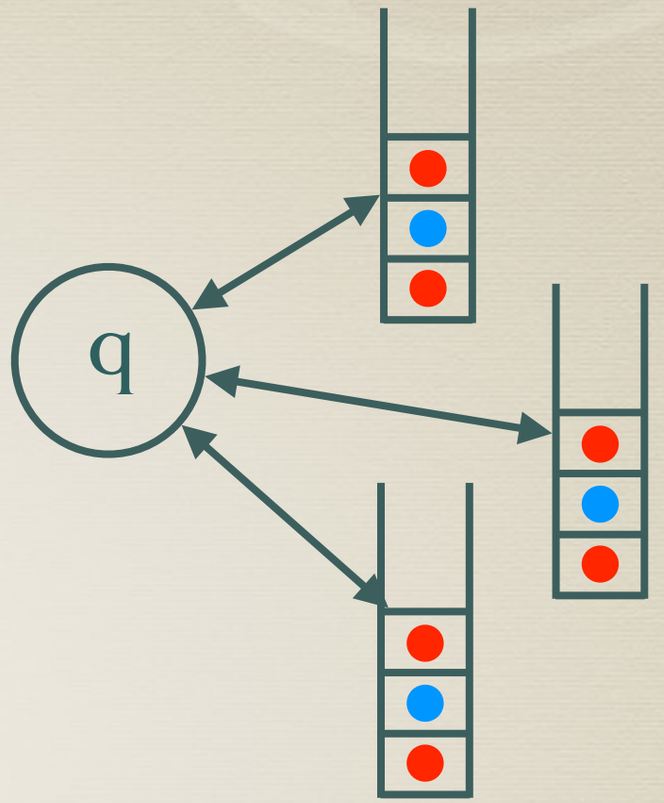
Special cases

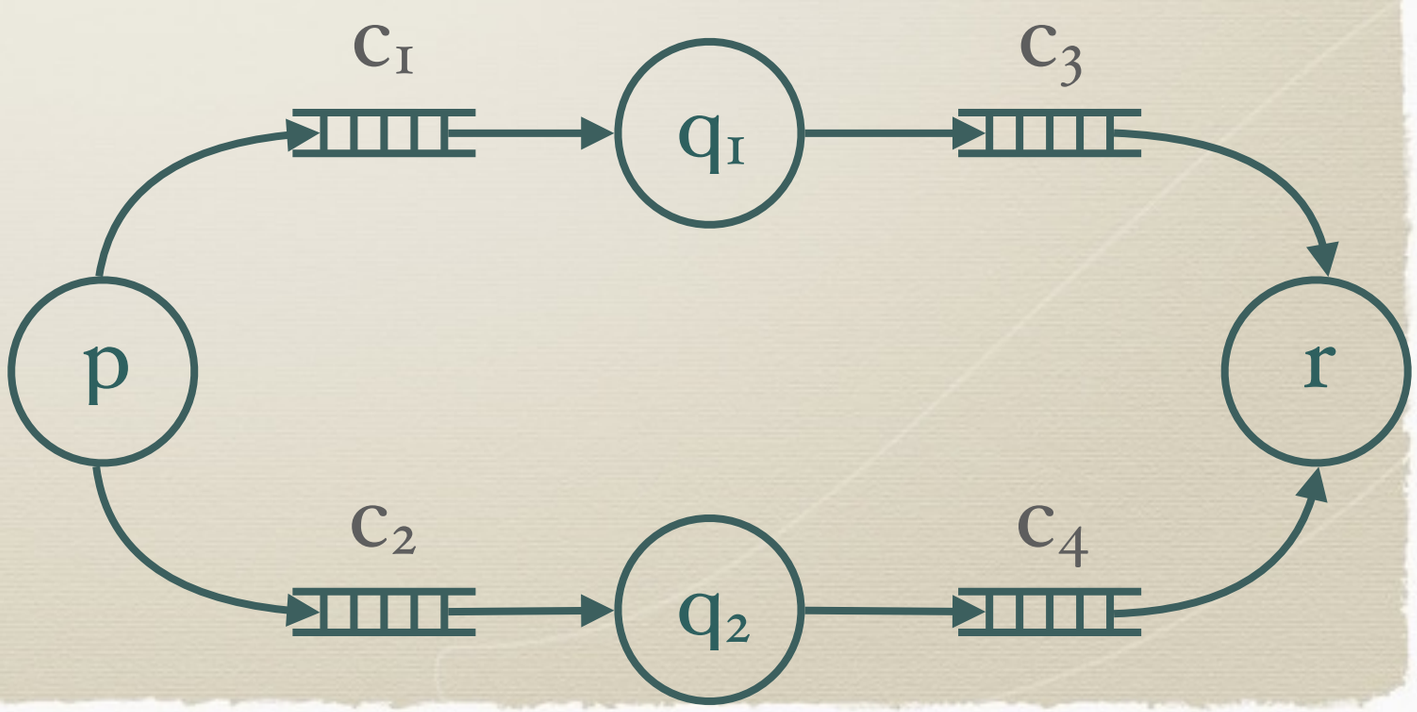
- PDA: Pushdown automata
  Recursive programs
- MPDA: Multi-pushdown automata
  Multi-threaded recursive programs



Special cases

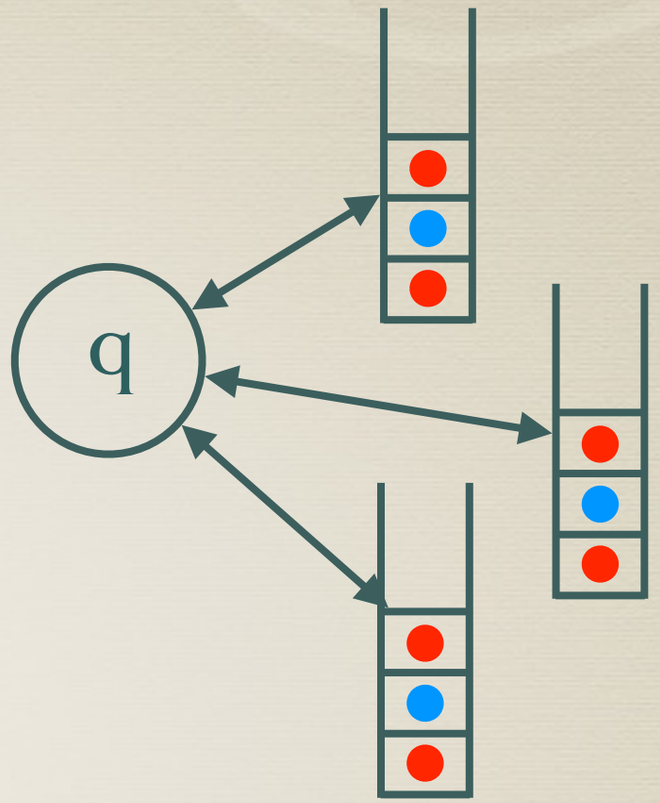
- PDA: Pushdown automata
  Recursive programs
- MPDA: Multi-pushdown automata
  Multi-threaded recursive programs
- MPA: Message passing automata
  Communicating finite state machines

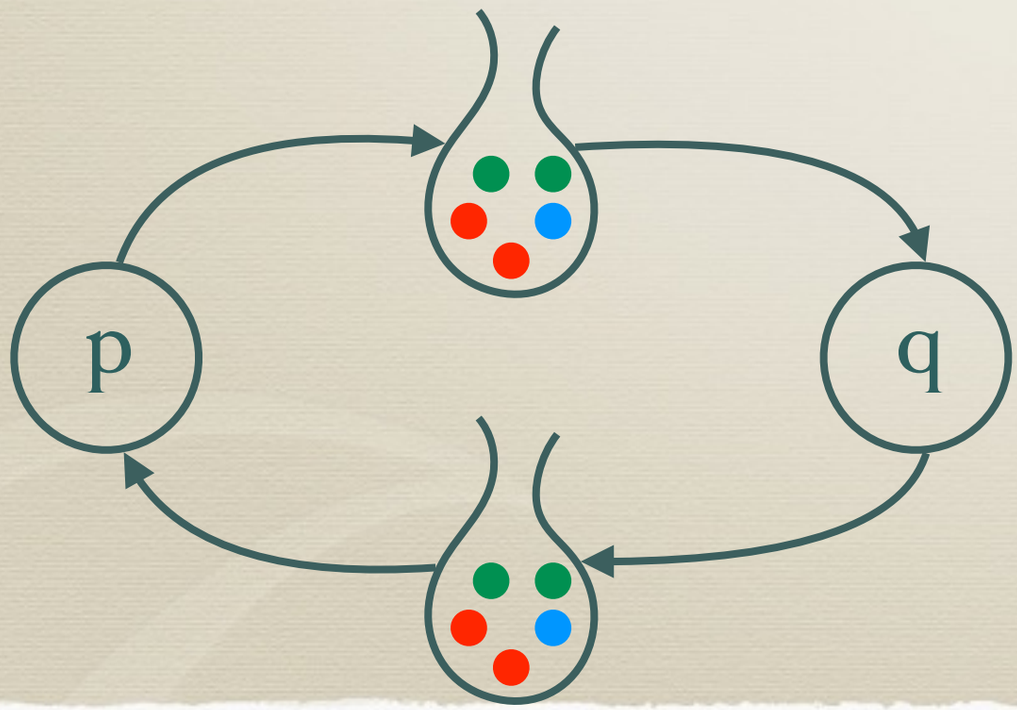


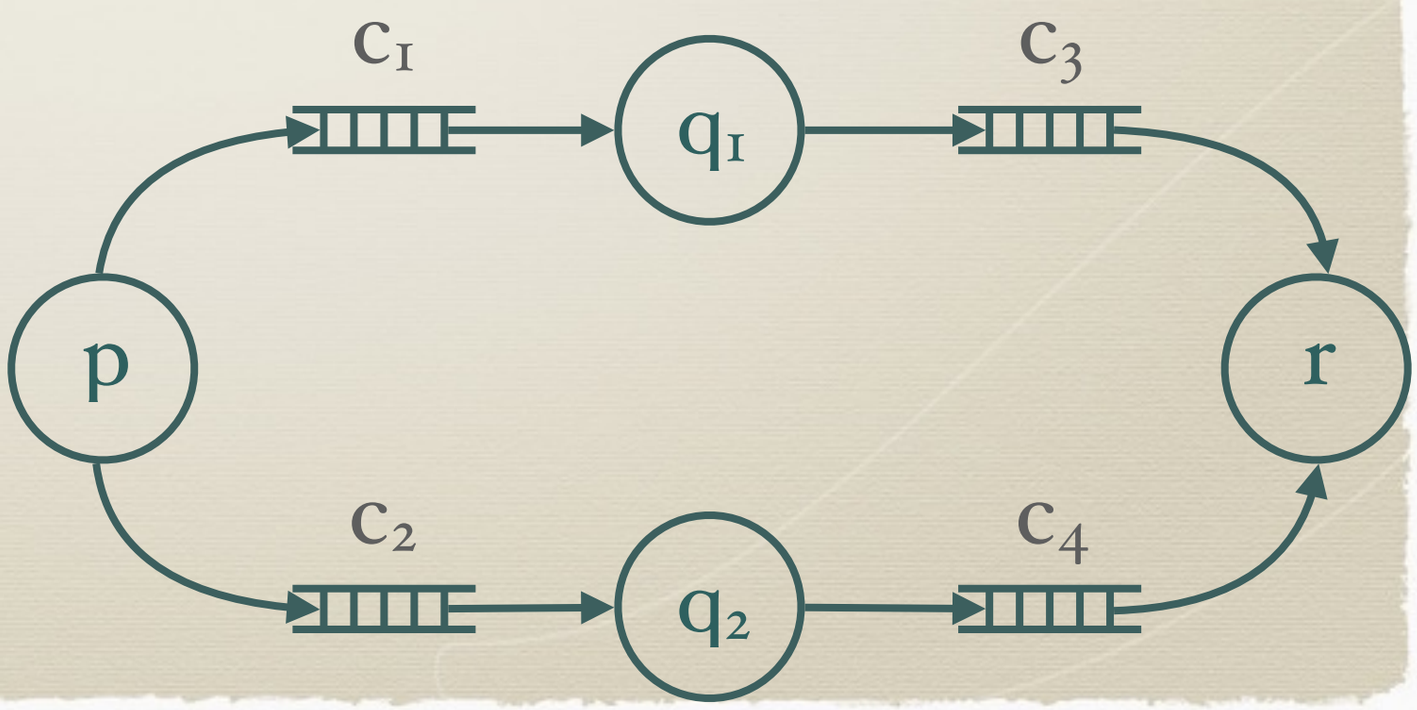


Special cases

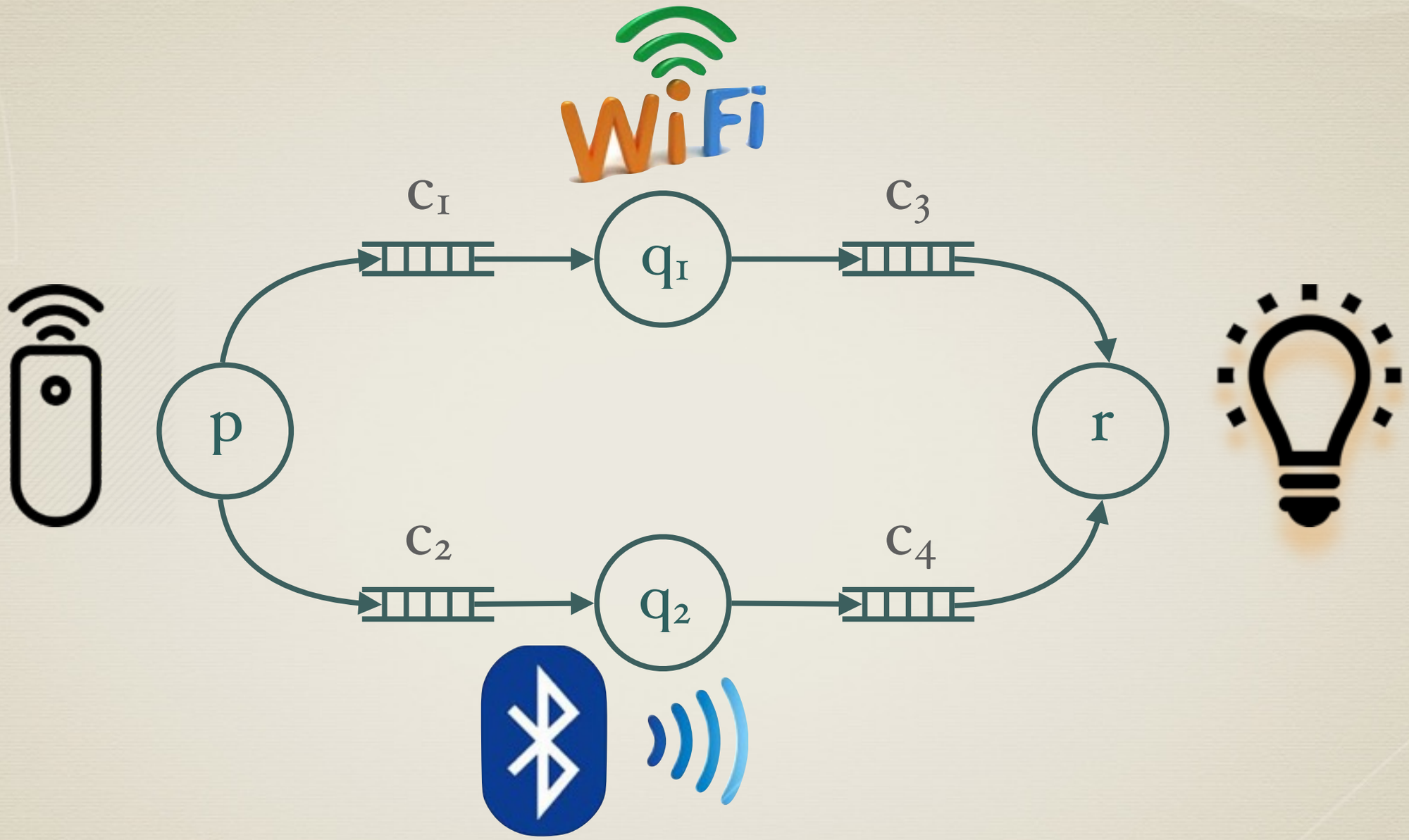
- PDA: Pushdown automata
  Recursive programs
- MPDA: Multi-pushdown automata
  Multi-threaded recursive programs
- MPA: Message passing automata
  Communicating finite state machines
- PN: Petri Nets : Only bags



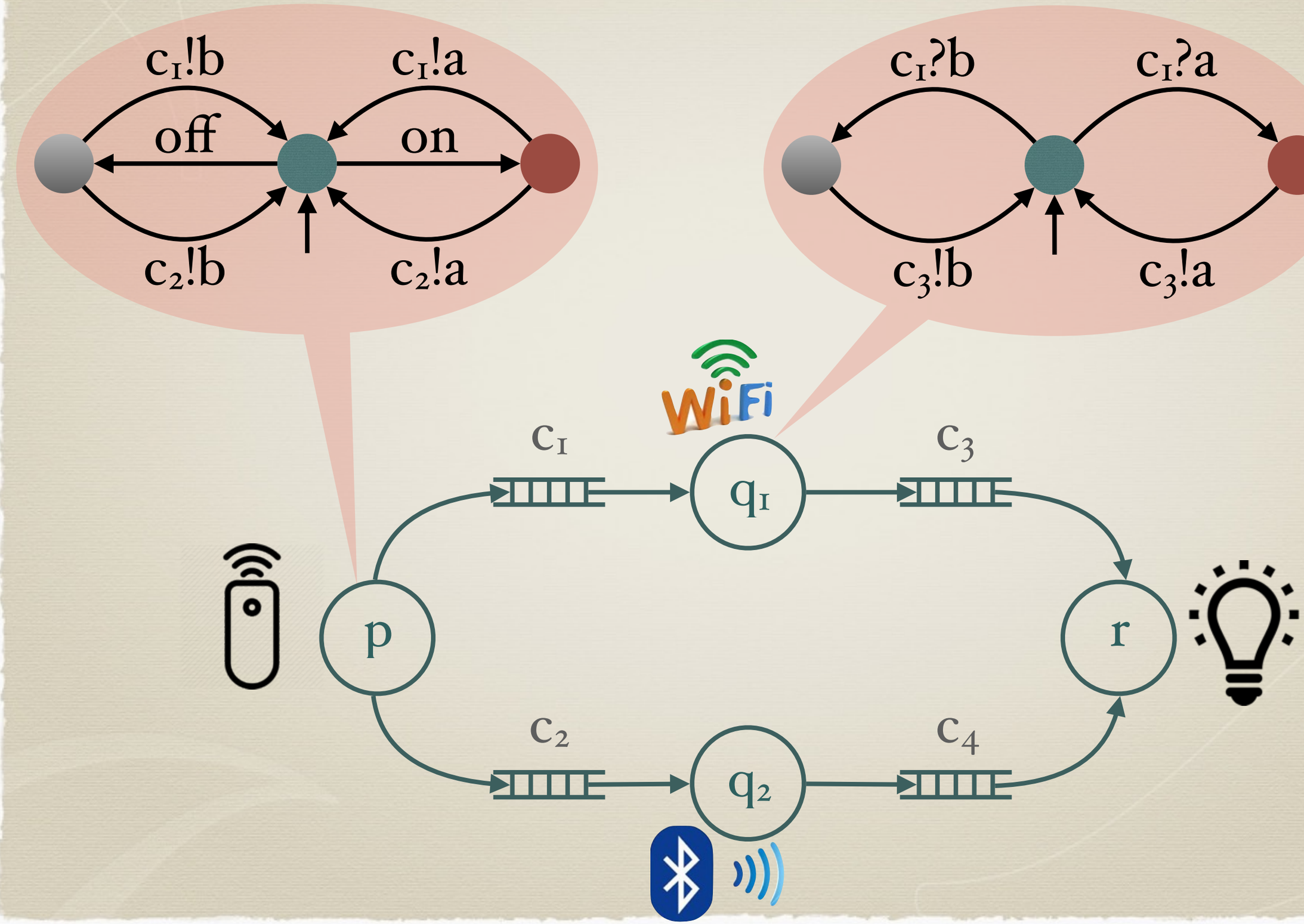




Remote on-off via 2 channels

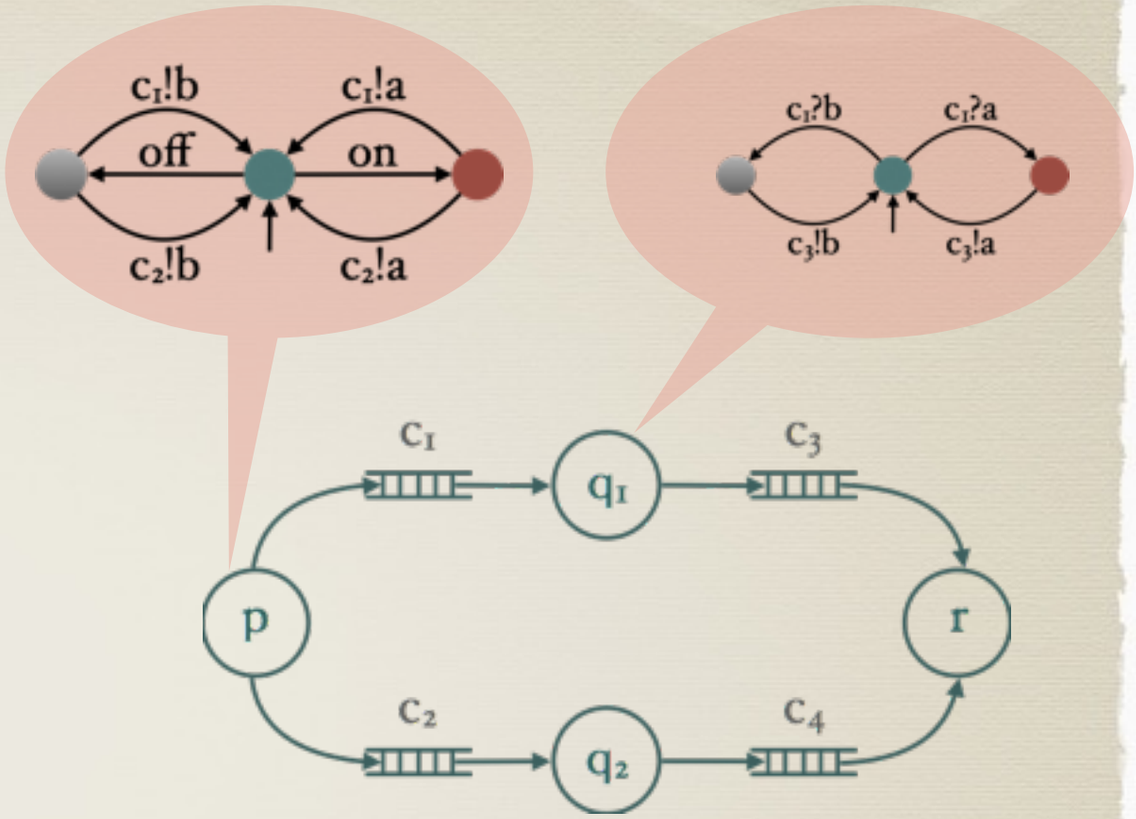


System: Architecture + Boolean Programs



Operational semantics

- Transition system TS
  - Configurations (infinite)
    - local states of processes
    - contents of data structures
- Transitions
  - Induced by the boolean programs



Outline

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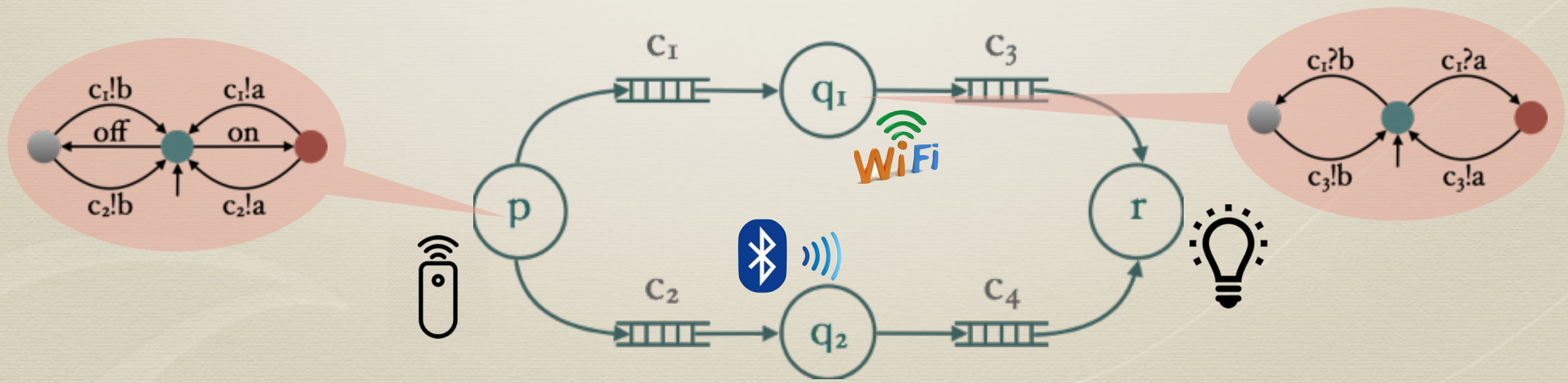
Linear Traces

$$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$$
$$(q_2, c_4!)(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(r, c_4?)(r, \mathsf{on})$$
$$(q_1, c_3!)(p, c_1!)(q_1, c_1?)(q_1, c_3!)(q_2, c_2?)(q_2, c_4!)$$
$$(r, c_4?)(r, \mathsf{on})(r, c_3?)(r, \mathsf{off}) \cdots$$



Linear Traces

$$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$$
$$(q_2, c_4!)(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(r, c_4?)(r, \mathsf{on})$$
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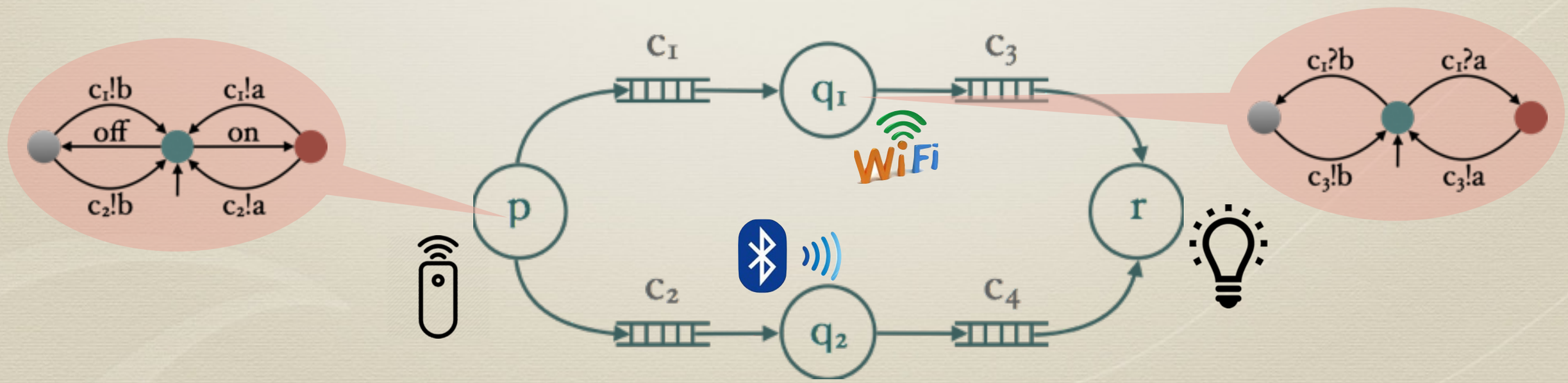

Linear Traces

$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$
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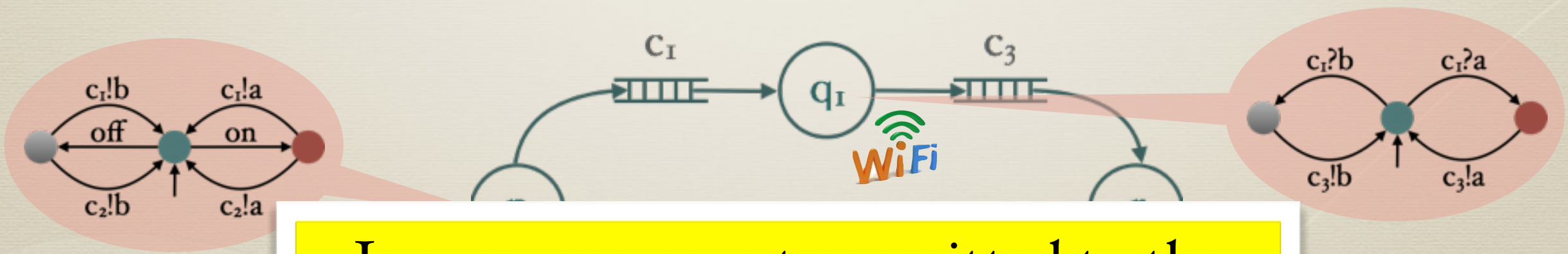

Is every message transmitted to the bulb?

Causal Response

$$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$$
$$(q_2, c_4!)(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(r, c_4?)(r, \mathsf{on})$$
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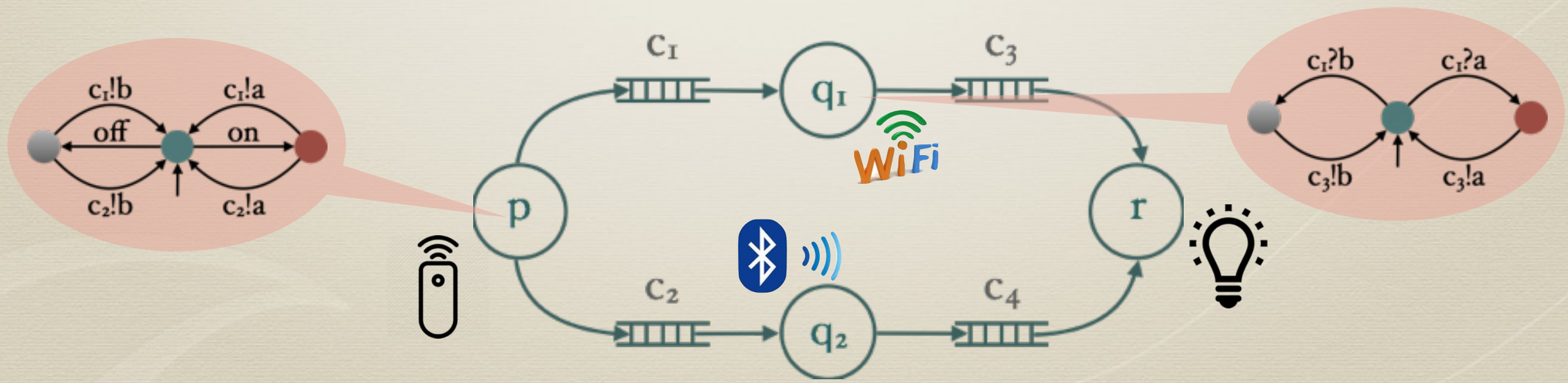

Causal Response

$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$
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$(r, c_4?)(r, \mathsf{on})(r, c_3?)(r, \mathsf{off}) \cdots$

Does it obey the latest order?

Behaviours as Graphs

$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$

$(q_2, c_4!)(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(r, c_4?)(r, \mathsf{on})$

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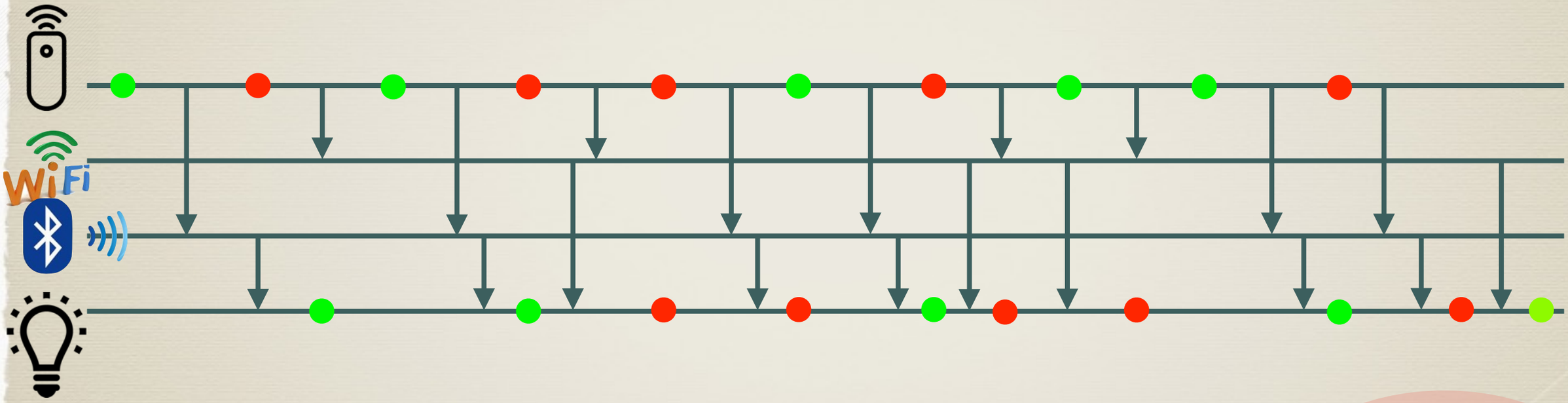

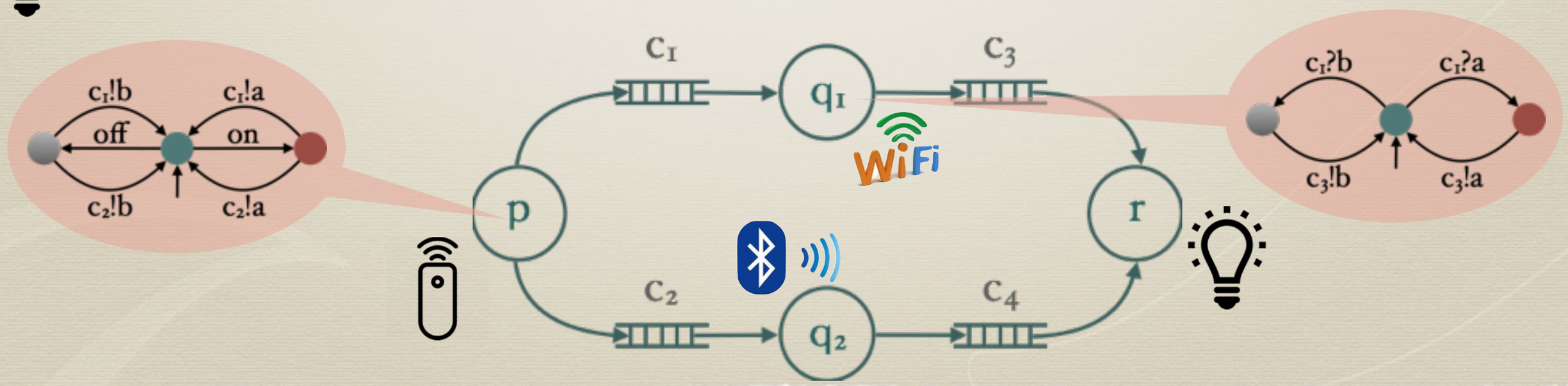

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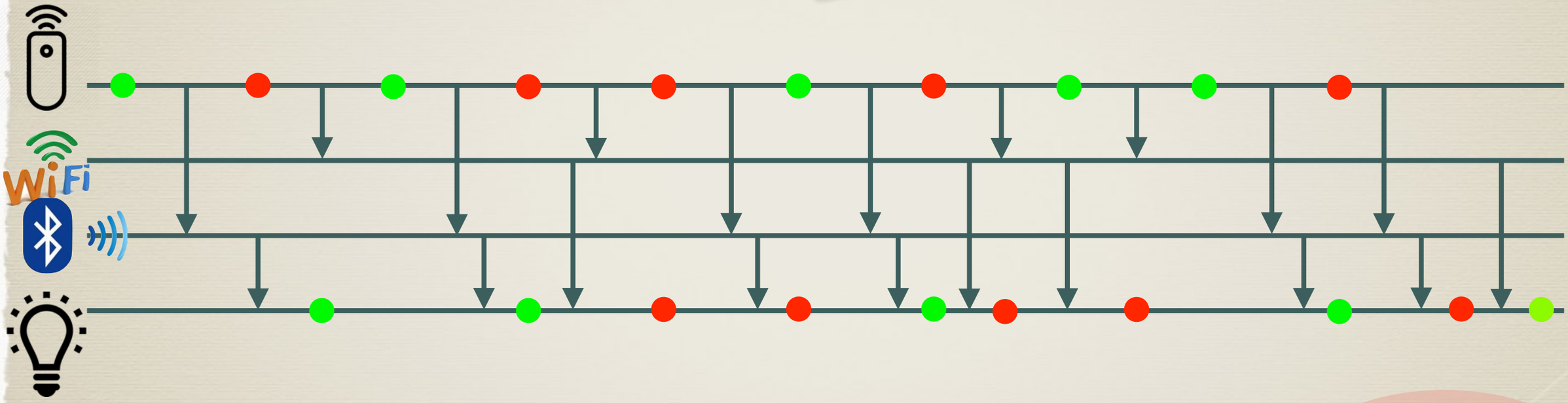
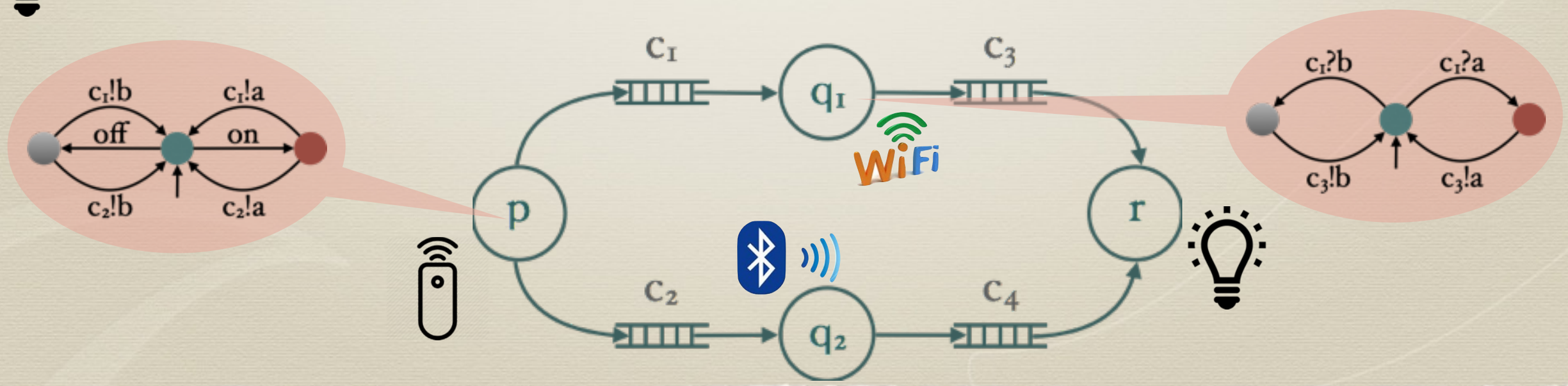
Message Sequence Charts
ITU Standard





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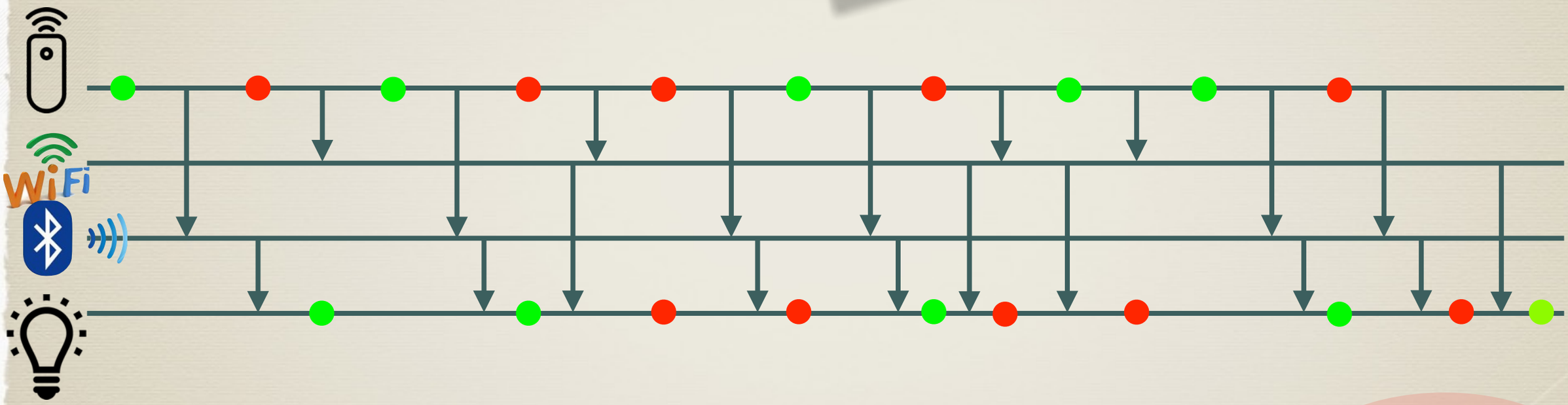
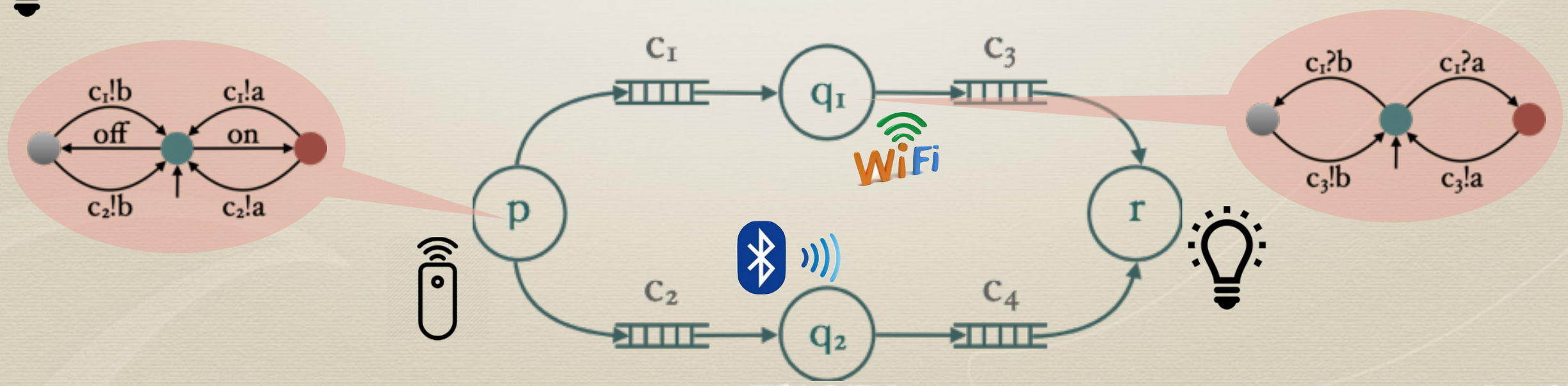
Does it obey the latest order?





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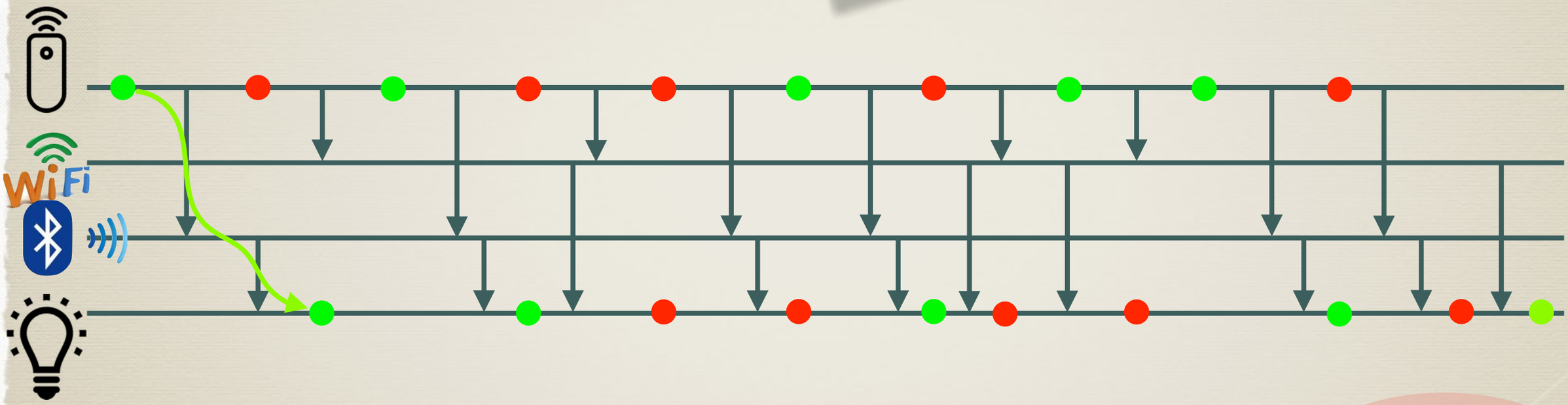

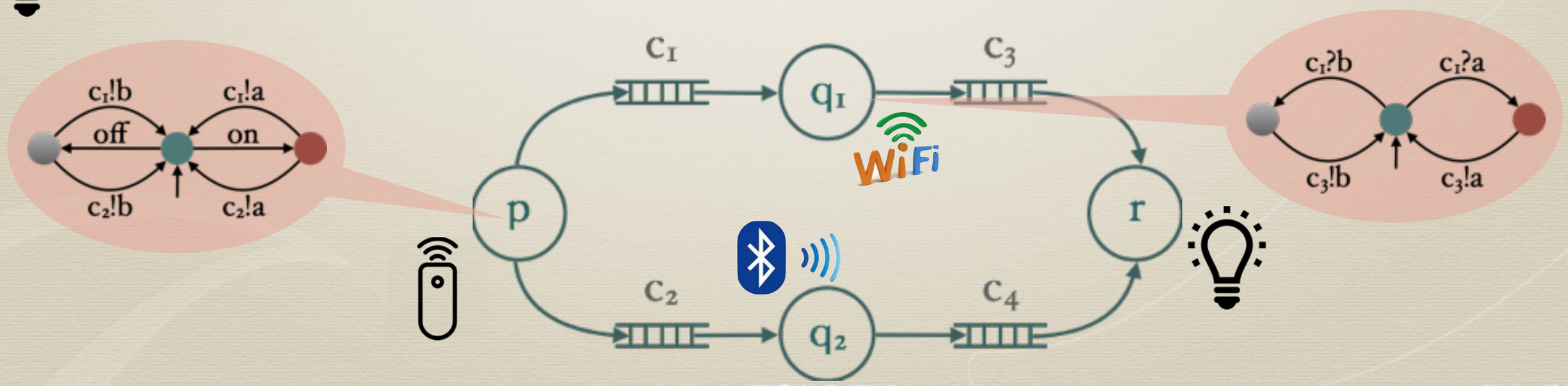

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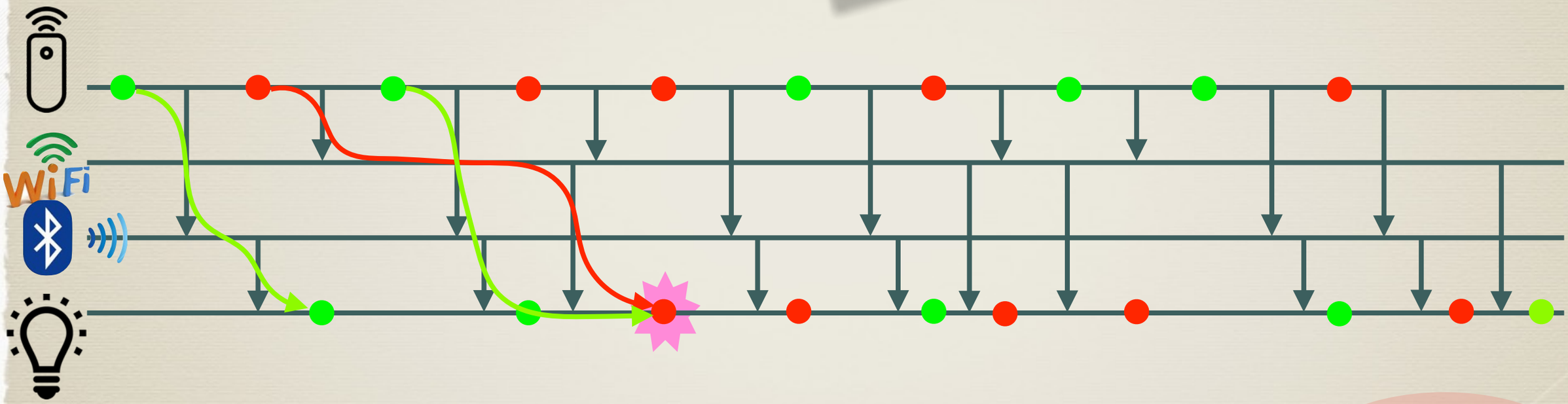
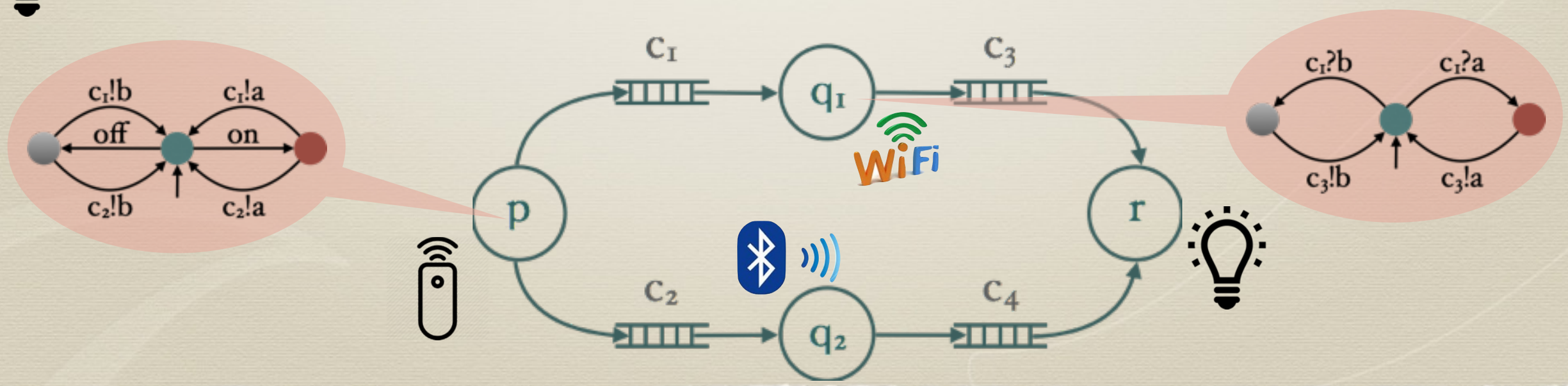
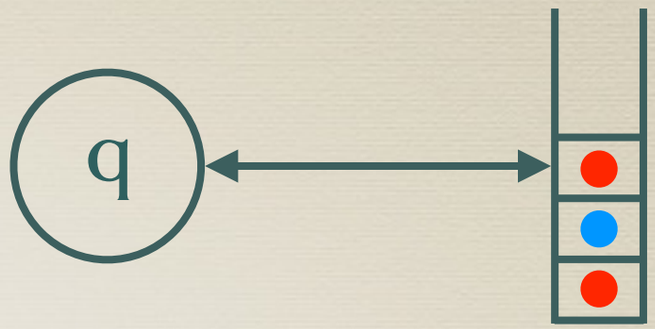


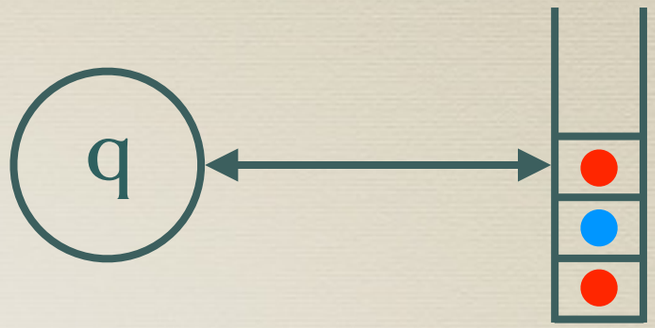


Graphs for Sequential Systems



a↑ba↑c↑aa↓↑b↓a↓cb↑↑a↓↓a↑b↑cb↓a↓↓ab↑b↑aa↑↑↓↑↓↓↓↓b

Graphs for Sequential Systems



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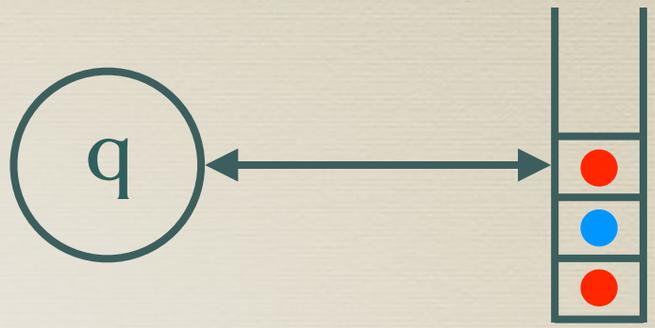
Letter before outermost call is the same as the letter after its return

Graphs for Sequential Systems



a↑ba↑c↑aa↓↑b↓a↓cb↑↑a↓↓a↑b↑cb↓a↓↓ab↑b↑aa↑↑↓↑↓↓↓↓b

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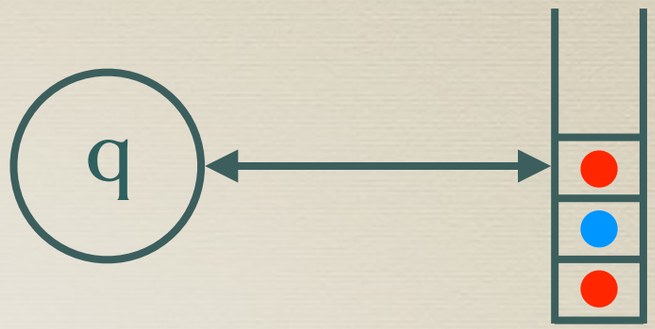
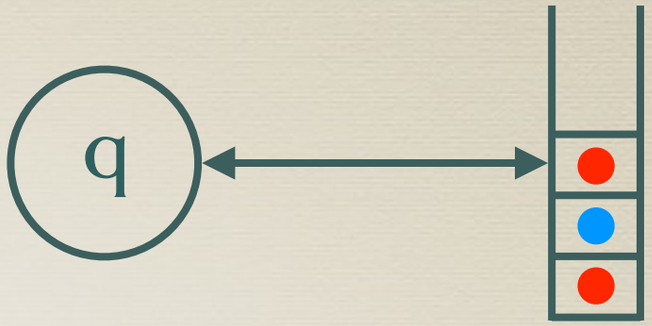


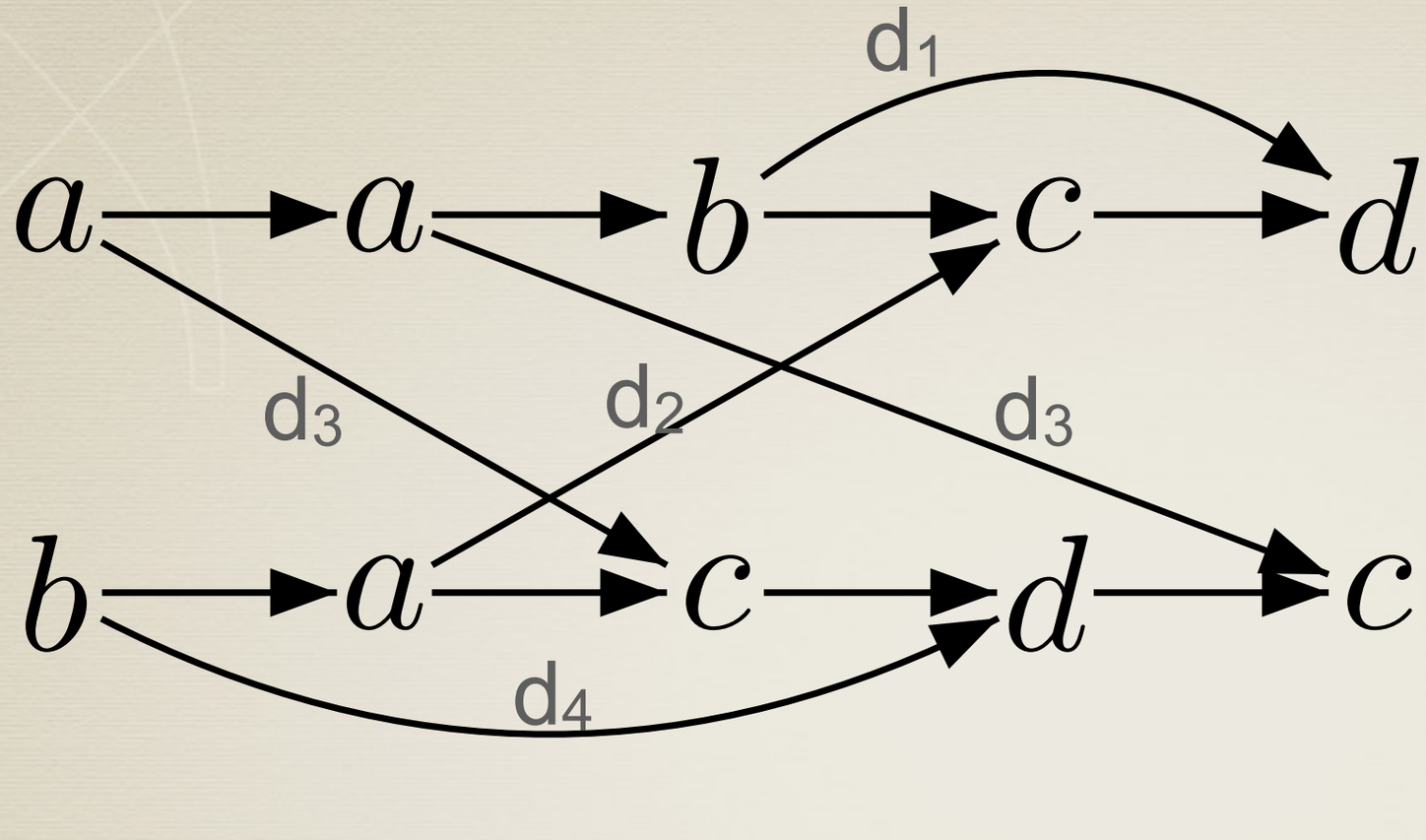
a↑ba↑c↑aa↓↑b↓a↓cb↑↑a↓↓a↑b↑cb↓a↓↓ab↑b↑aa↑↑↓↑↓↓↓↓b

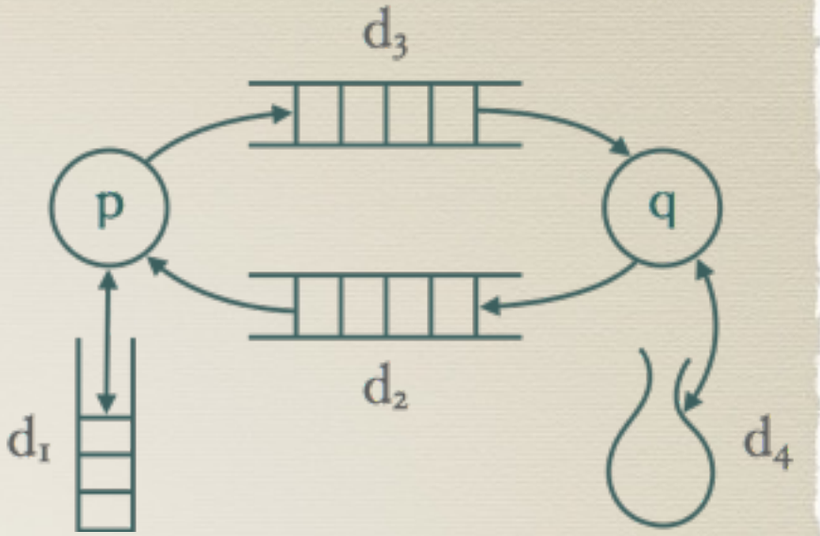
Graphs for Sequential Systems



Nested Words
Alur, Madhusudan, 2009

Concurrent Behaviour with Matching (CBM)





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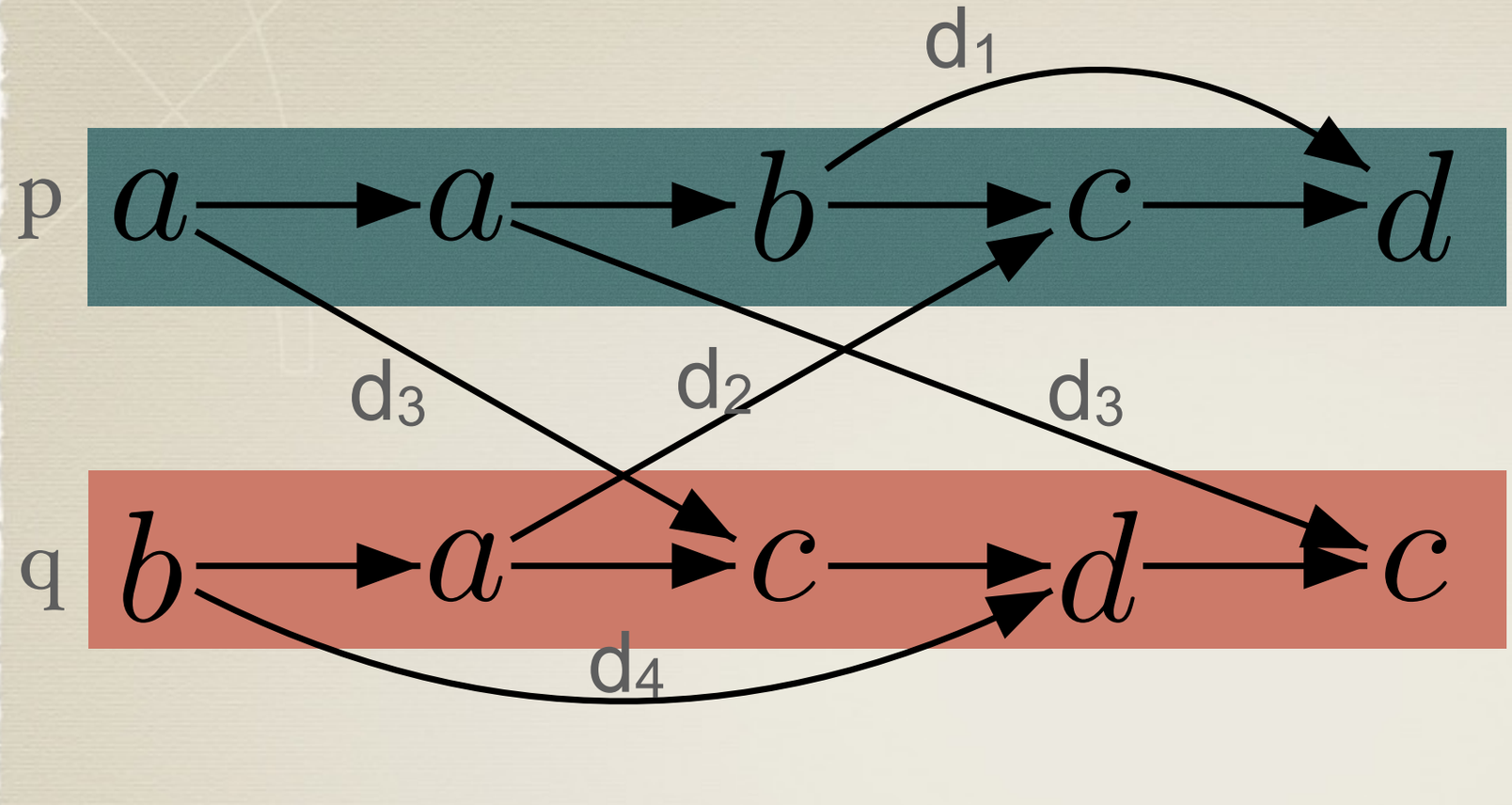
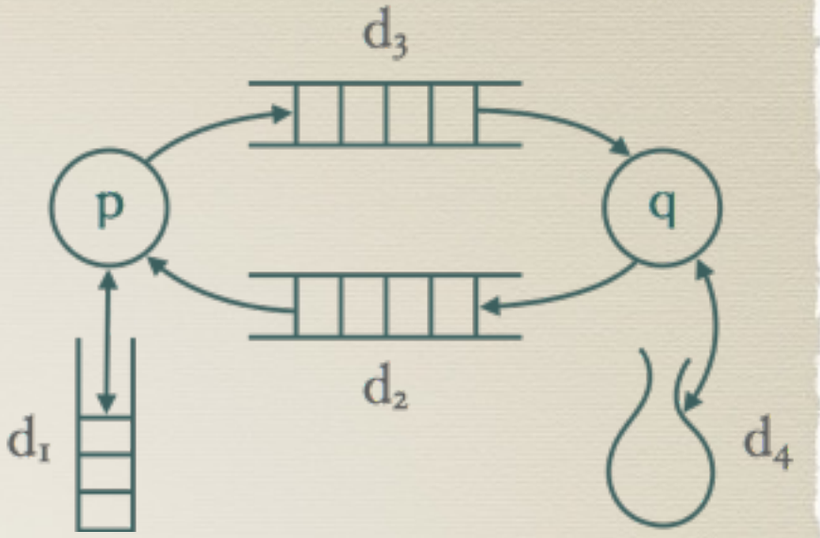
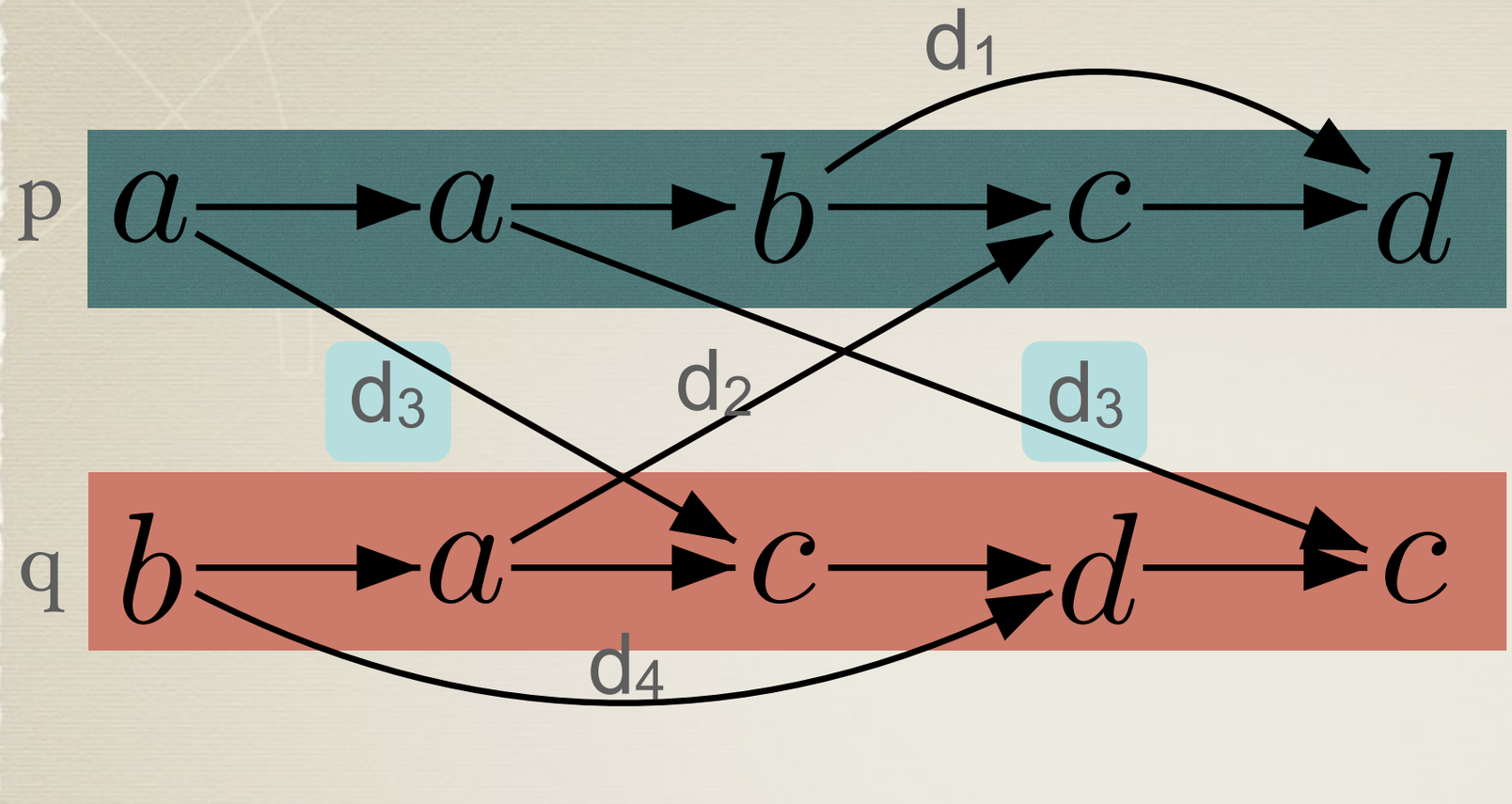
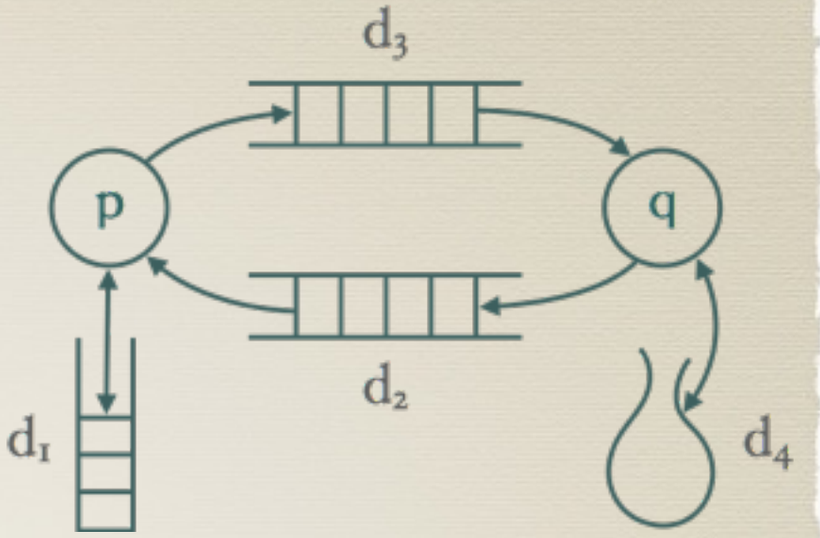




* A linear order (or path) for each process
* Edges labeled with data structures

Concurrent Behaviour with Matching (CBM)





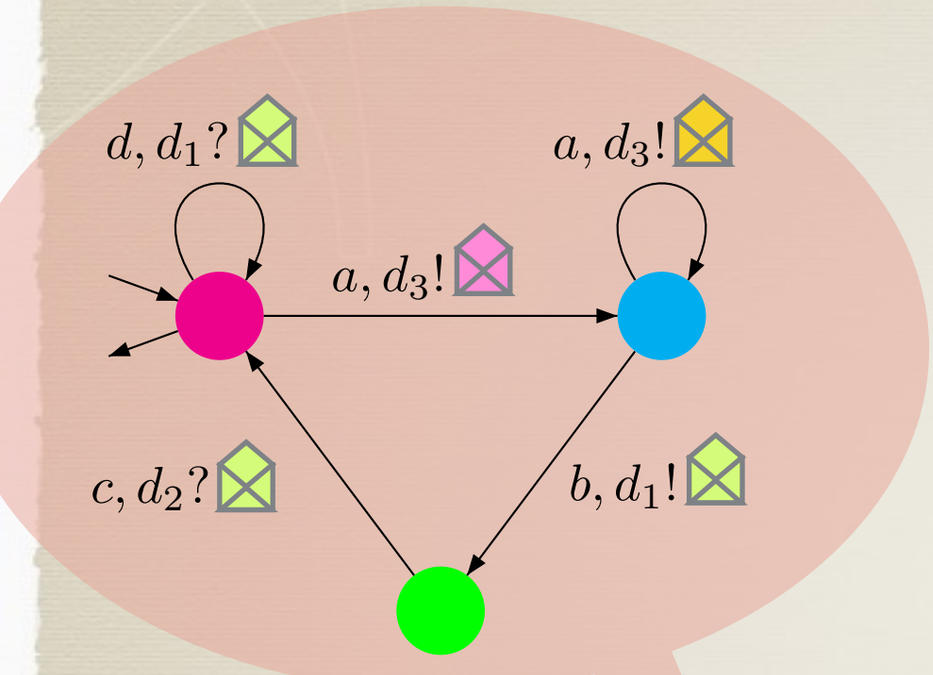
* A linear order (or path) for each process
* Edges labeled with data structures
  * Communication edges form a matching
  * Edge labelled d relates the writer and reader of d
  * Edges follow the discipline of the data structure
    * LIFO/FIFO/Bag

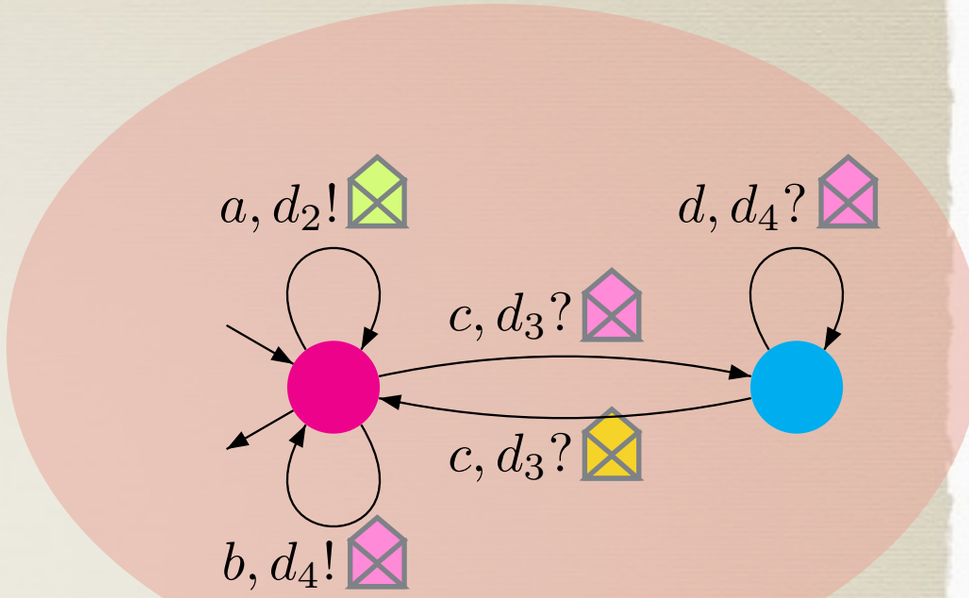
Graphs vs Linear Traces

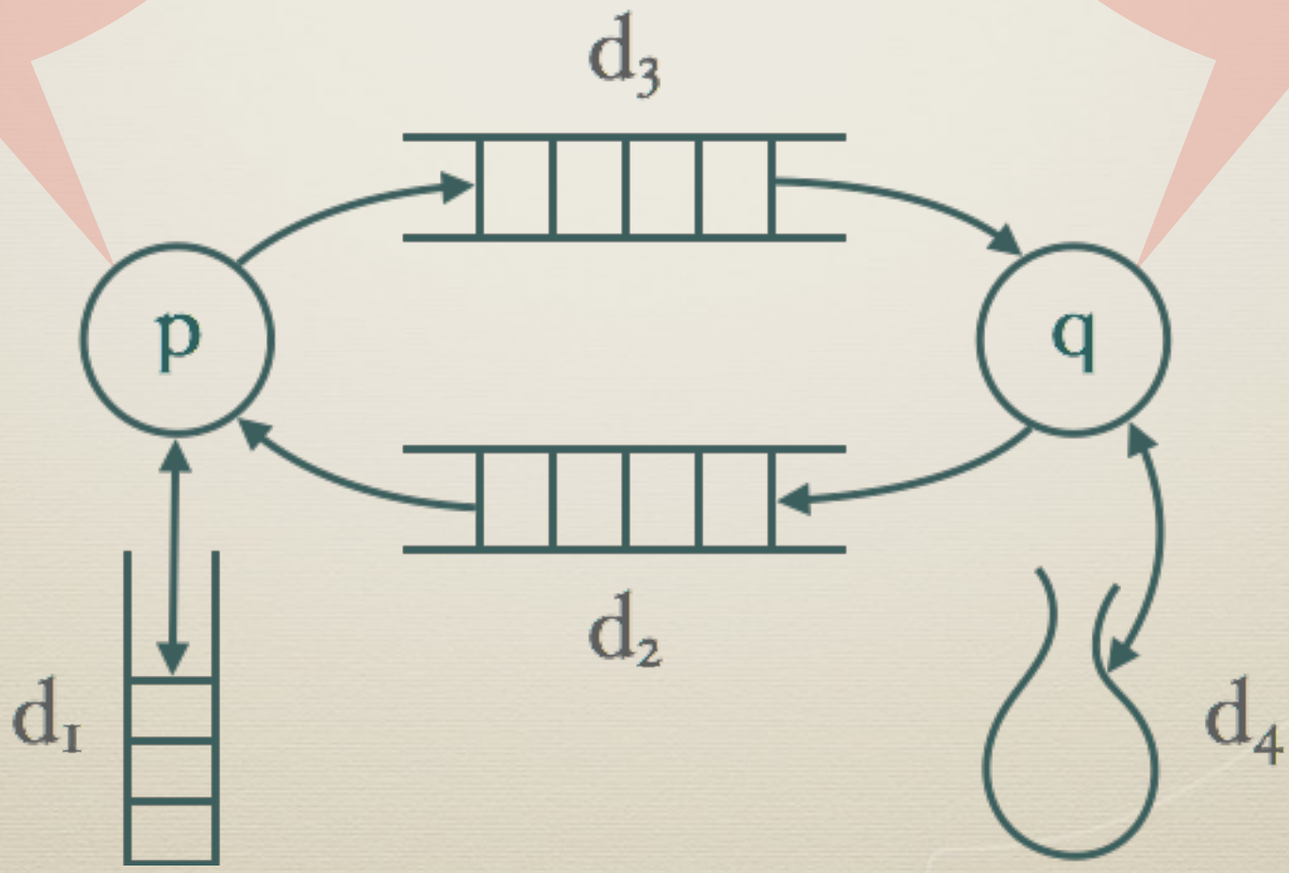
Understanding Behaviors

Linear Traces	Graphs (CBMs)
• Interactions are obfuscated and very difficult to recover. • Successor relation not meaningful • Combinatorial explosion single distributed behavior results in a huge number of linear traces	• Interactions are visible • no combinatorial explosion

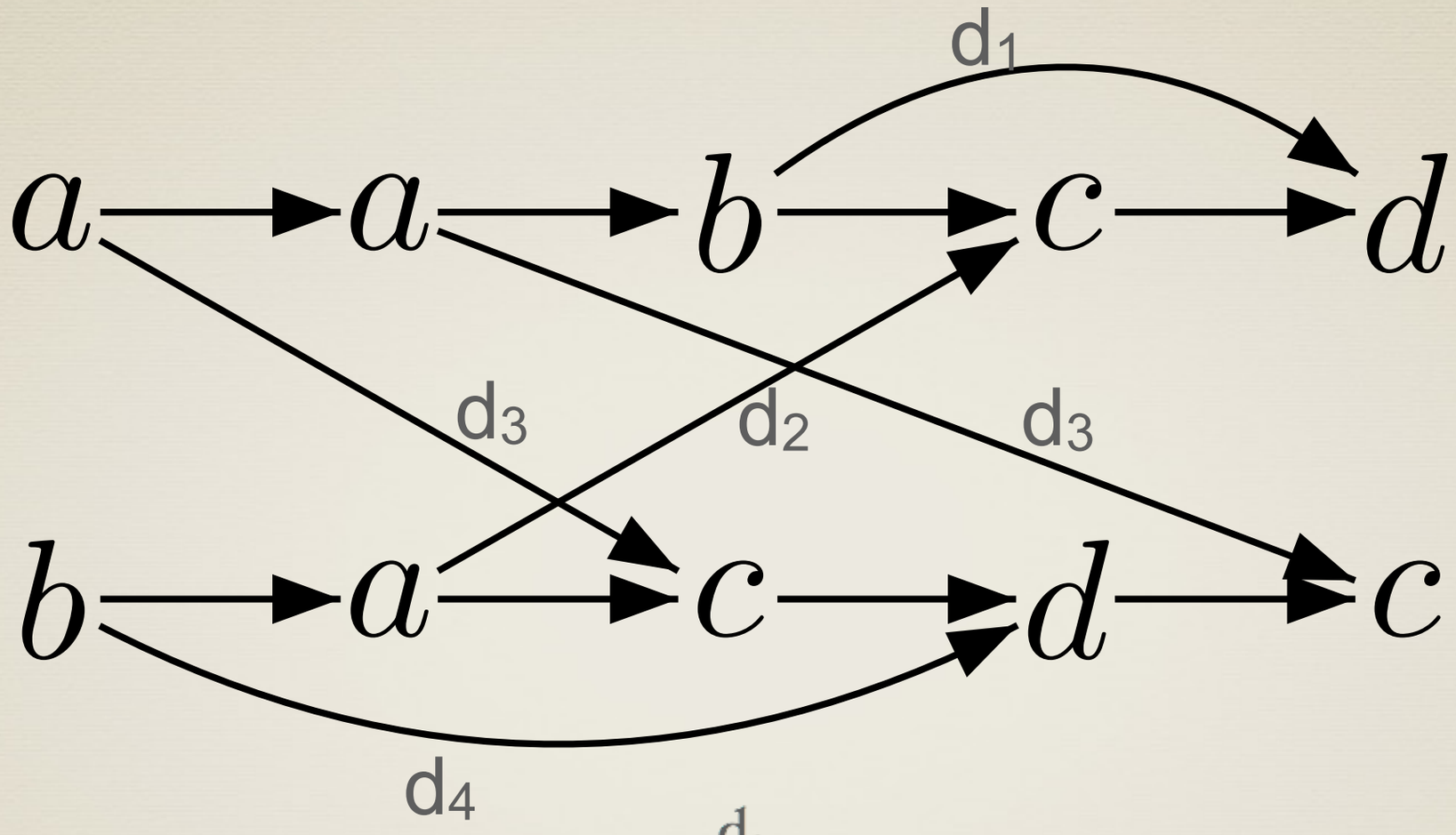
Semantics of CPDS on CBMs

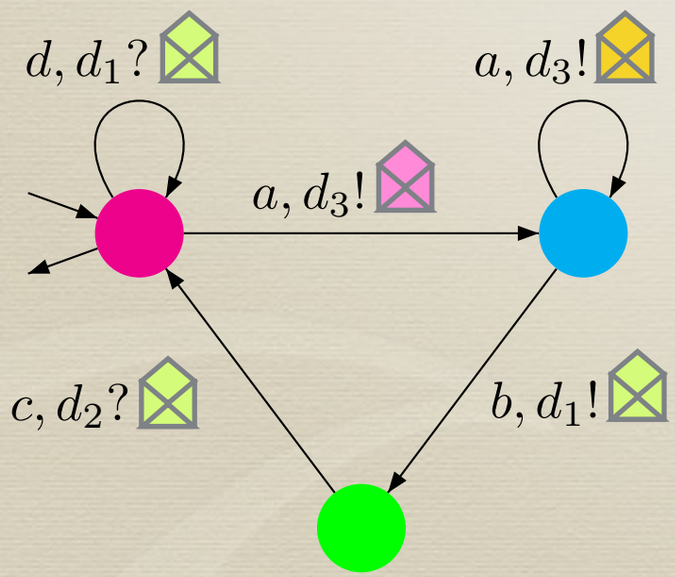


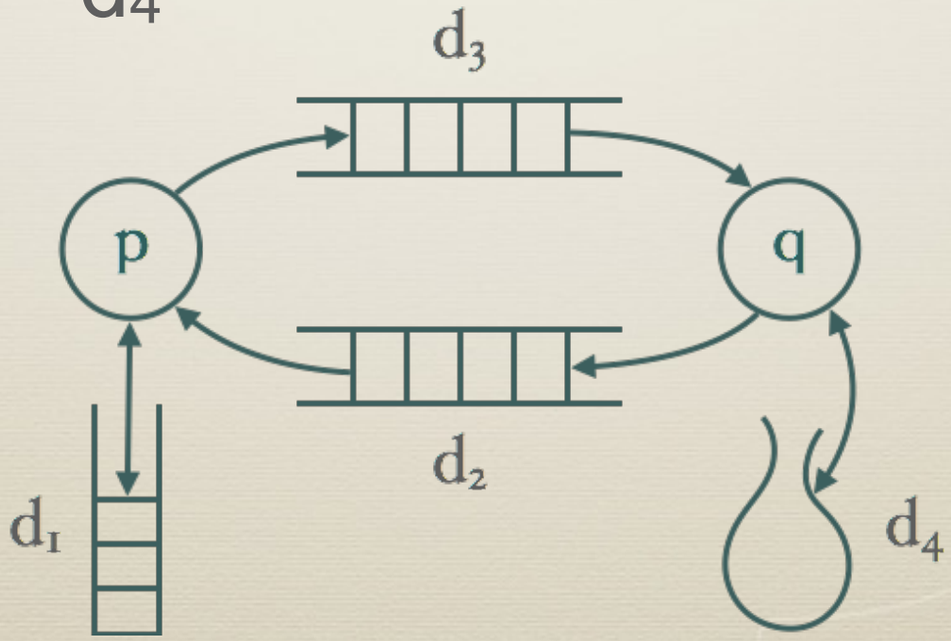


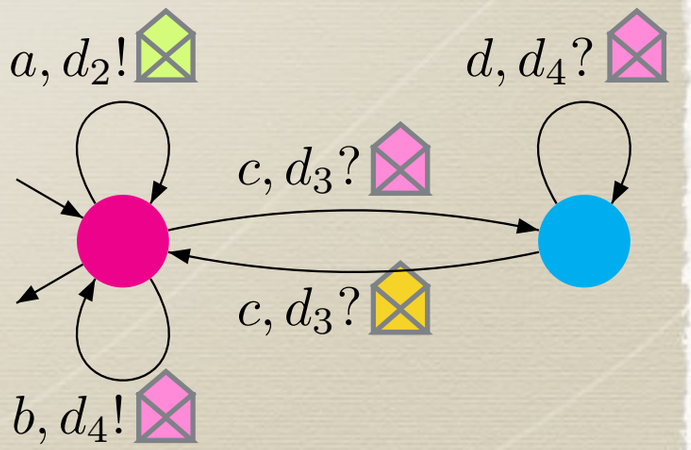


Semantics of CPDS on CBMs

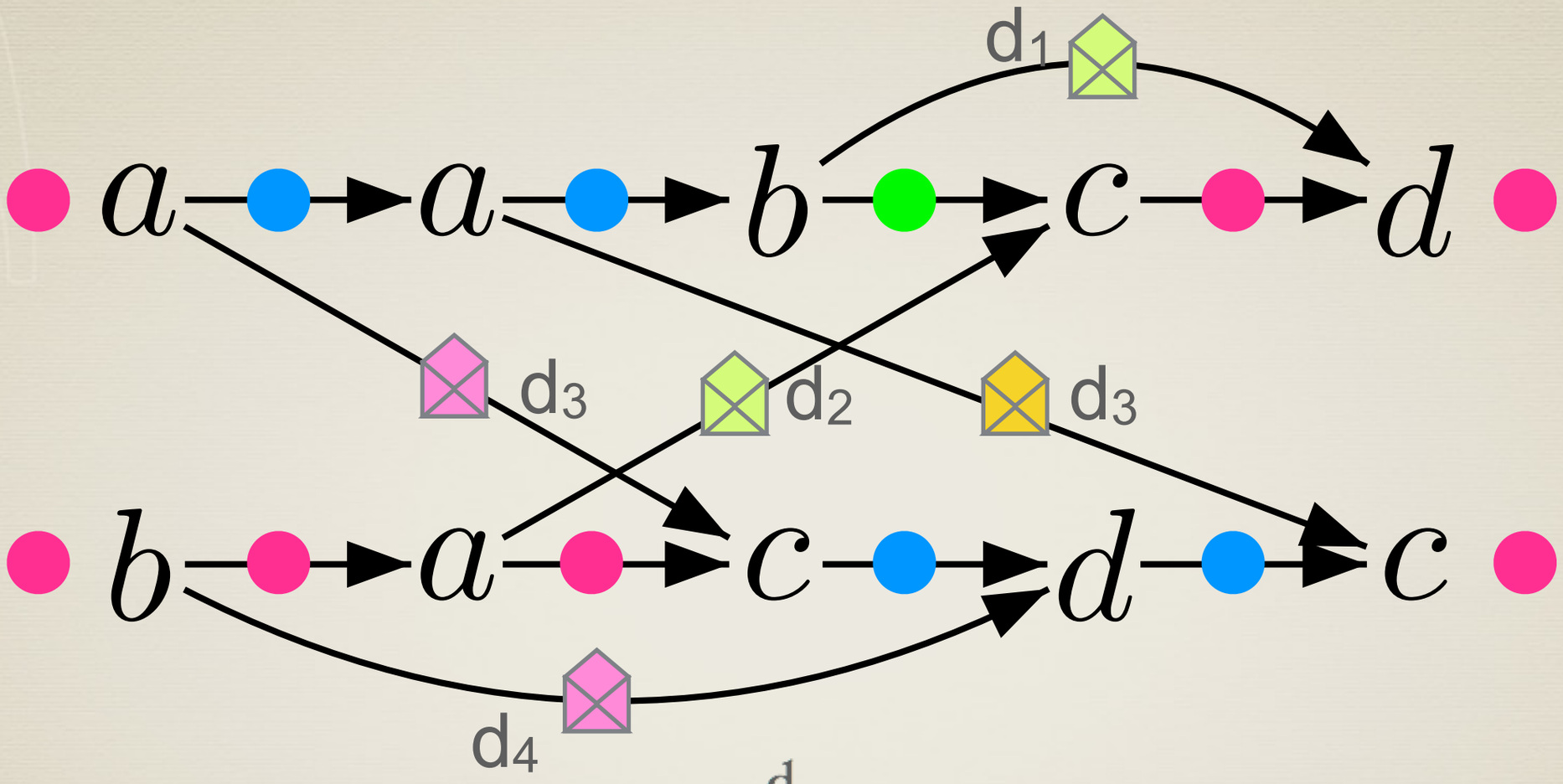


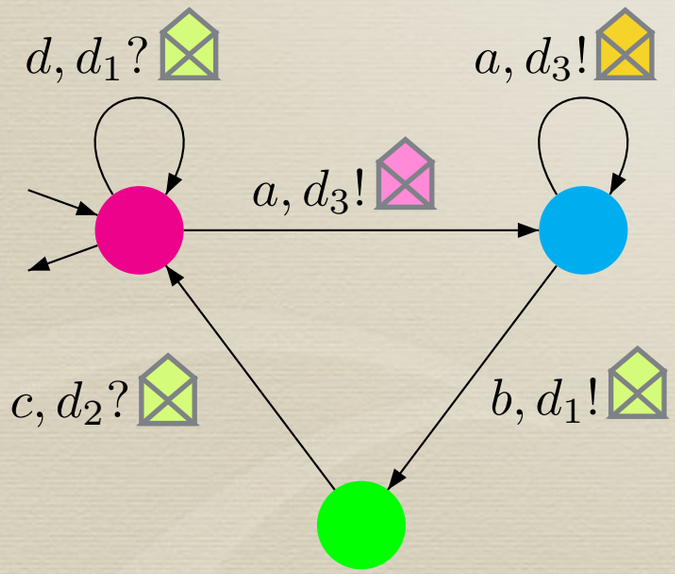


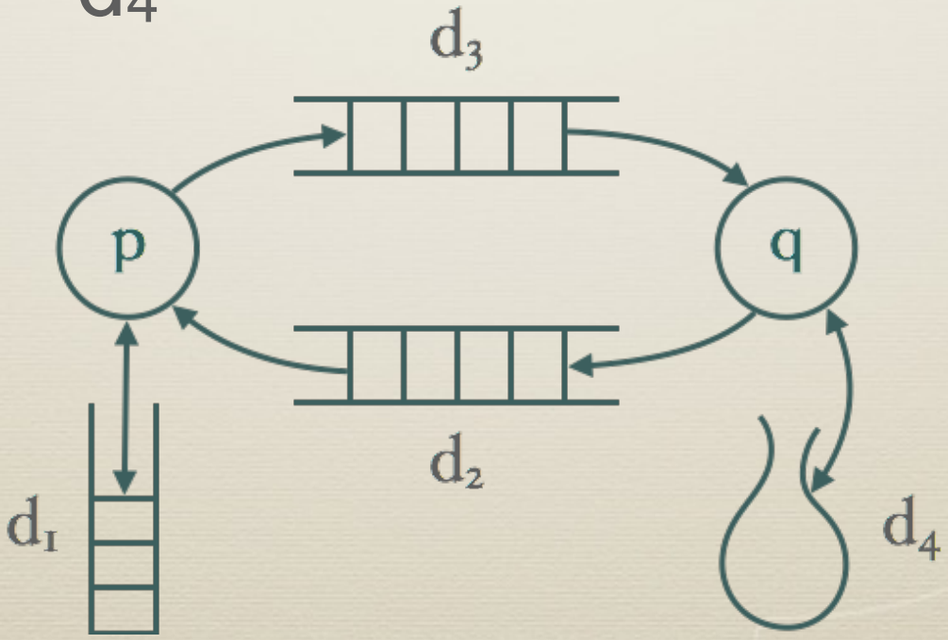


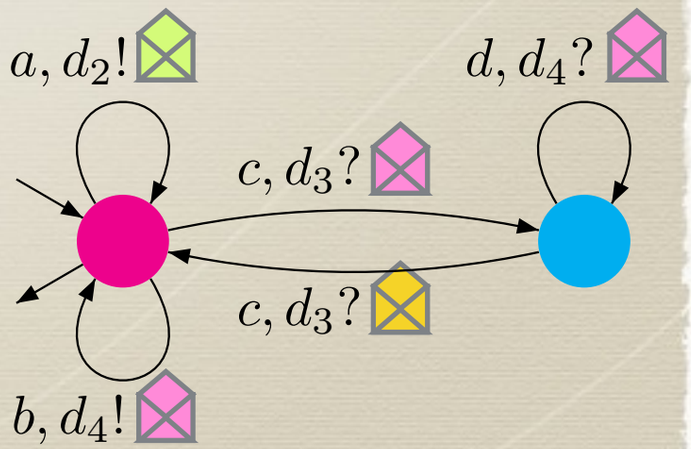


Semantics of CPDS on CBMs









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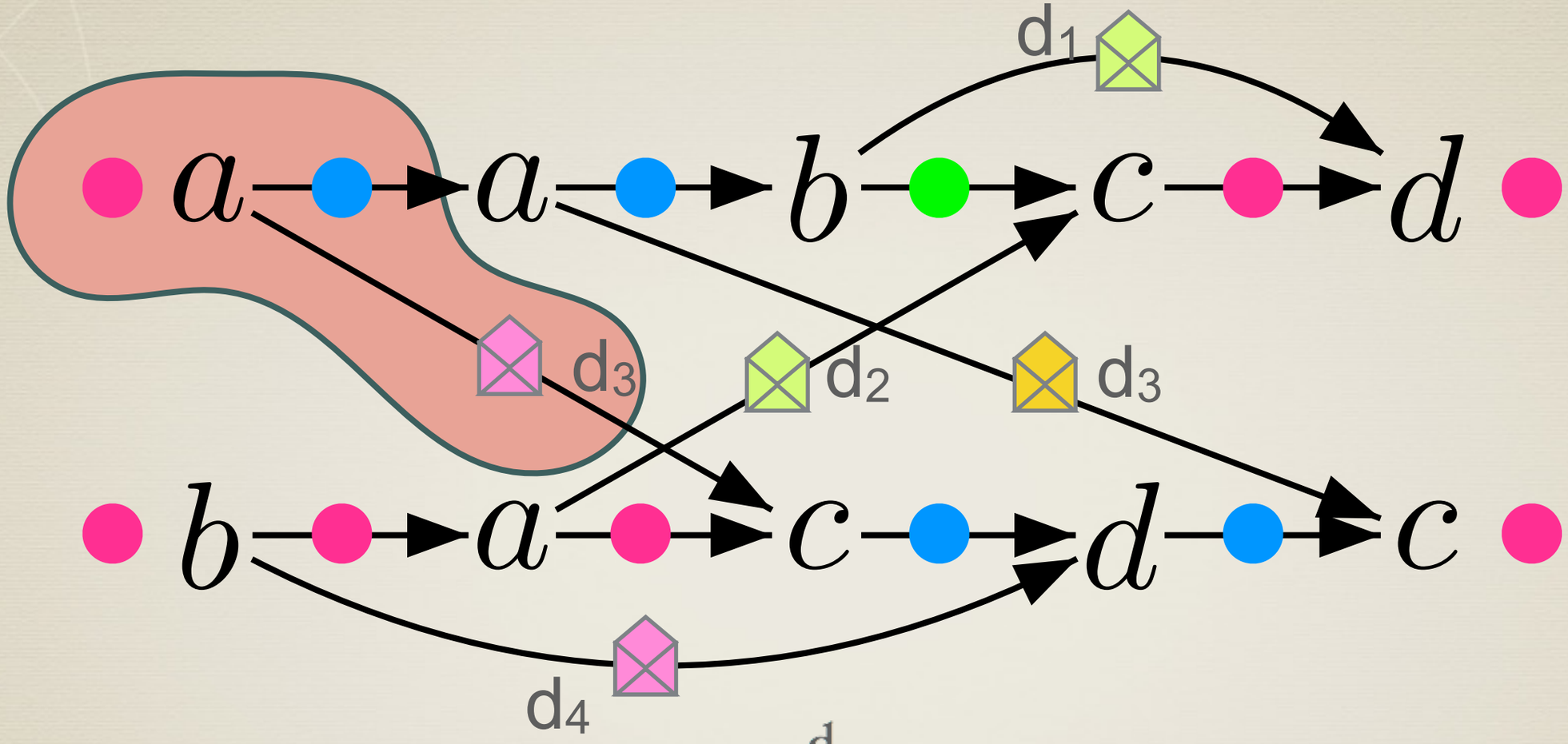

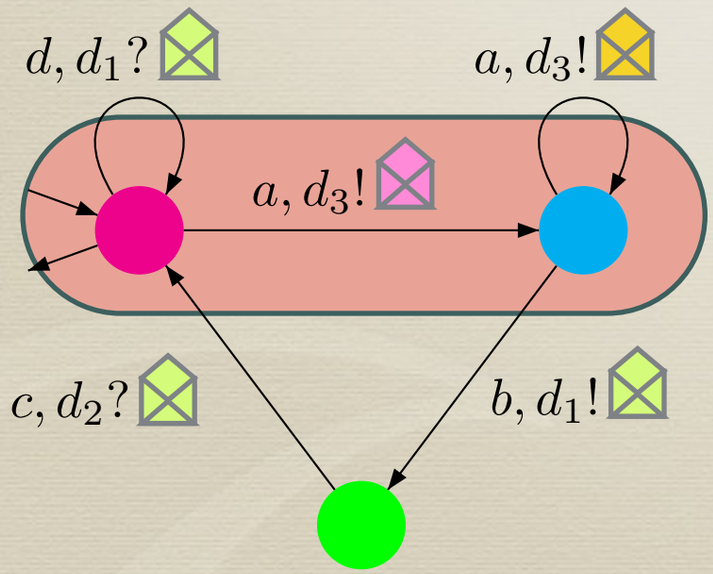

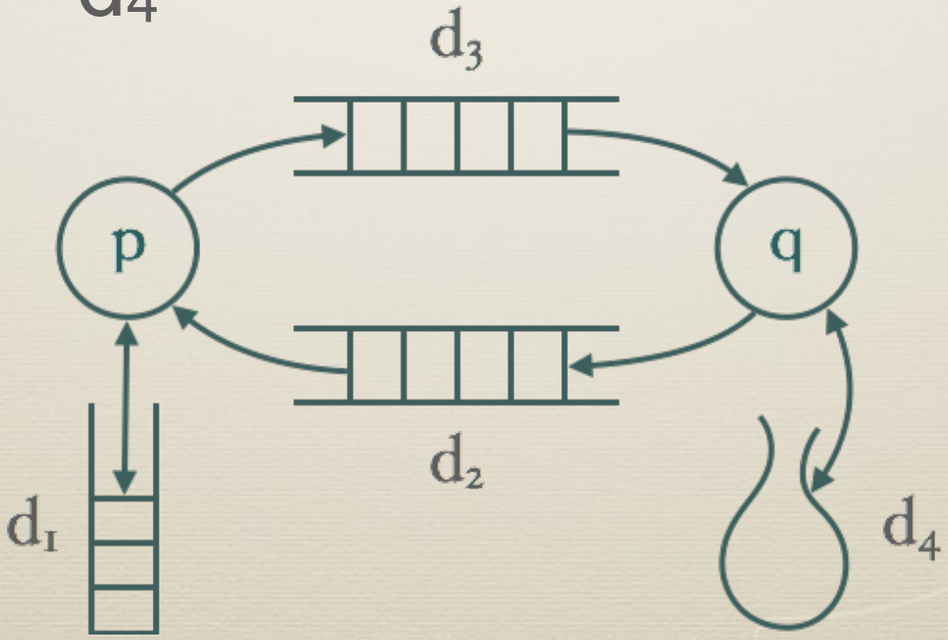

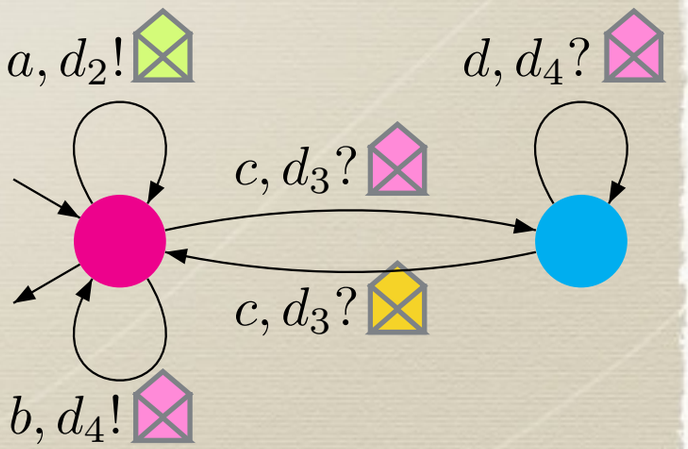

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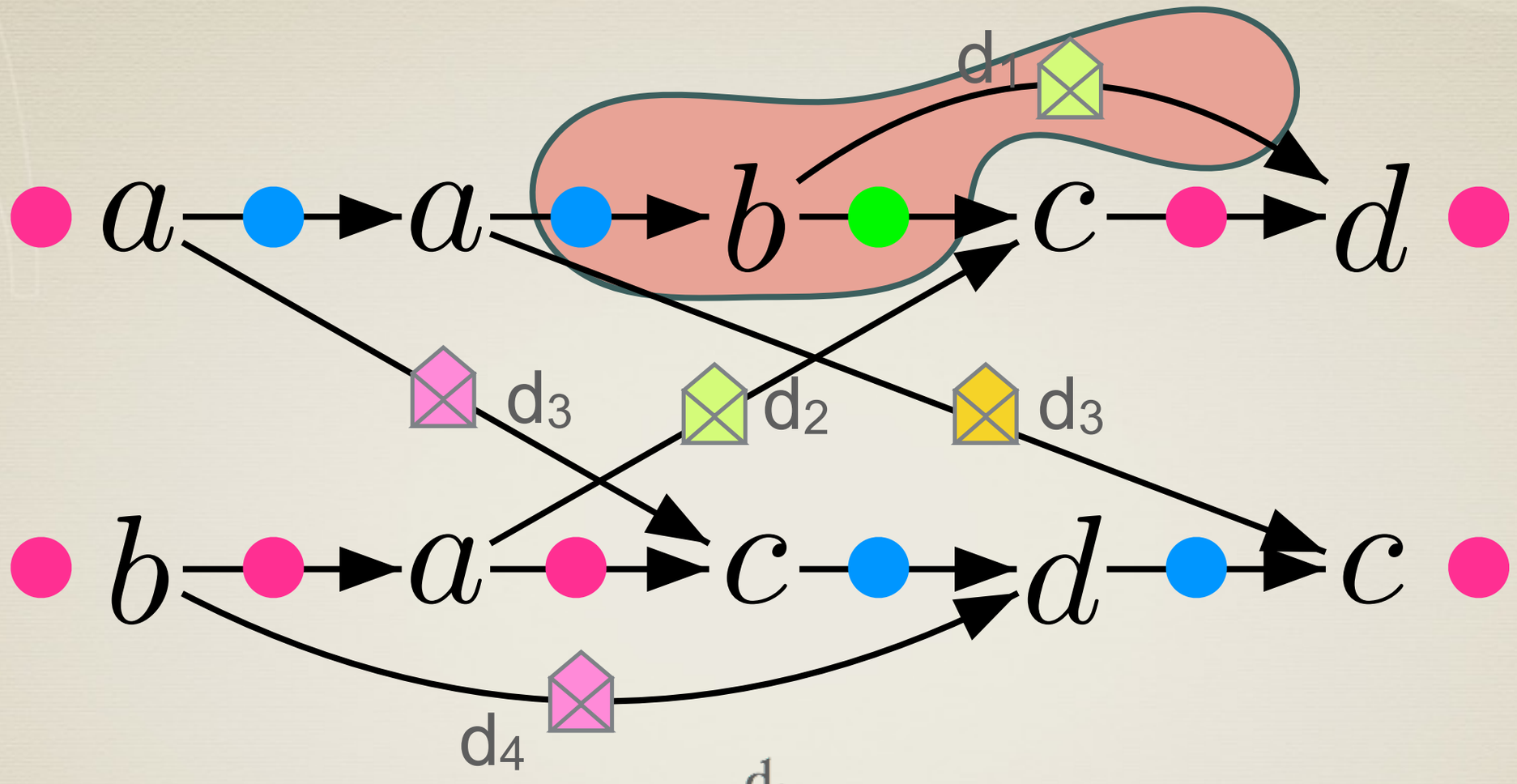

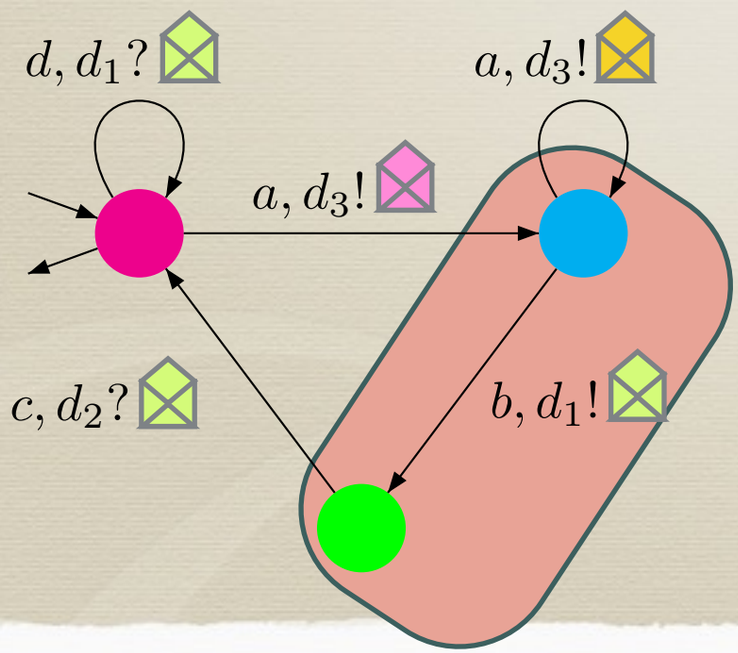

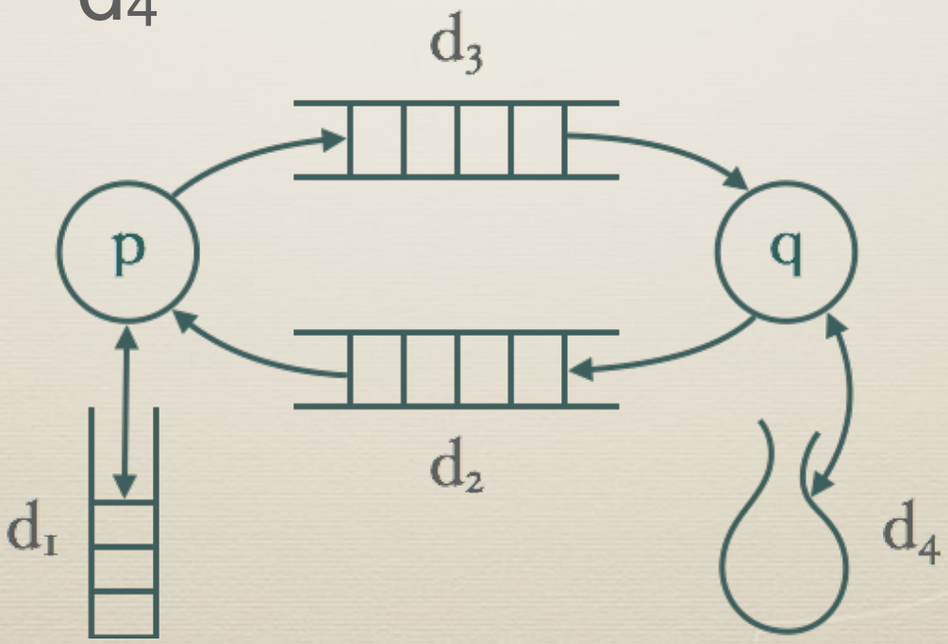

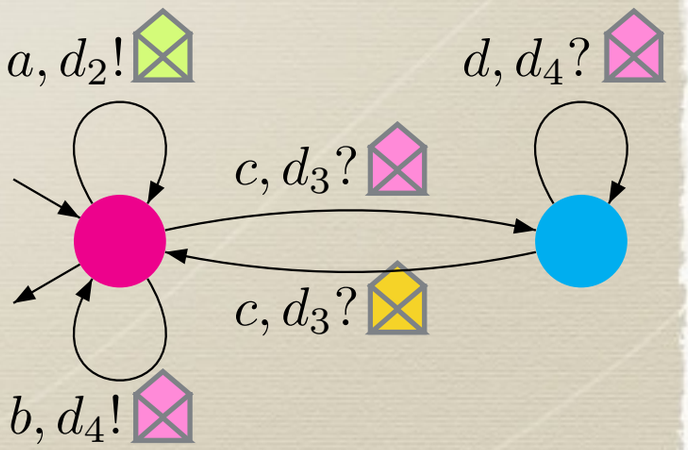

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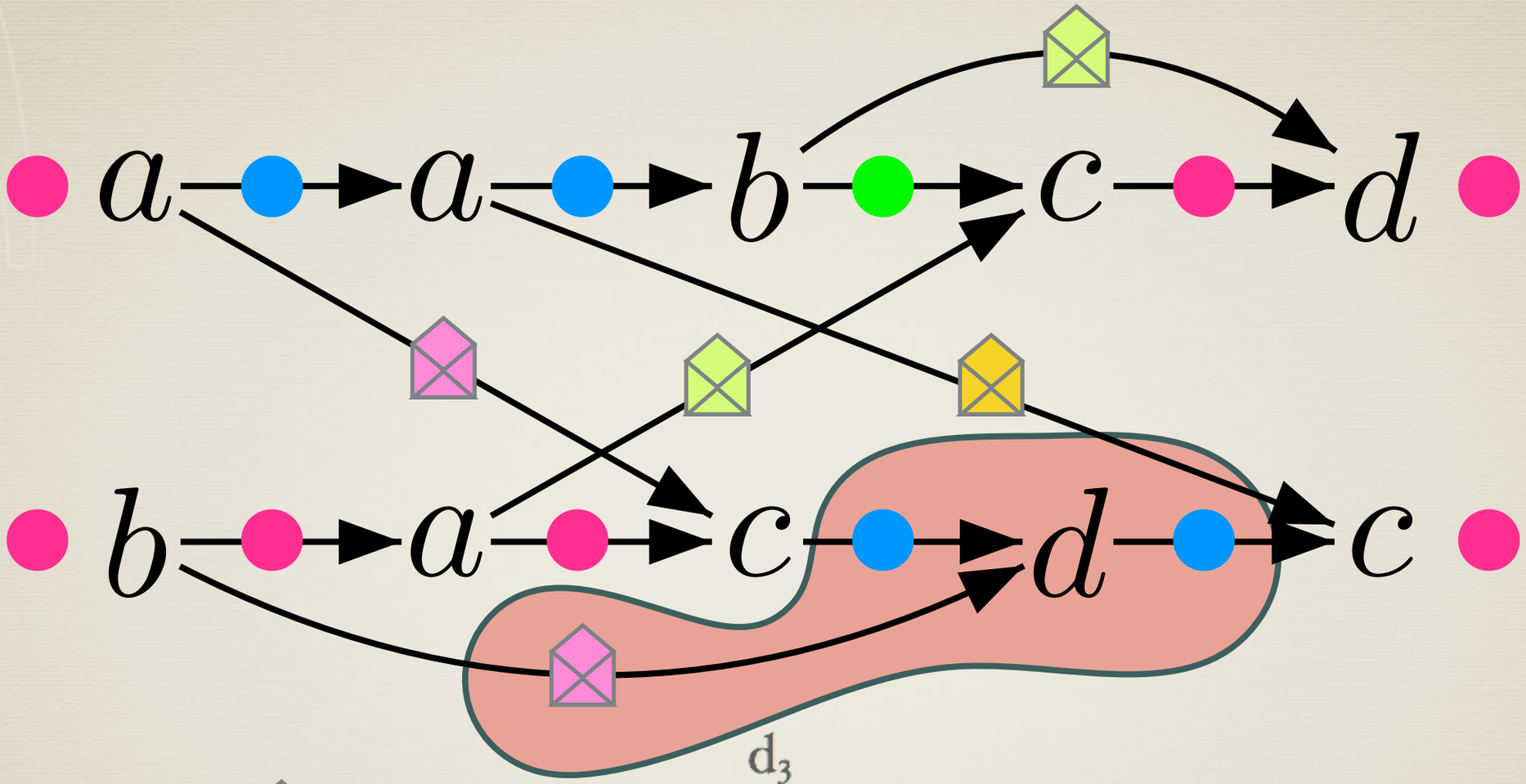

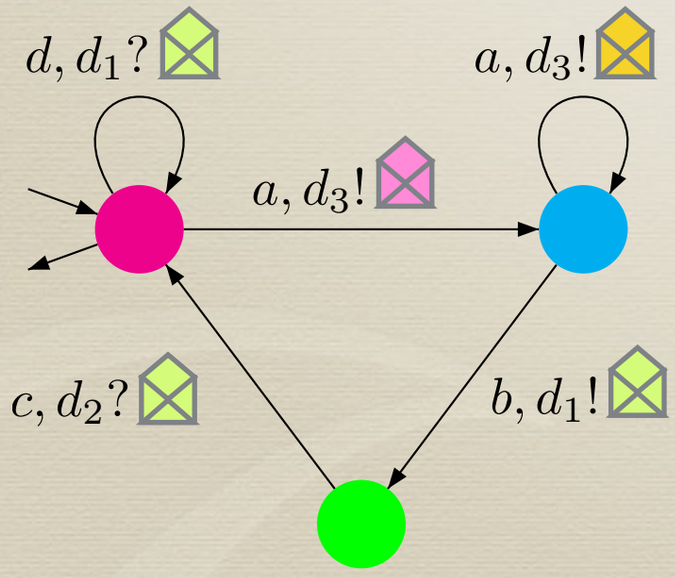

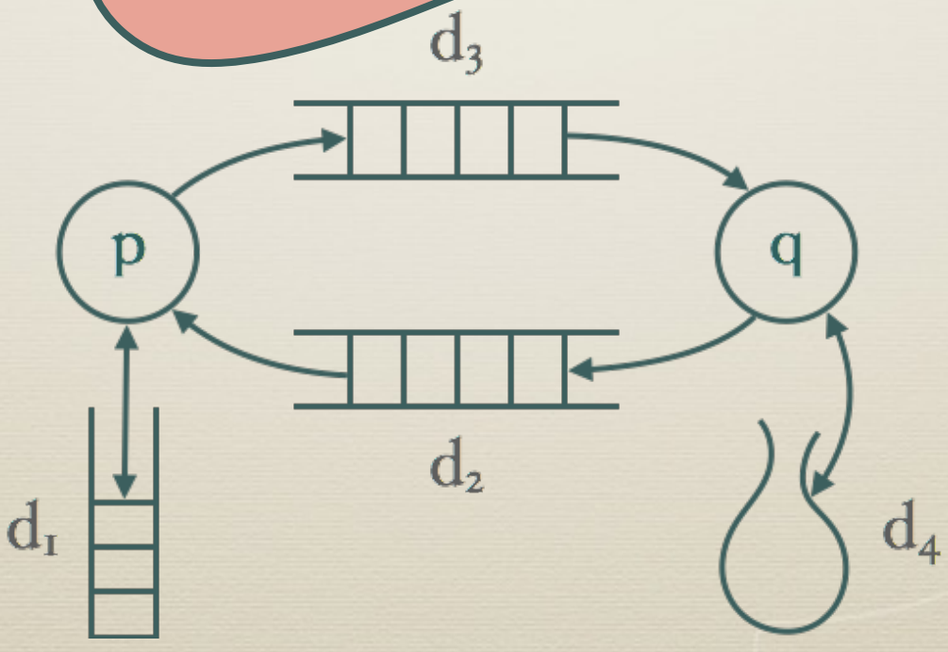

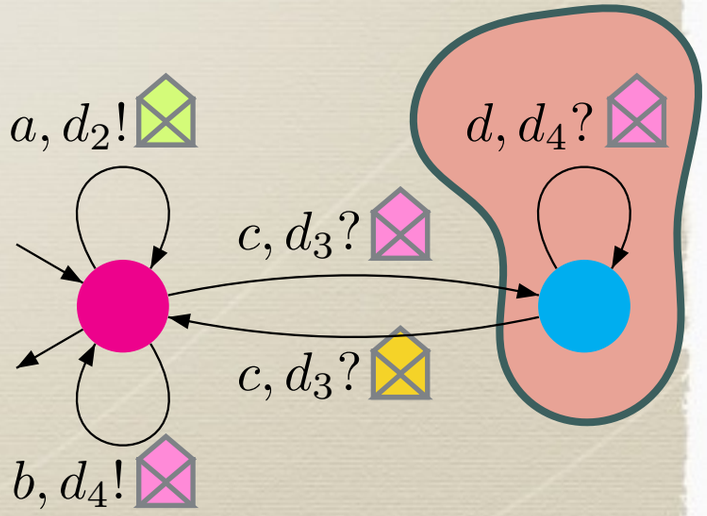

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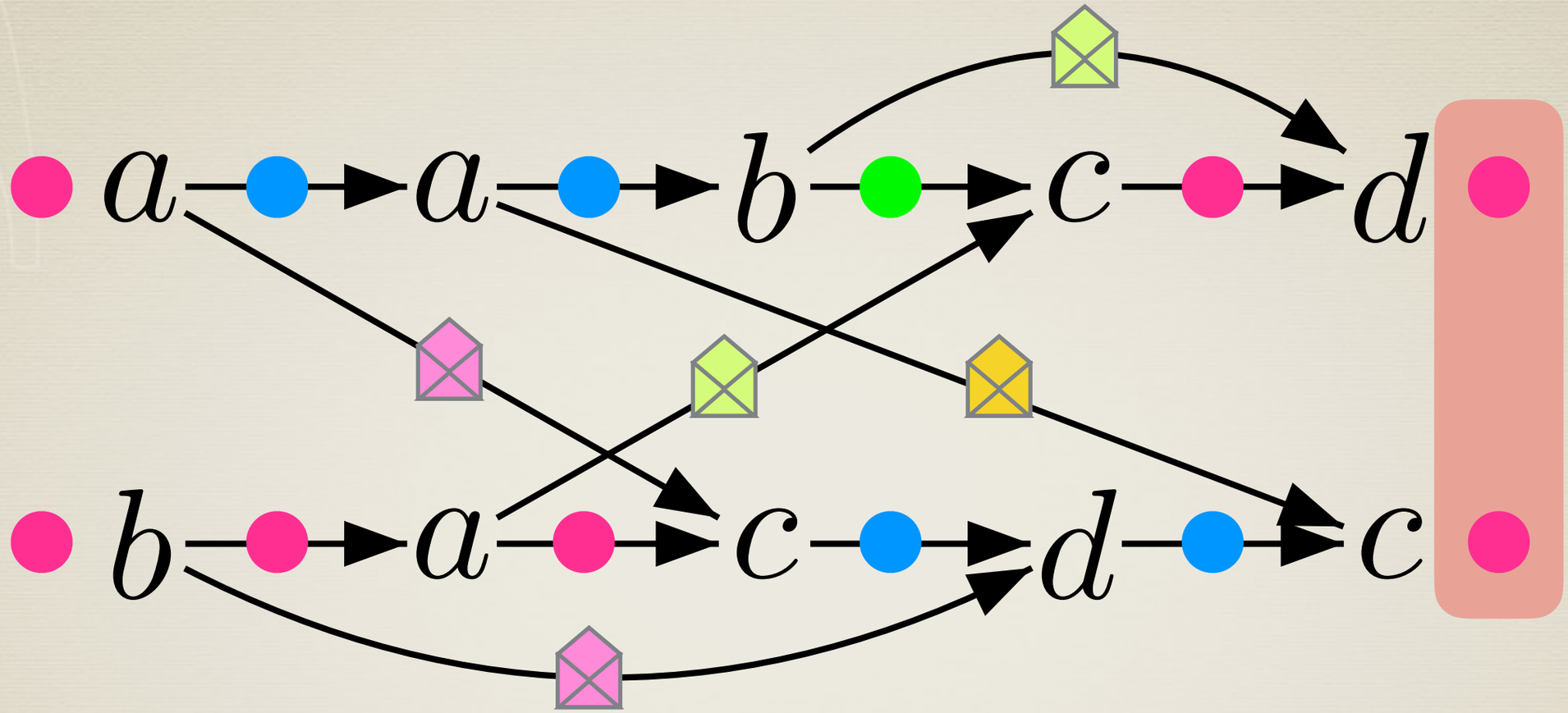

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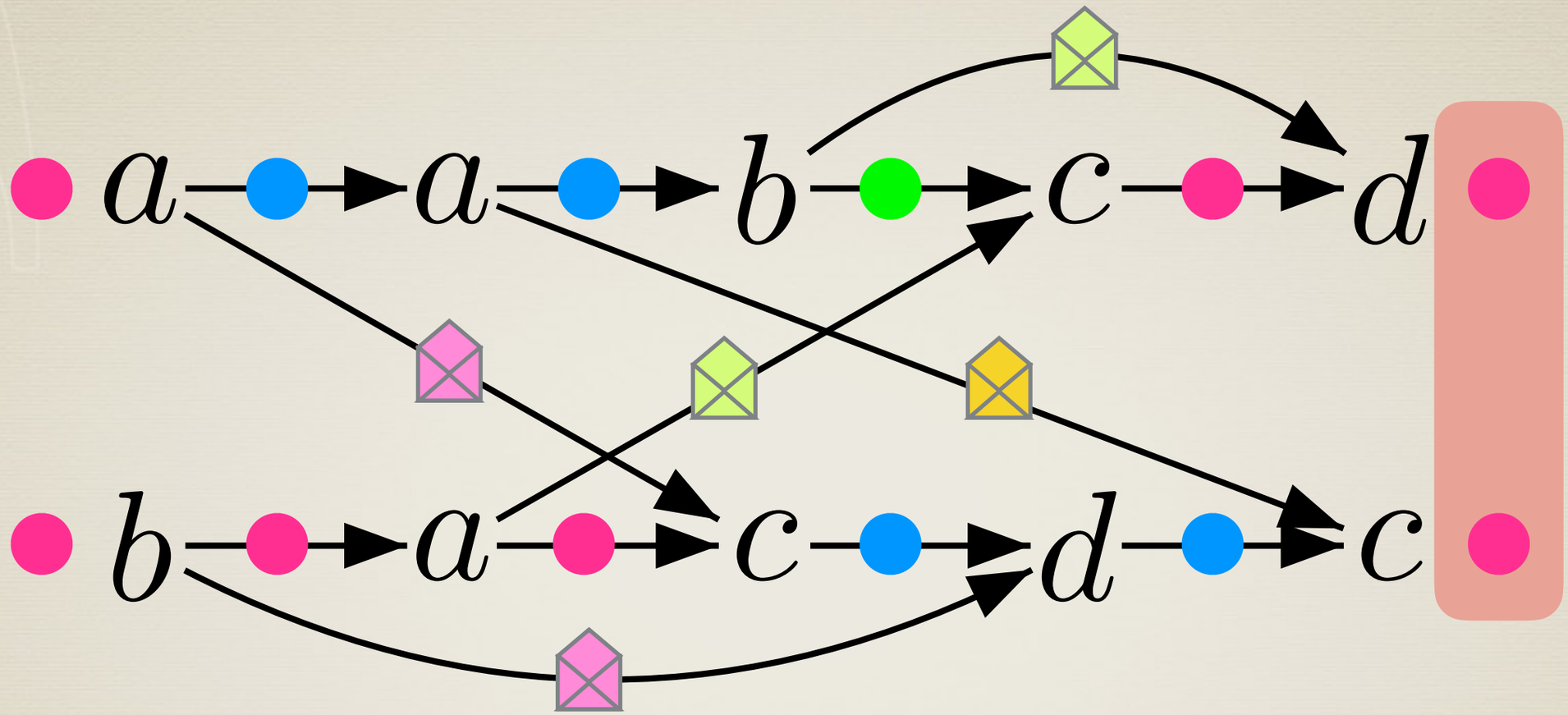
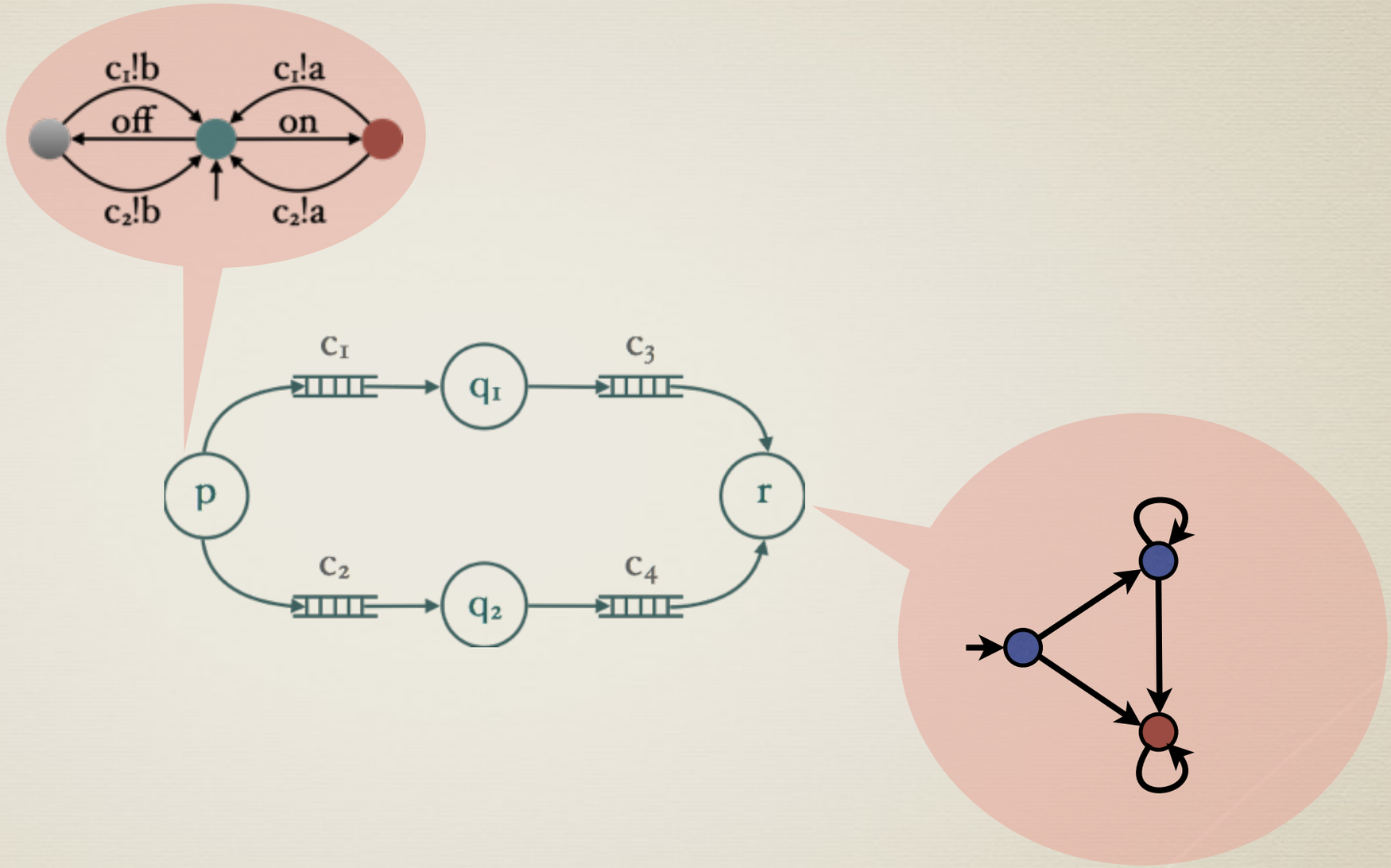


Accepting states to turn them into language acceptors

Outline

* Models
* Behaviours
* Specifications
* Verification via graph decompositions
* Conclusion

Reachability



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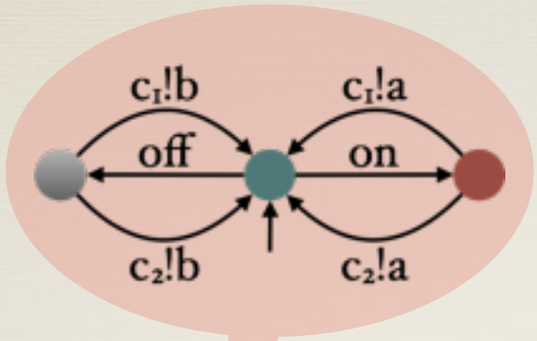
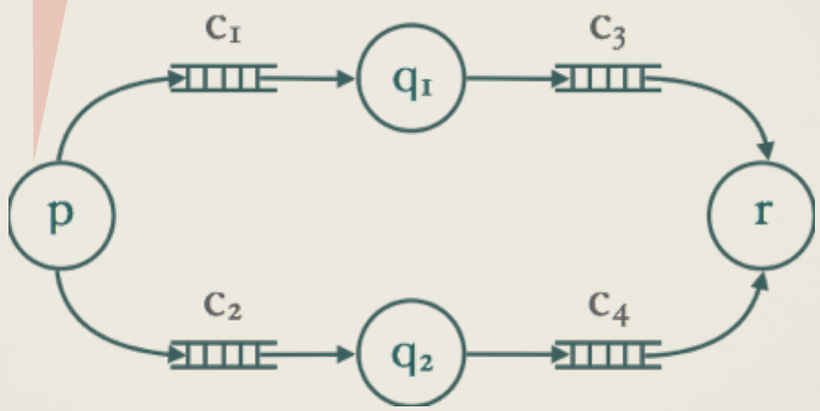
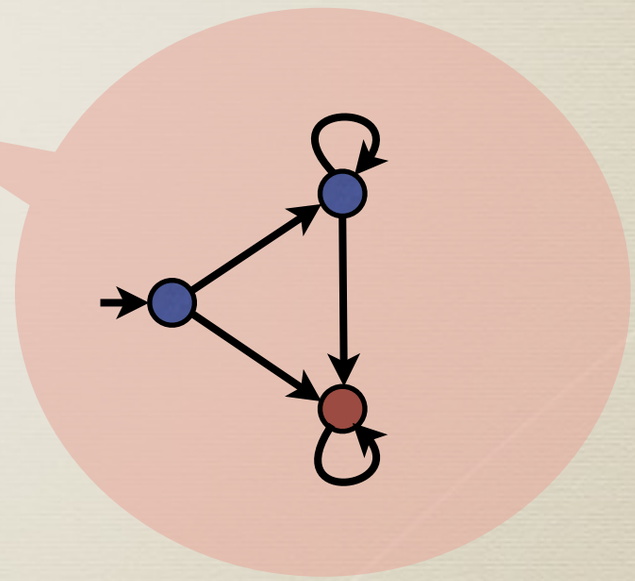
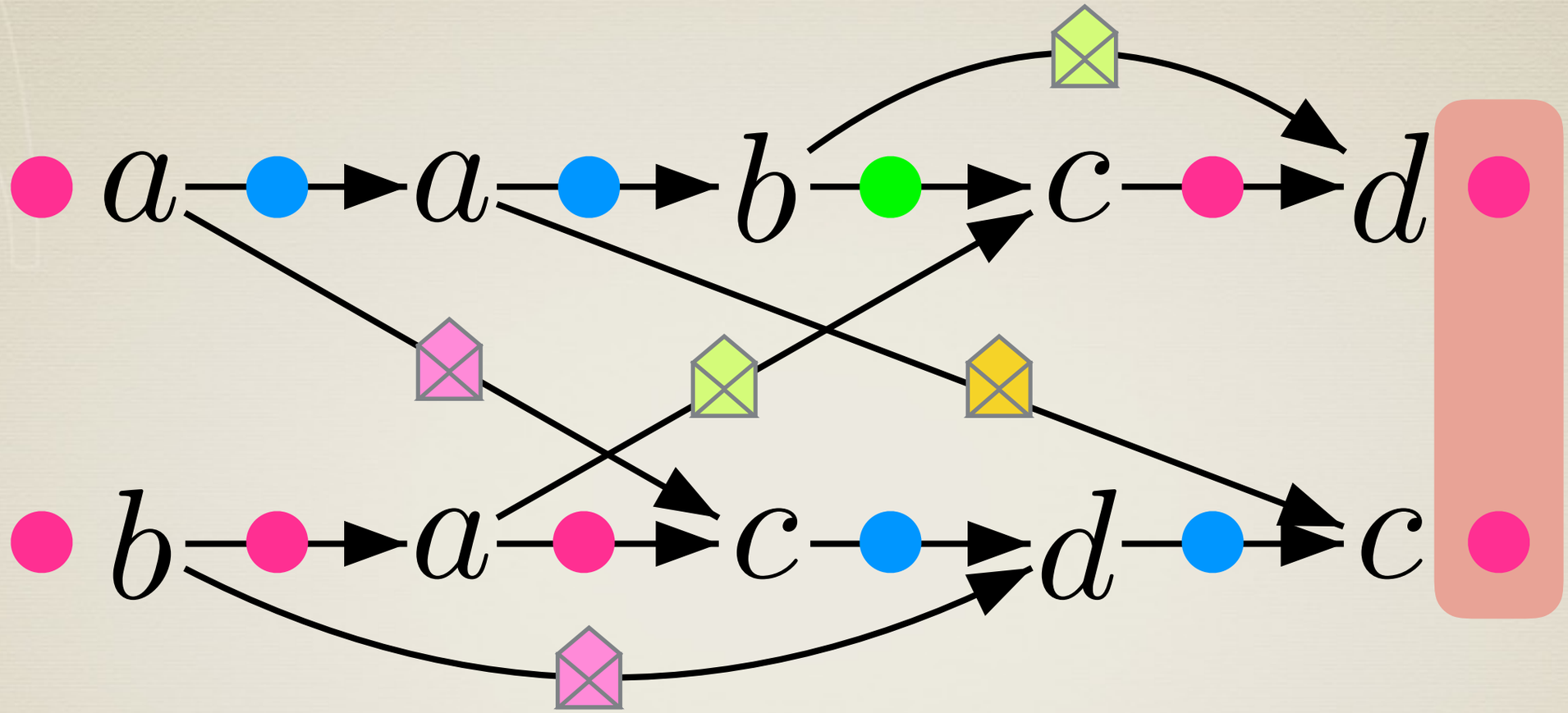






Is the bad state reachable?

Semantics of CPDS on CBMs



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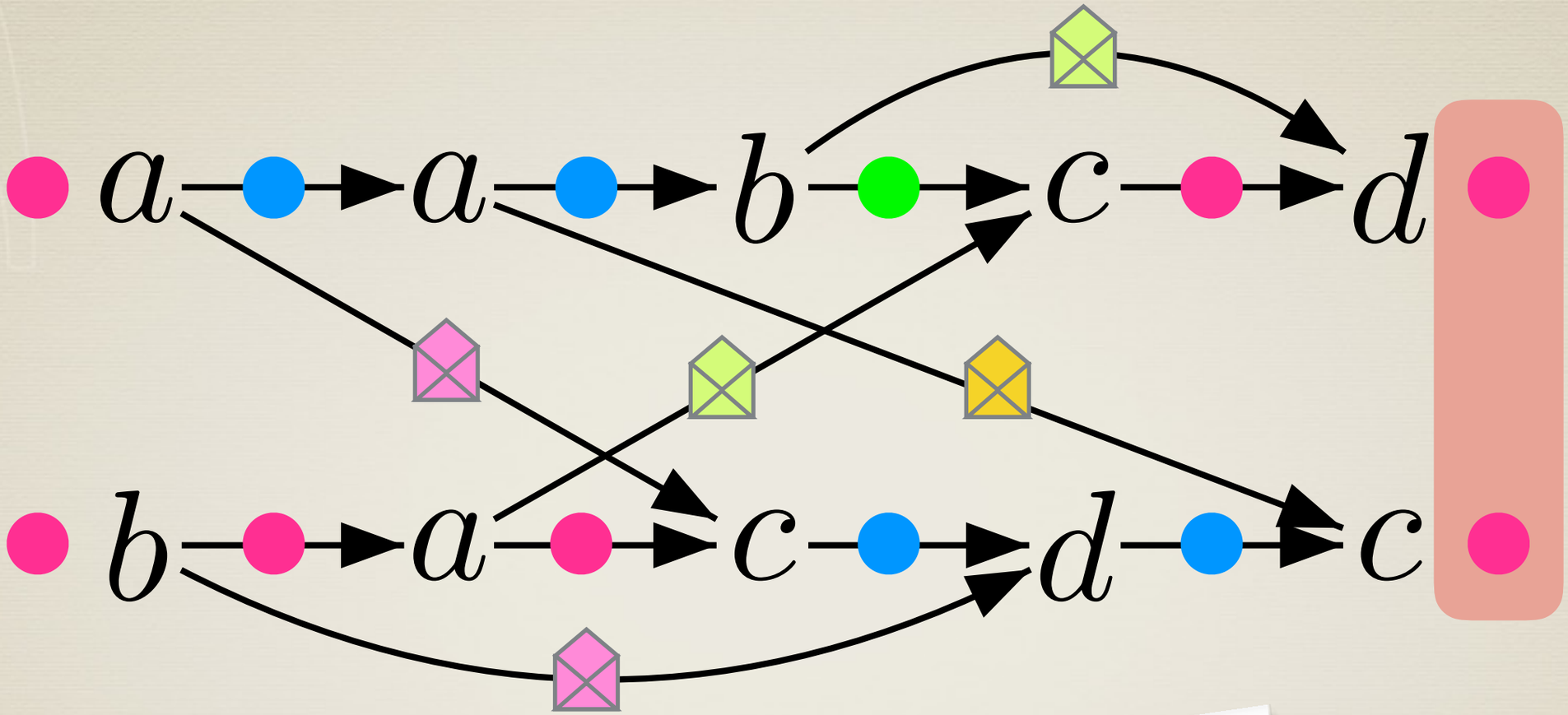
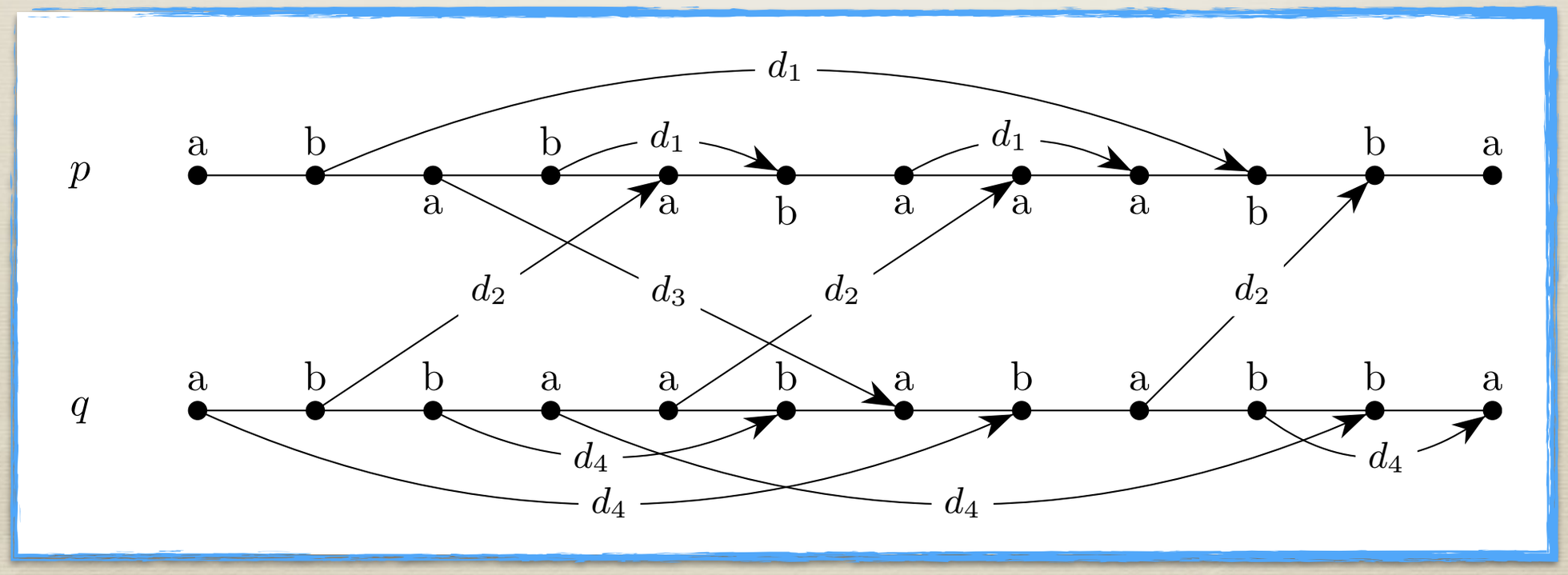


Reachability reduces to Language Emptiness

Specification over CBMs

MSO: Monadic Second Order Logic

$$\varphi ::= \mathsf{false} \mid a(x) \mid p(x) \mid x \leq y \mid x \rhd^{d} y \mid x \to y$$
$$\mid x \in X \mid \varphi \vee \varphi \mid \neg\varphi \mid \exists x\, \varphi \mid \exists X\, \varphi$$



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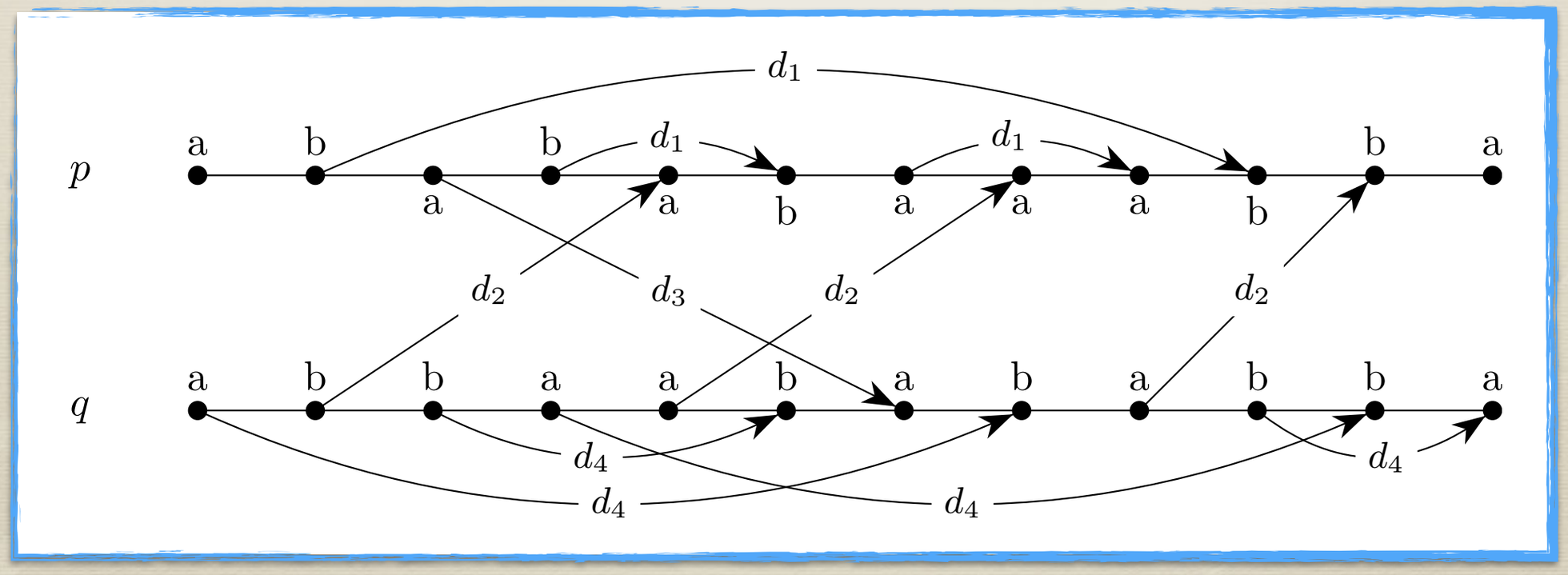

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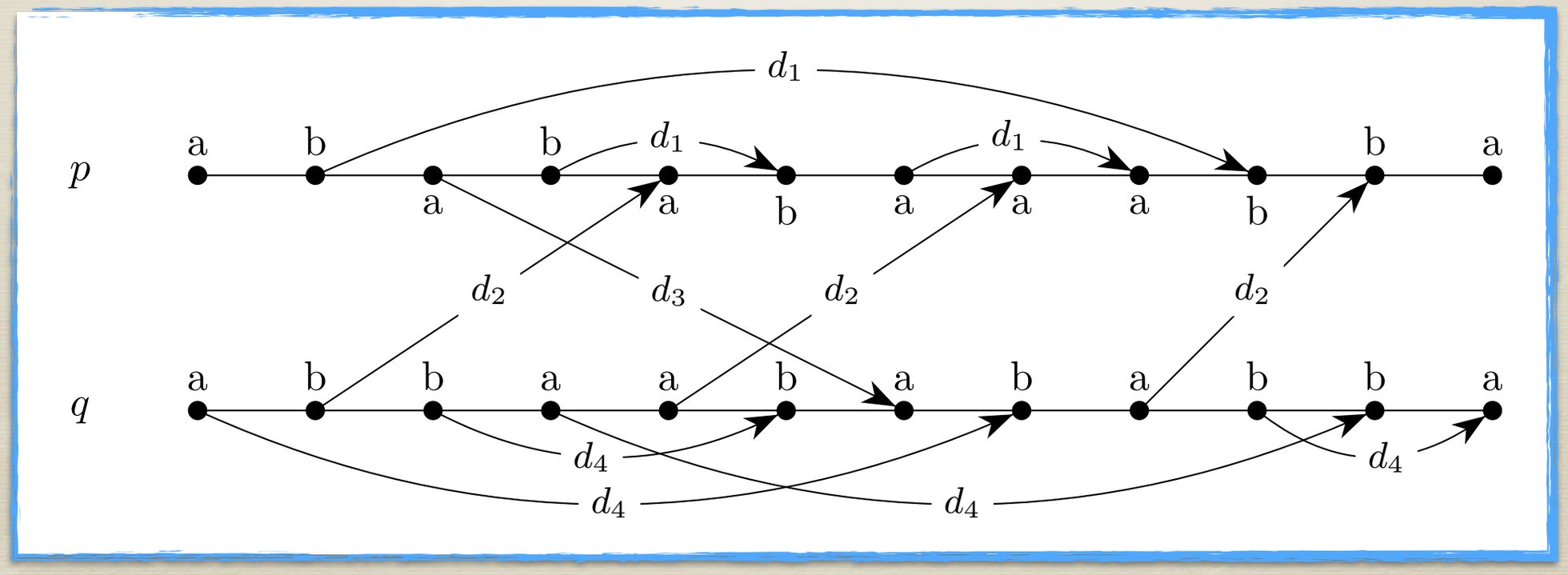
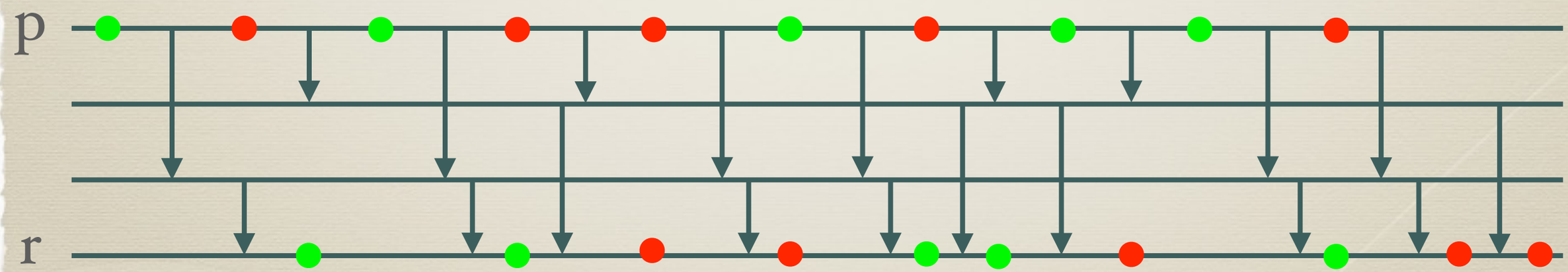
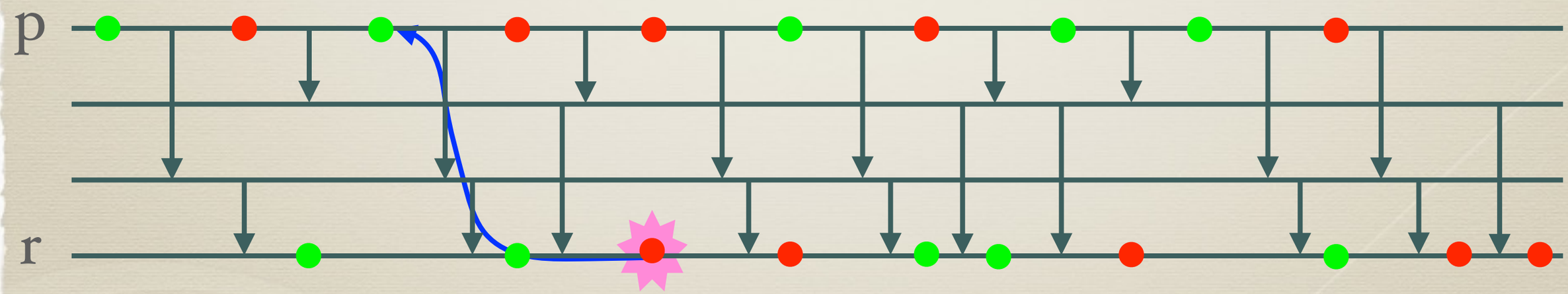


Specification over CBMs
Obey the latest order



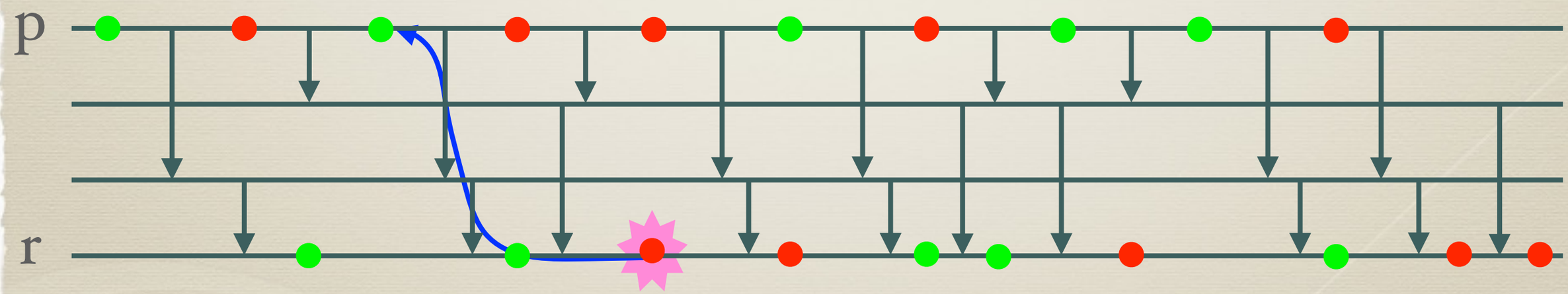
Specification over CBMs
Obey the latest order



Specification over CBMs
Obey the latest order

FO

$$\forall z\,(r(z) \wedge \mathsf{on}(z)) \Rightarrow \exists y\,(p(y) \wedge y < z$$
$$\wedge\, \forall x\,(x < z \wedge p(x) \Rightarrow x \leq y)$$
$$\wedge\, \exists x\,(x \to y \wedge \mathsf{on}(x)))$$



Specification over Linear Traces

$$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_1!)(q_1, c_1?)$$
$$(q_2, c_4!)(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(r, c_4?)(r, \mathsf{on})$$
$$(q_1, c_3!)(p, c_1!)(q_1, c_1?)(q_1, c_3!)(q_2, c_2?)(q_2, c_4!)$$
$$(r, c_4?)(r, \mathsf{on})(r, c_3?)(r, \mathsf{off}) \cdots$$

* Based on the word successor relation, and the word total order
* LTL over words, MSO over words

Specification over Linear Traces

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Process successor can be recovered
Data edges cannot in general

Specification over Linear Traces

$(p, \mathsf{on})(p, c_2!)(p, \mathsf{off})(q_2, c_2?)(p, c_4!)$

$(q_2, c_4!)(p, \mathsf{on})($

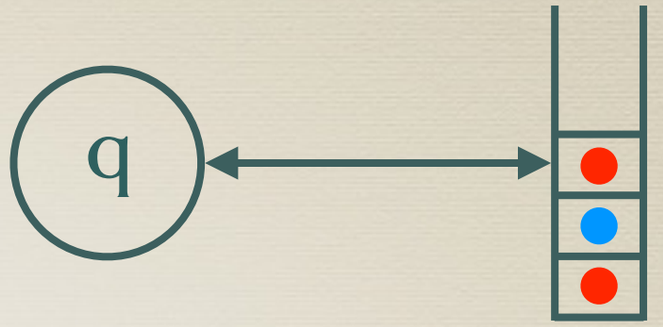
* Based o… …e word total order
* LTL ove… …ds, MSO over words

Obey the latest order
not expressible
in MSO over Linear Traces

Process successor can be recovered
Data edges cannot in general

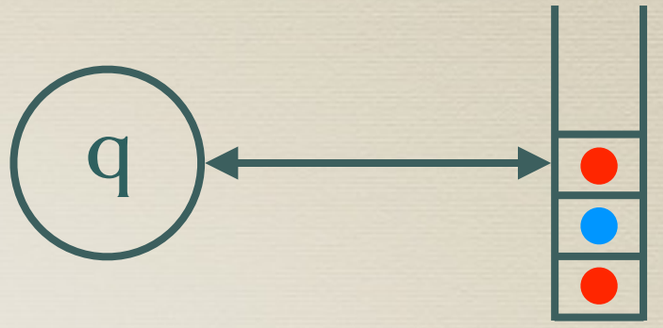
Graphs for Sequential Systems

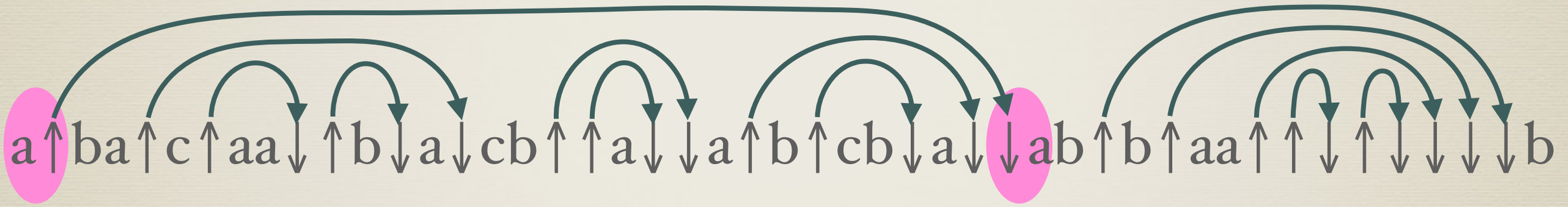
Relate outer most call and returns



Graphs for Sequential Systems

Relate outer most call and returns

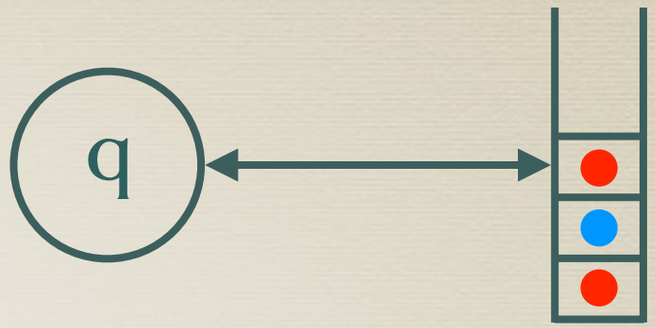




$$\forall x, y \left(\begin{array}{c} a(x-1) \wedge x \rhd y \wedge \\ \neg \exists z, z' \left(z \rhd z' \wedge z < x < z' \right) \end{array} \right) \Rightarrow a(y+1)$$

Graphs for Sequential Systems

Relate outer most call and returns



a↑ba↑c↑aa↓↑b↓a↓cb↑↑a↓↓a↑b↑cb↓a↓↓ab↑b↑aa↑↑↓↑↓↓↓↓b

Graphs for Sequential Systems

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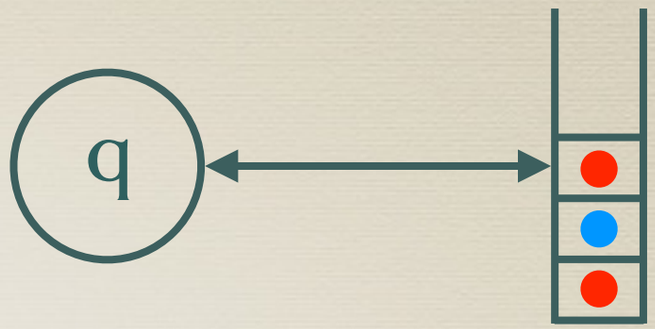

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Not expressible in MSO over Linear Traces
without nesting relation
even with visible alphabet

Graphs vs Linear Traces

Expressiveness of Specifications

Linear Traces	Graphs (CBMs)
• Too weak for many natural specifications • Difficult to write/understand • Requires syntactical or semantical restrictions to be meaningful	• Powerful specifications • Easy to write/understand • meaningful, interactions built-in

Outline

* Models
* Behaviours
* Specifications
* Verification via graph decompositions
* Conclusion

Verification problems

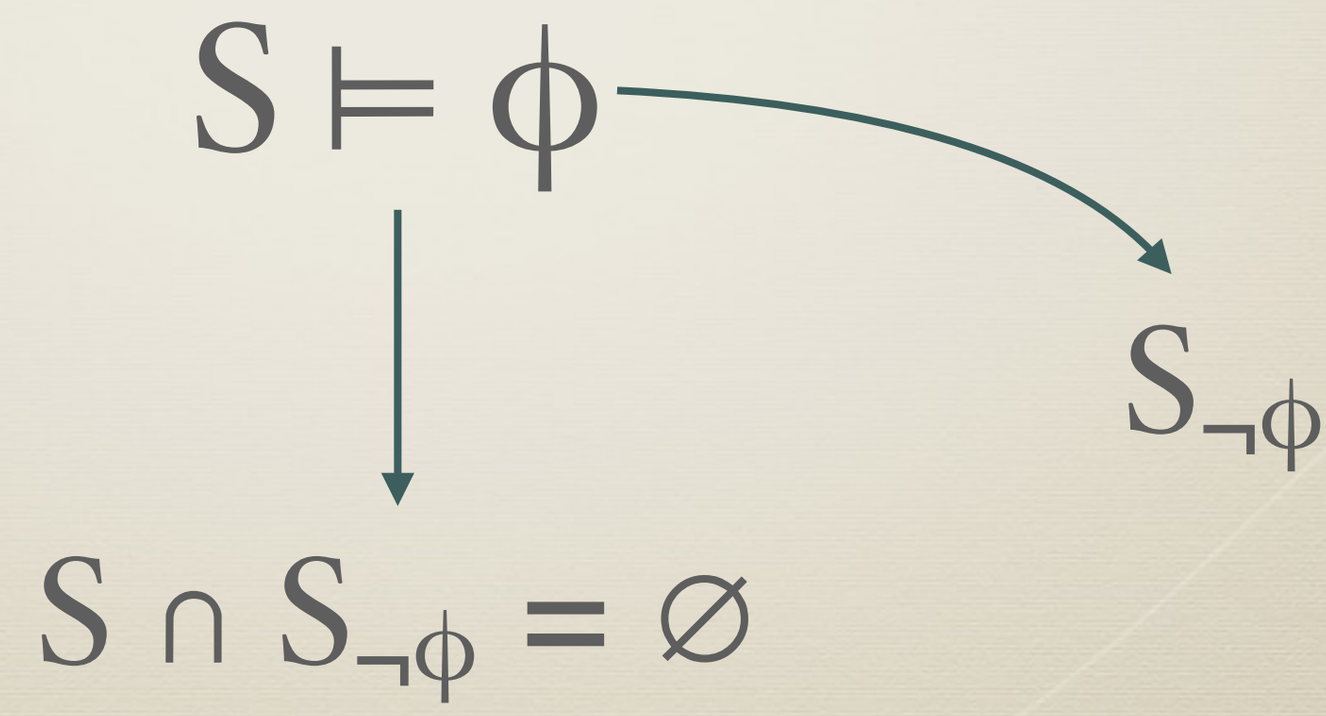
* Emptiness or Reachability for CPDS
* Inclusion or Universality for CPDS
* Satisfiability ϕ: Is there a CBM that satisfies ϕ?
* Model Checking: $S \models \phi$: Does every CBM accepted by S satisfy ϕ?
  * Monadic second order logic
  * Propositional dynamic logics
  * Temporal logics

Model Checking vs Reachability

* Reachability reduces to model checking

Model Checking vs Reachability

* Reachability reduces to model checking
* Model checking reduces to Reachability ...
  ... when specifications can be translated to systems
  ... this is not possible in general for graphs



CPDS to MSO

Theorem:
From any CPDS S we can construct a MSO formula ϕ_S such that a CBM satisfies ϕ_S iff it is accepted by S.

i.e. $L(S) = L(\phi_S)$

* Emptiness of CPDS reduces to satisfiability for MSO

  S accepts a CBM iff ϕ_S is satisfiable

* Model Checking reduces to satisfiability for MSO.

  $S \models \phi$ iff $\neg\phi \wedge \phi_S$ is not satisfiable

* Similarly for language containment and universality

Verification problems

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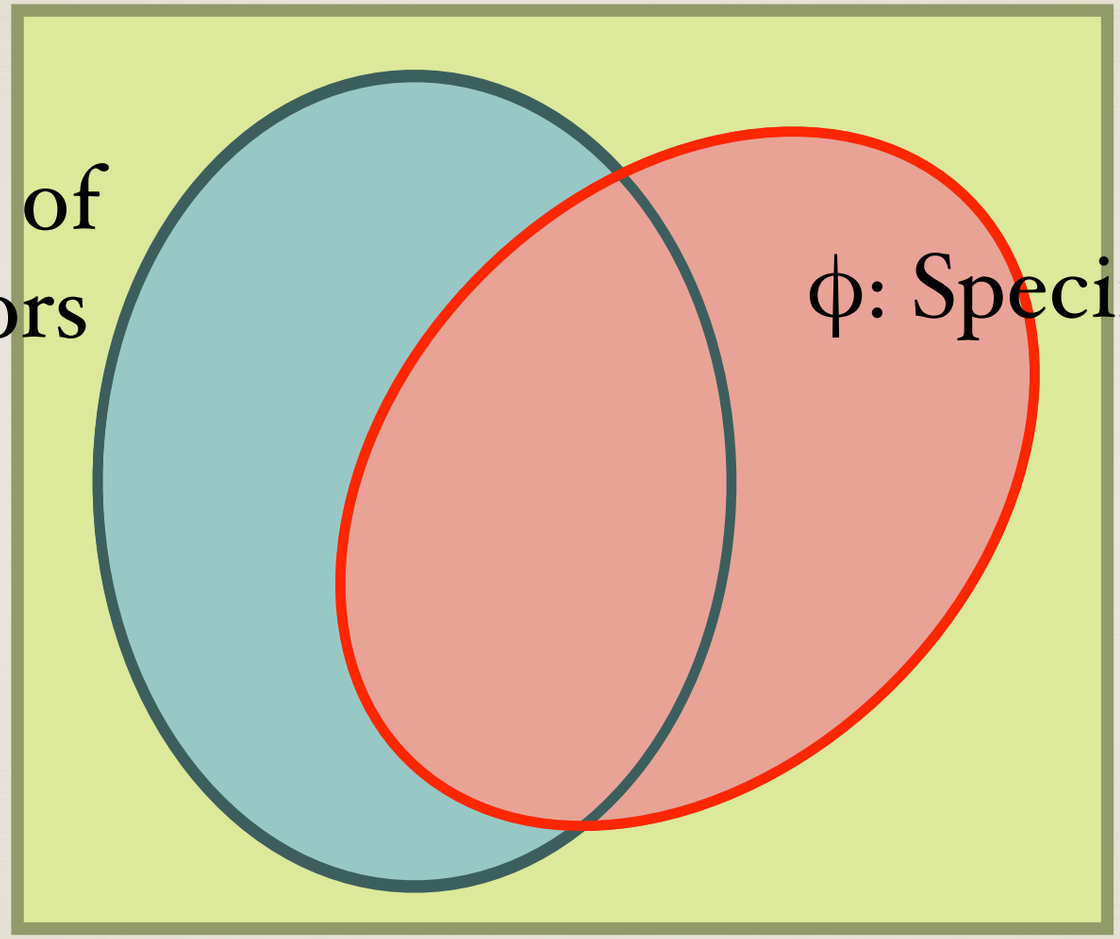
  * Pr
  * Mo logic

undecidable in general

Under-approximate Verification

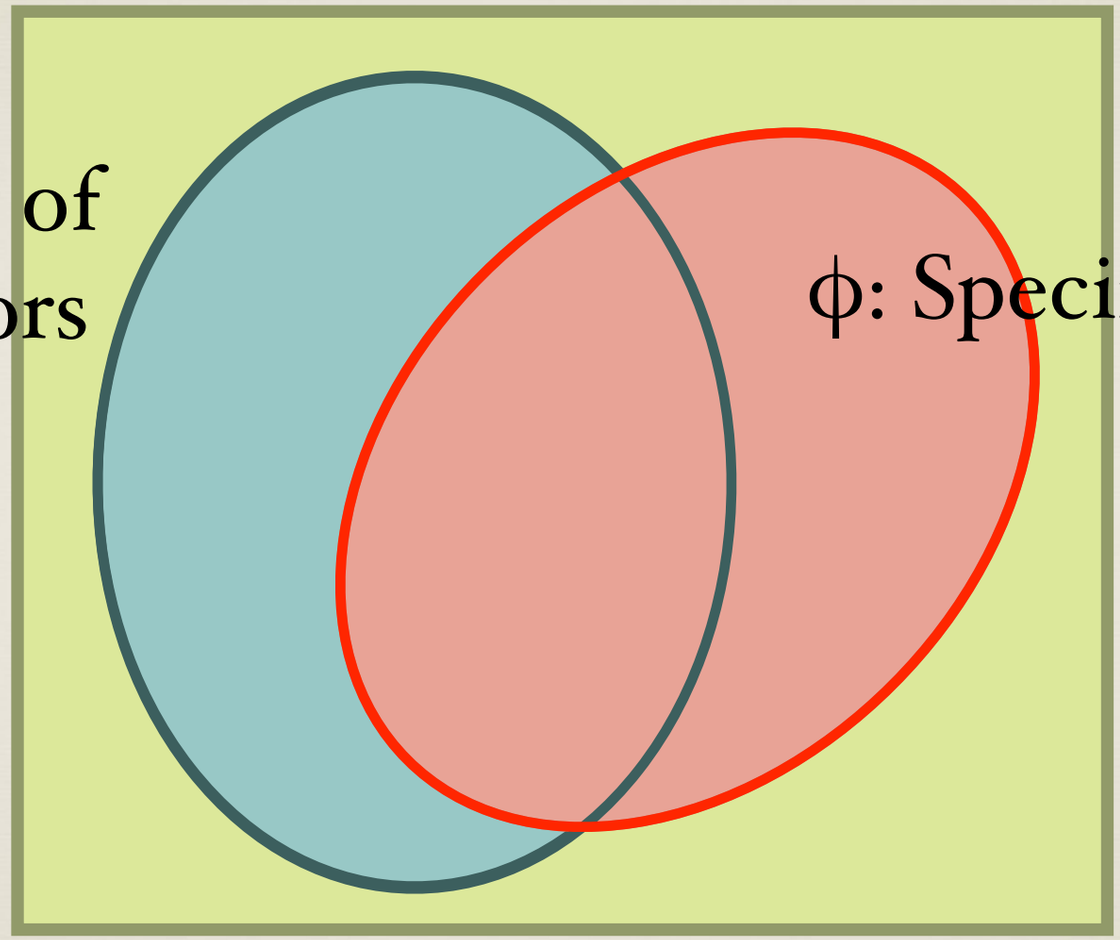
Satisfiability problem:



Under-approximate Verification

Satisfiability problem:

C: class of behaviors

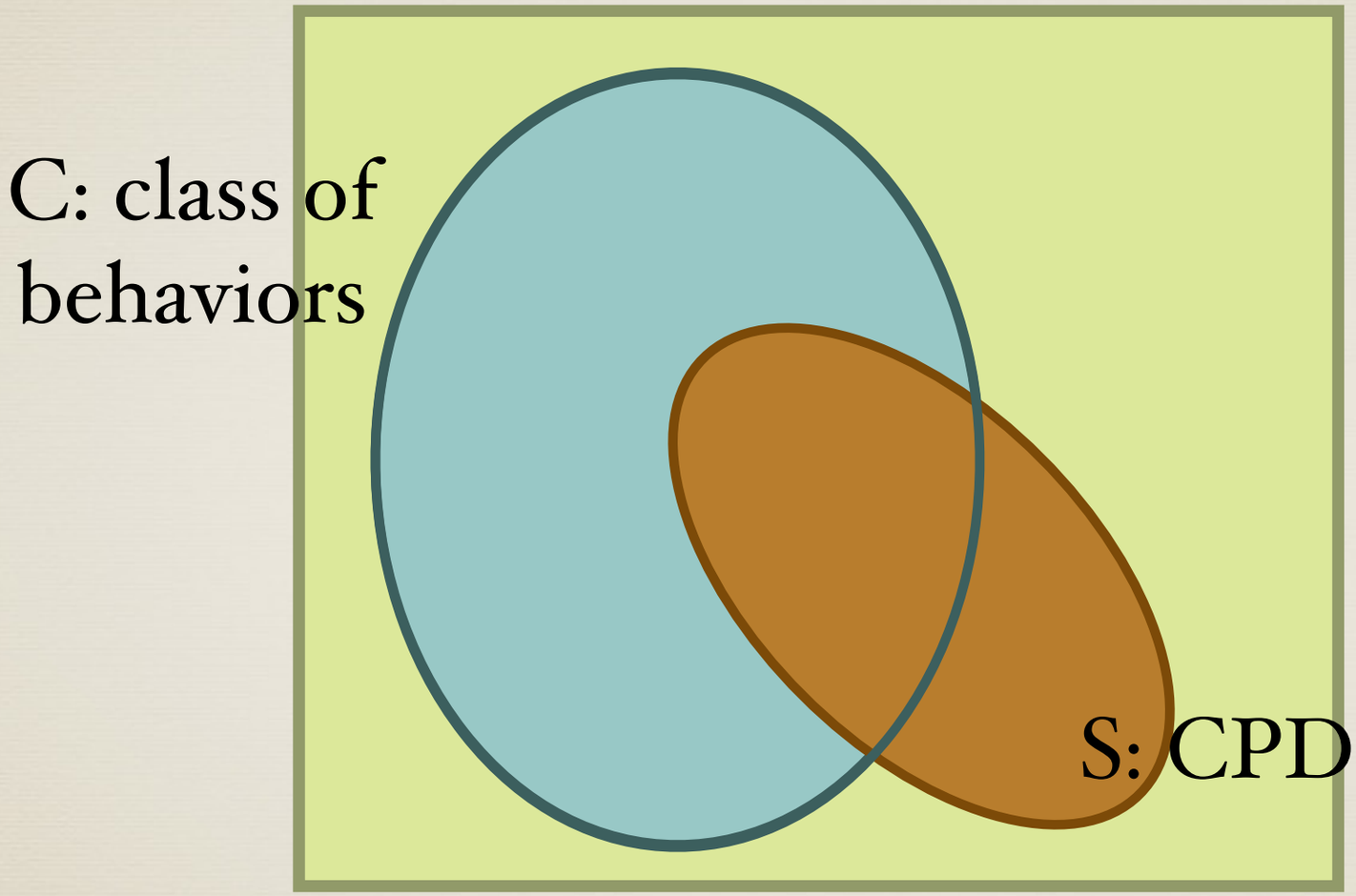


ϕ: Specification

Is ϕ satisfiable in C?

Under-approximate Verification

Emptiness or reachability problem:



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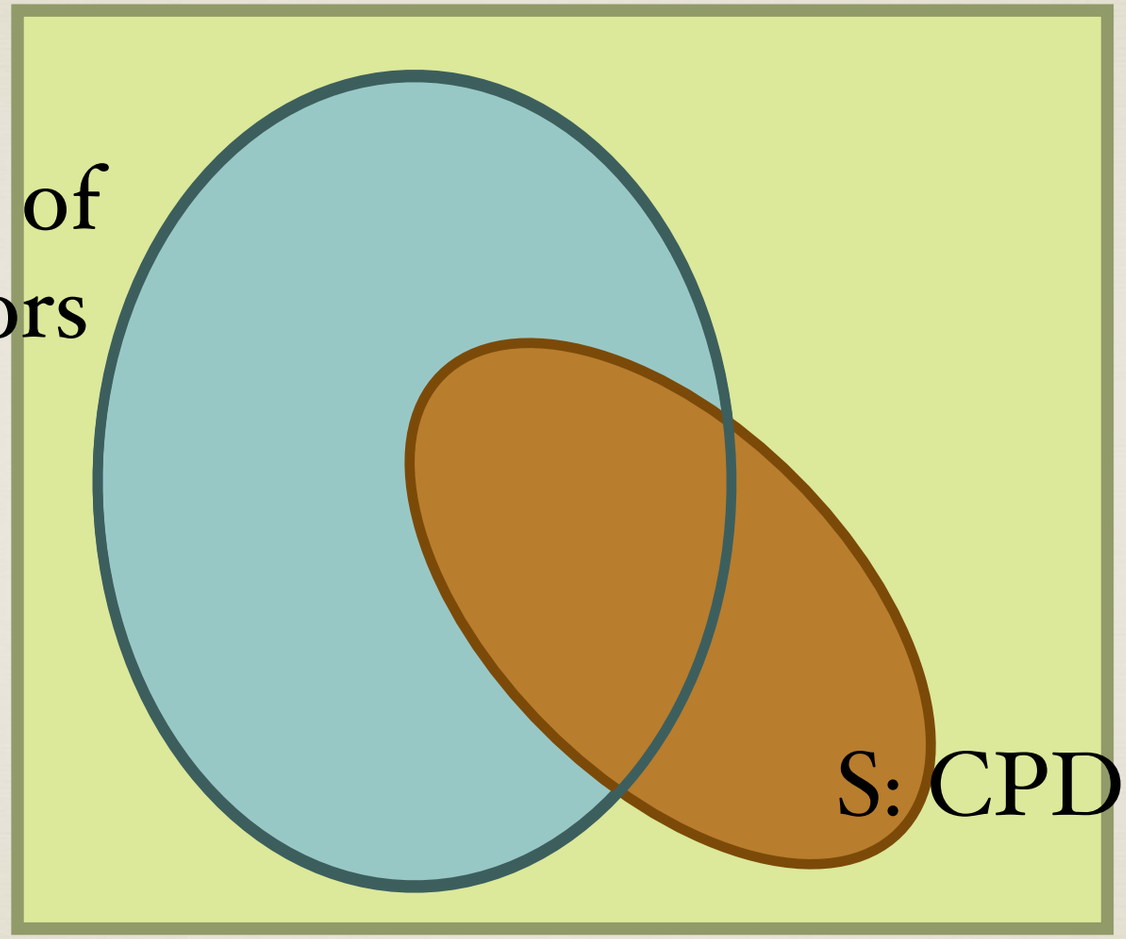
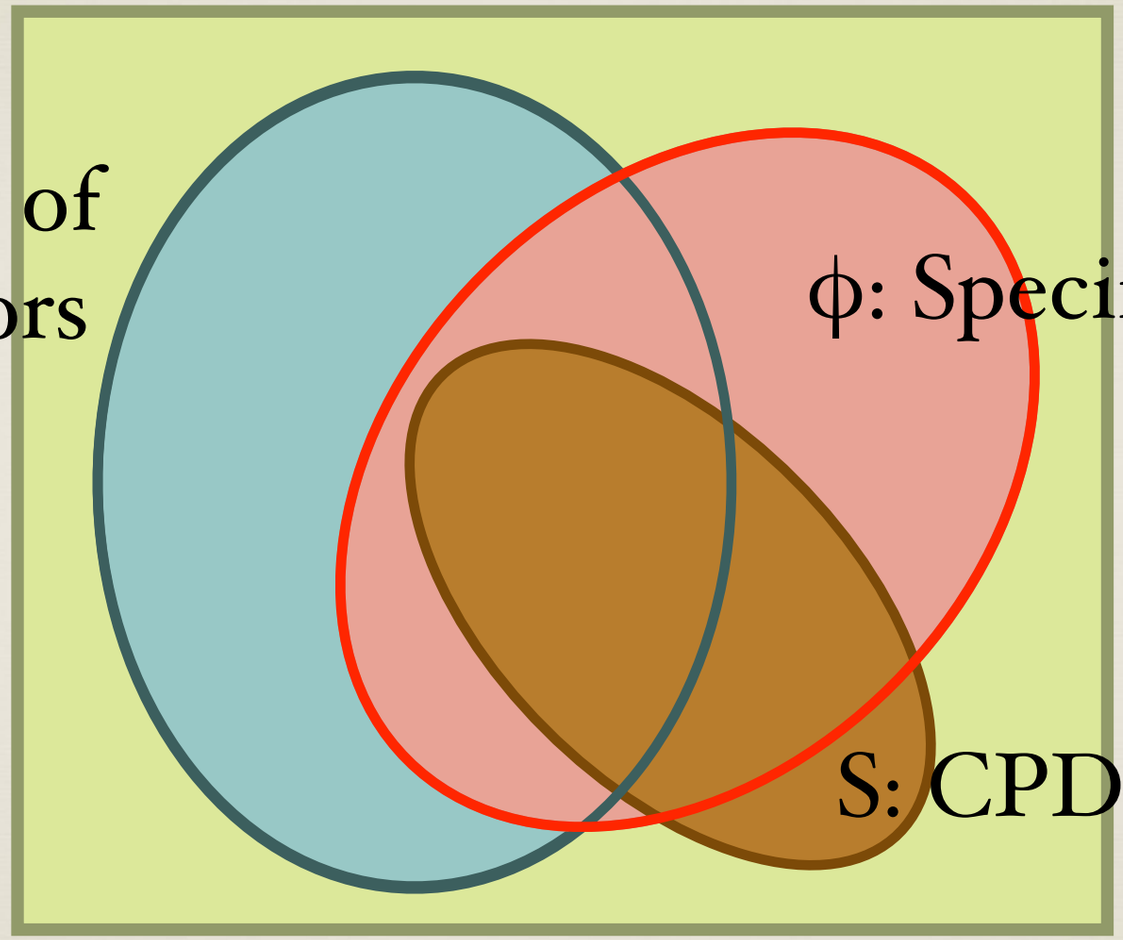


Is there an accepting run of S on some behavior from C?

Under-approximate Verification

Model checking problem: $S \models_C \phi$



Under-approximate Verification

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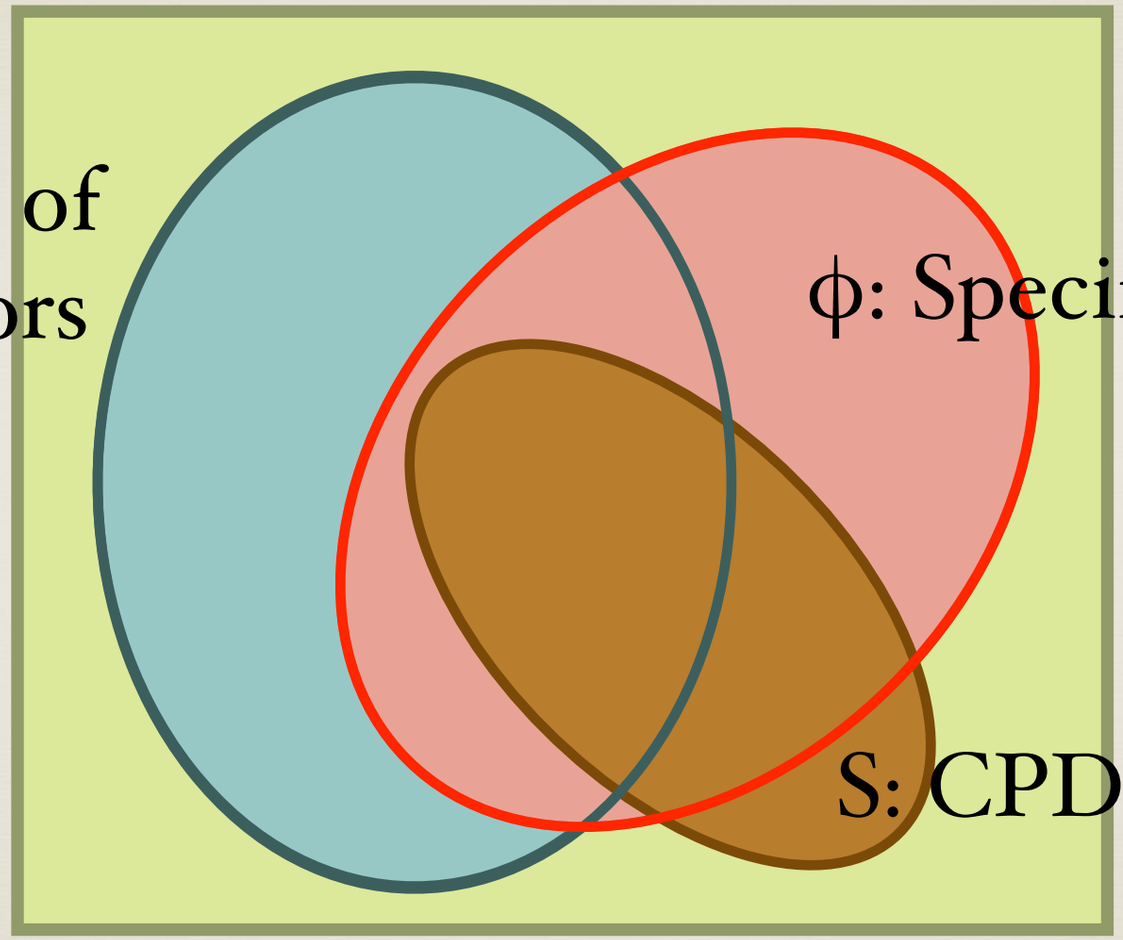

Do all behaviors from C accepted by S satisfy ϕ?

Under-approximate Verification

Mainly for reachability

undecidable

* Emptiness or Reachability
* Inclusion or Univ
* g. $S \models \phi$
* Temporal logics
* Propositional dynamic logics
* Monadic second order logic

* Bounded data structures
* Existentially bounded [Genest et al.]
* Acyclic Architectures [La Torre et al., Heußner et al. Clemente et al.]
* Bounded context switching [Qadeer, Rehof], [LaTorre et al.], ...
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* Bounded scope [LaTorre et al.]
* Priority ordering [Atig et al., Saivasan et al.]

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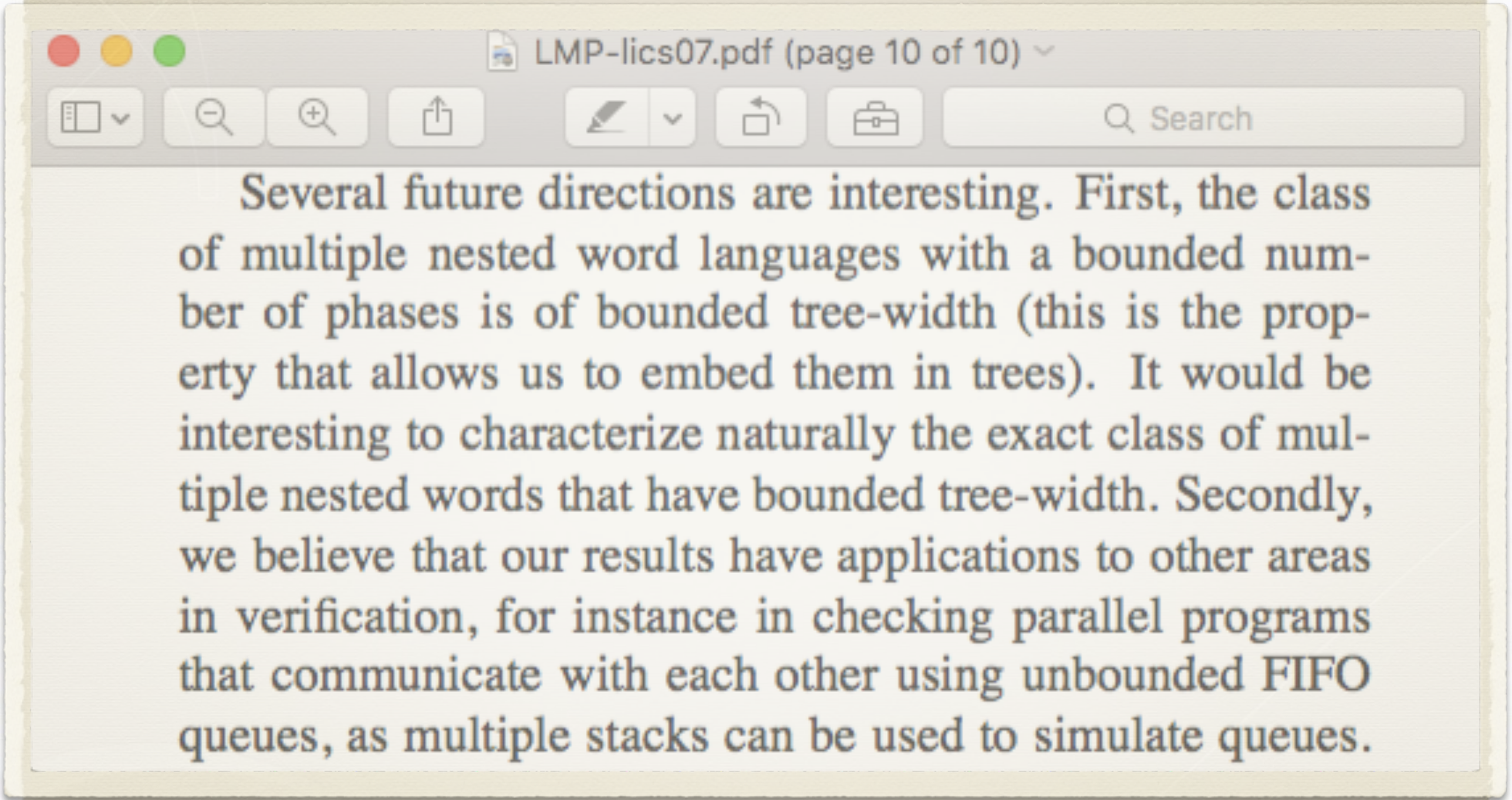
Reduction to MSO/ Automata over trees.

Bounded-phase to Tree-width



Several future directions are interesting. First, the class of multiple nested word languages with a bounded number of phases is of bounded tree-width (this is the property that allows us to embed them in trees). It would be interesting to characterize naturally the exact class of multiple nested words that have bounded tree-width. Secondly, we believe that our results have applications to other areas in verification, for instance in checking parallel programs that communicate with each other using unbounded FIFO queues, as multiple stacks can be used to simulate queues.

Outline

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Under-approximate Verification

The Tree Width of Auxiliary Storage

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ostract

e propose a generalization of results on the decidability of empti-
ss for several restricted classes of sequential and distributed au-
nata with auxiliary storage (stacks, queues) that have recently
en proved. Our generalization relies on reducing emptiness of
se automata to finite-state *graph automata* (without storage)
tricted to monadic second-order (MSO) definable graphs of
unded tree-width, where the graph structure encodes the mech-

However, the various identified decidable restrictions on the
automata are, for the most part, *awkward* in their definitions
e.g. emptiness of multi-stack pushdown automata where push
to any stack is allowed at any time, but popping is restricted
the first non-empty stack is decidable! [8]. Yet, relaxing the
definitions to more natural ones seems to either destroy decidabil
or their power. It is hence natural to ask: why do these automa
have decidable emptiness problems? Is there a common underlyi

Tree-width of Graphs

* A measure of the connectivity of a graph.
* How close is the graph to being a tree?

St. Jude of CS

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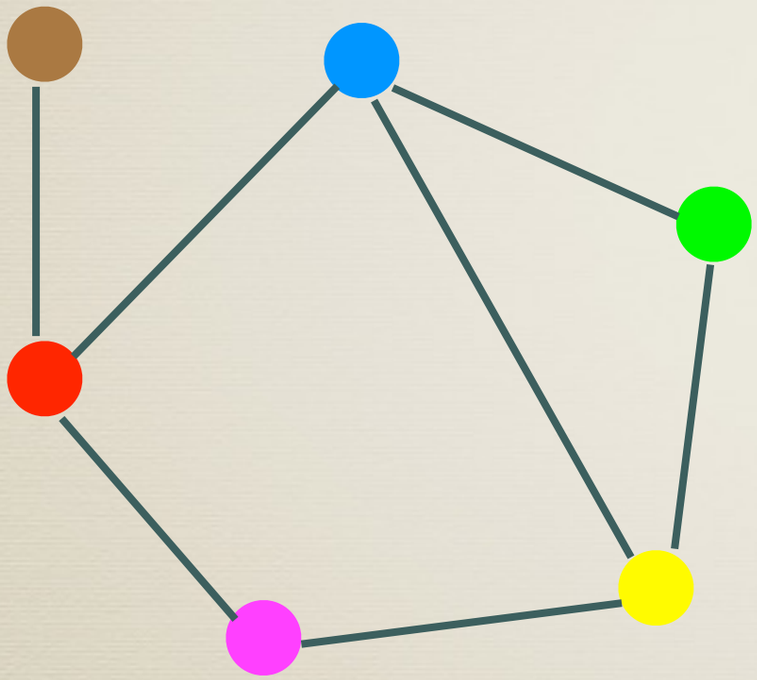
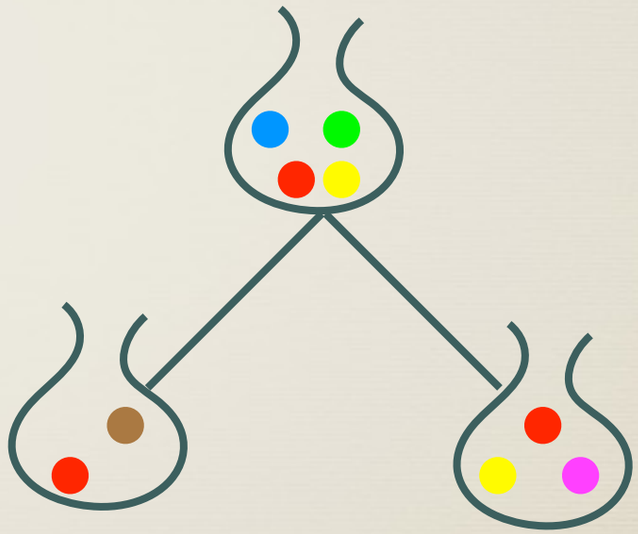
A concept from Graph theory that is very useful computer science

St. Jude of CS

Tree-decompositions of Graphs

* A tree whose nodes are labeled by sets of vertices (bags)
  * Every vertex appears in some bag.
  * (u,v) is an edge then there is a bag containing u,v
  * For any u the bags containing u form connected part of the tree.





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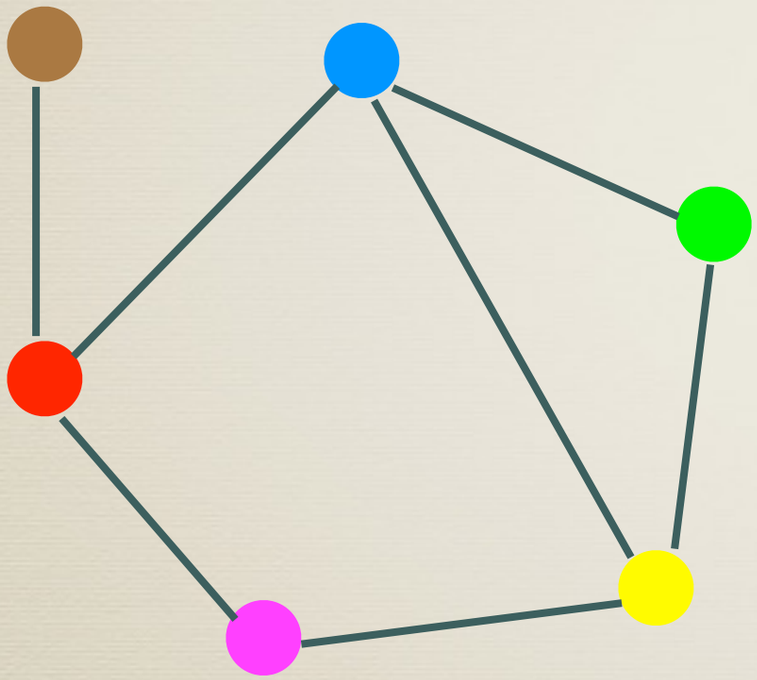

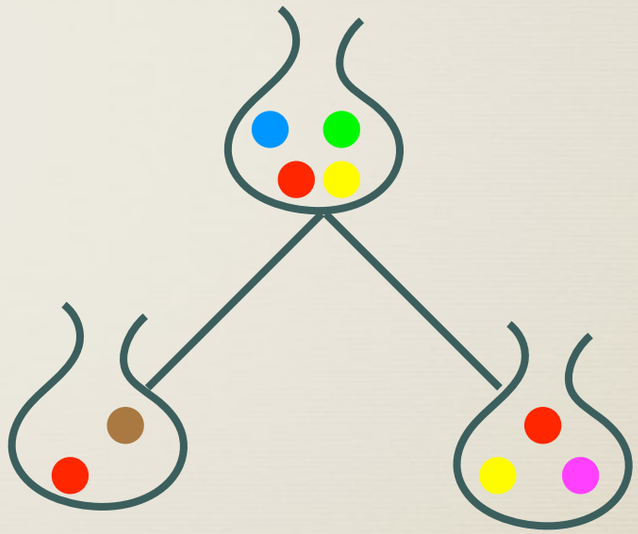

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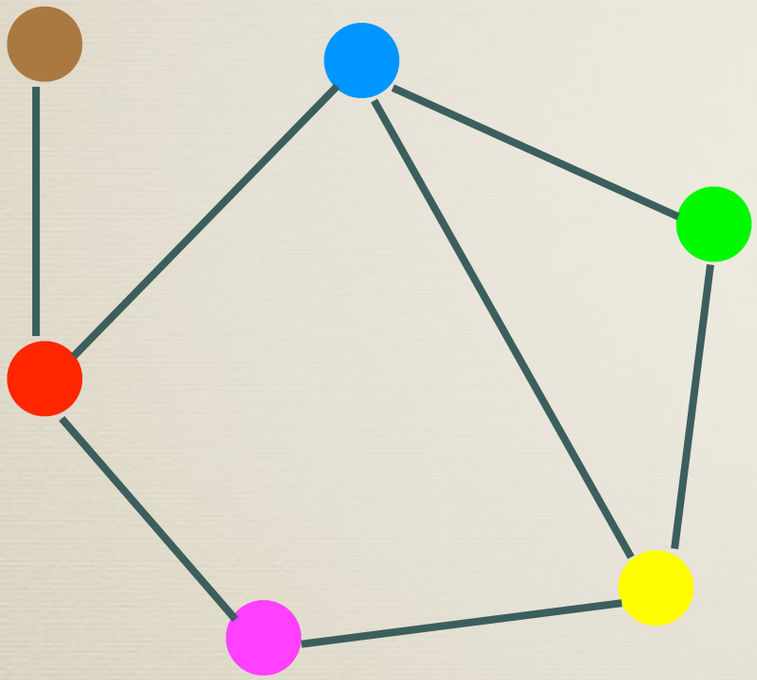

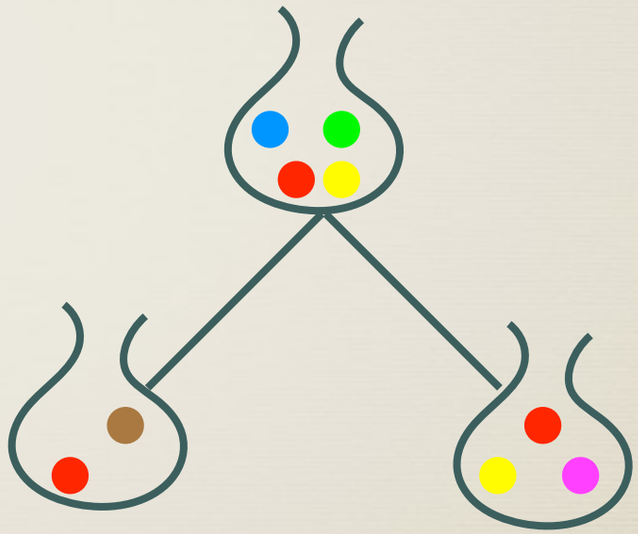

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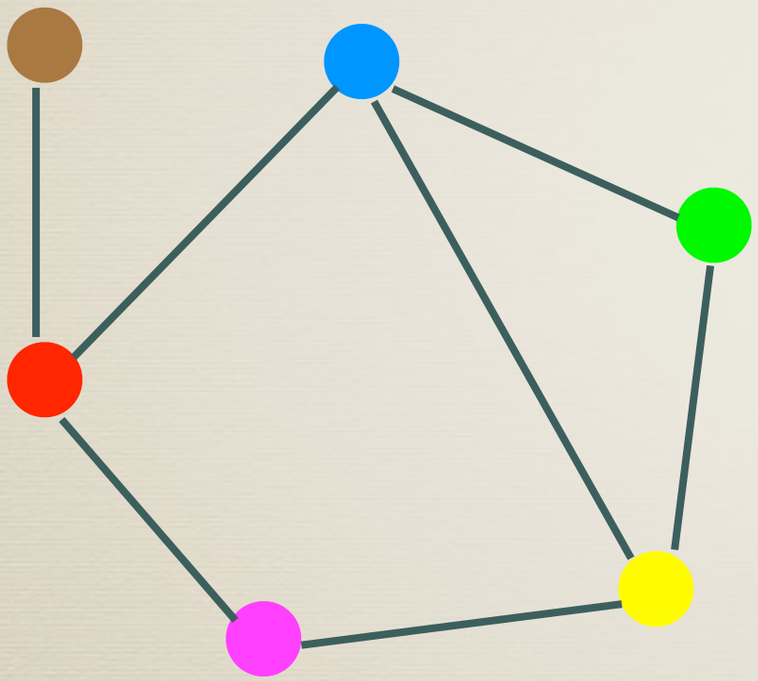

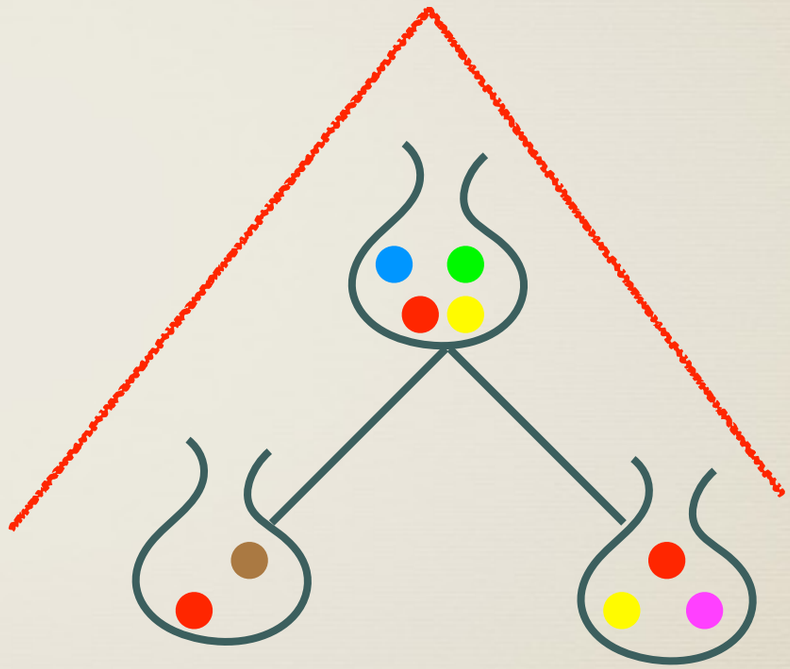

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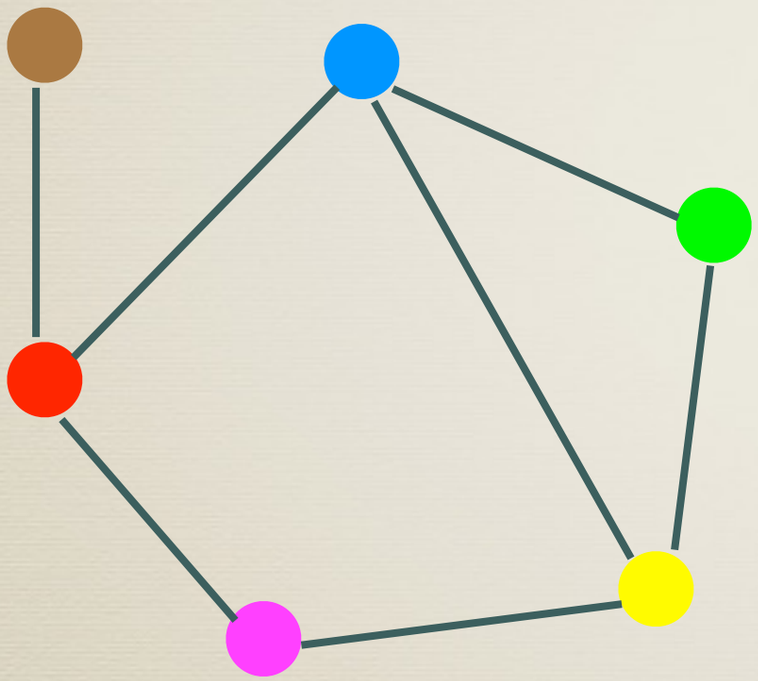
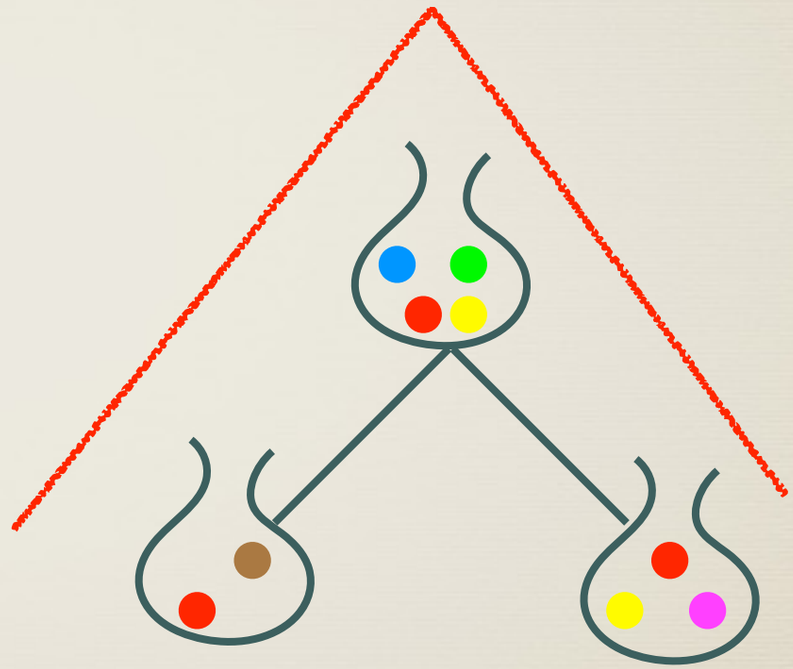
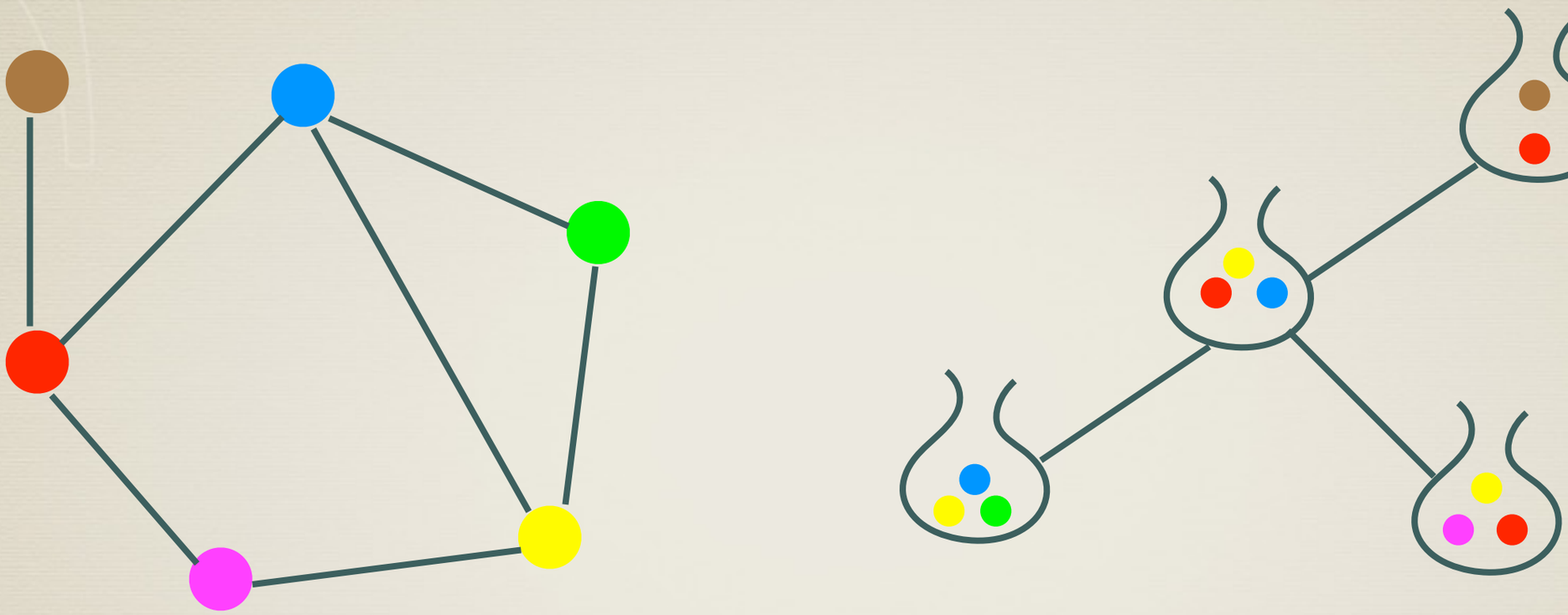




Size of the Decomposition = Size of largest bag - 1

Tree-decompositions of Graphs



A tree-decomposition of size 2

Tree-width of Graphs

The smallest n such that the graph has a decomposition of size n

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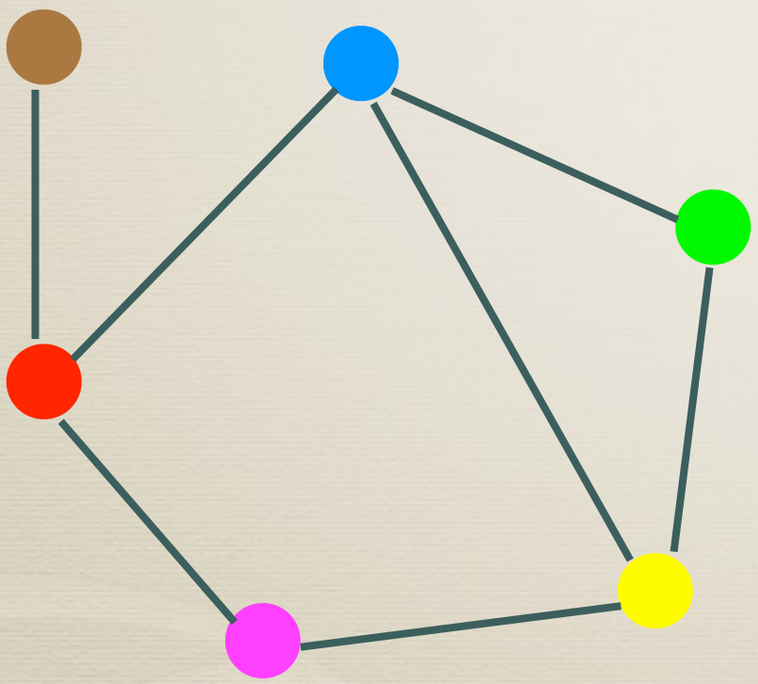


A graph with tree-width 2

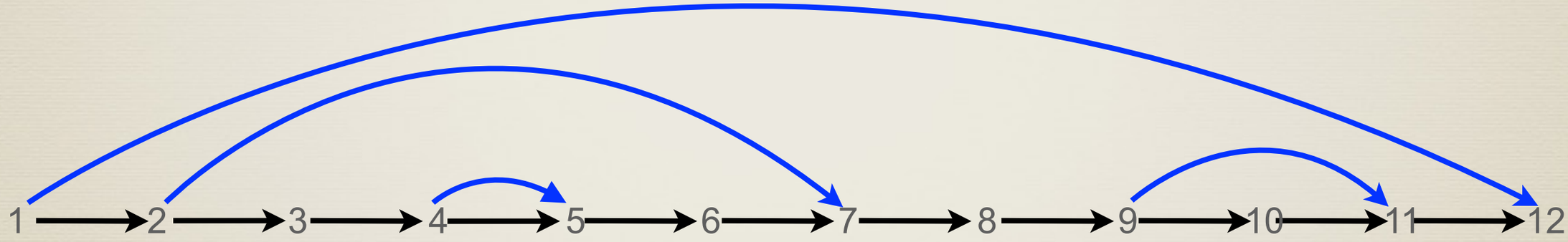
Classes with bounded tree-width

A class of graphs has tree-width bounded by k if every graph in the class has tree-width bounded by k.

A class of graphs has bounded tree-width if it has tree-width bounded by some k.

An Example: Nested Words

The class of nested words has tree-width bounded by 3



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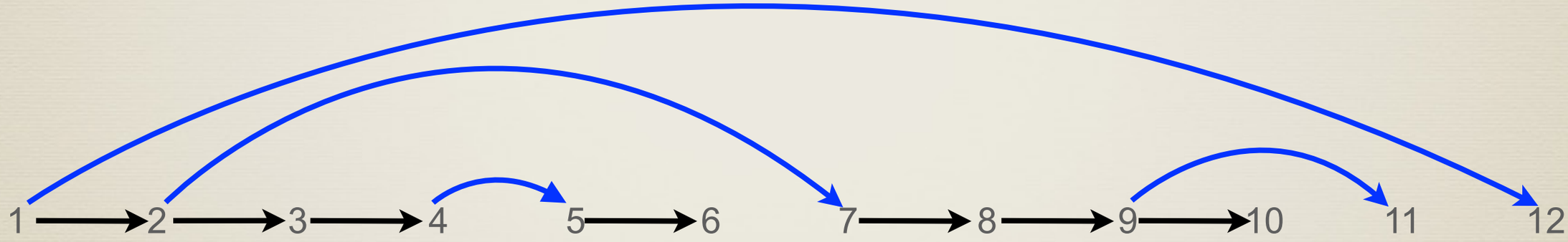

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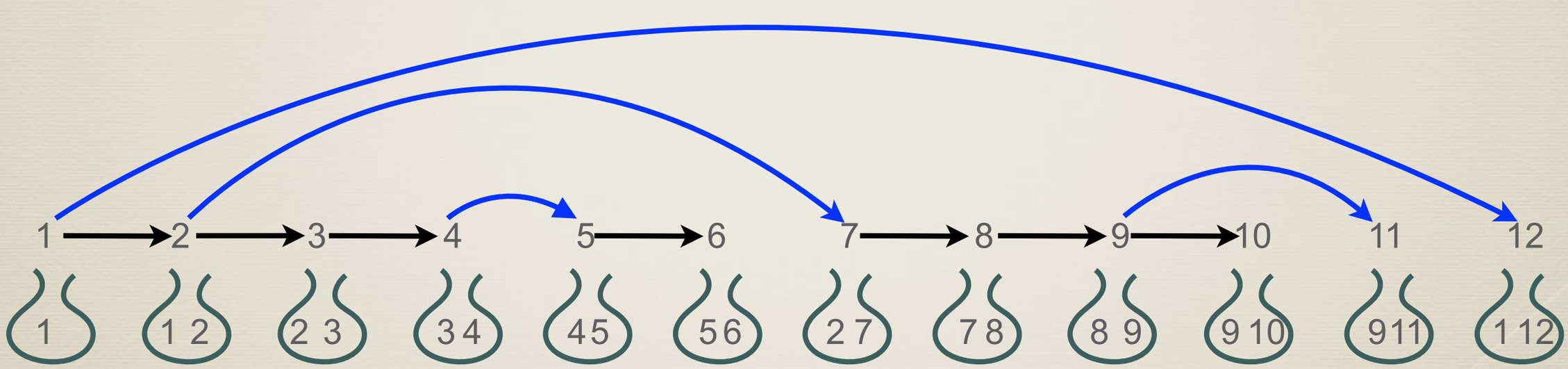

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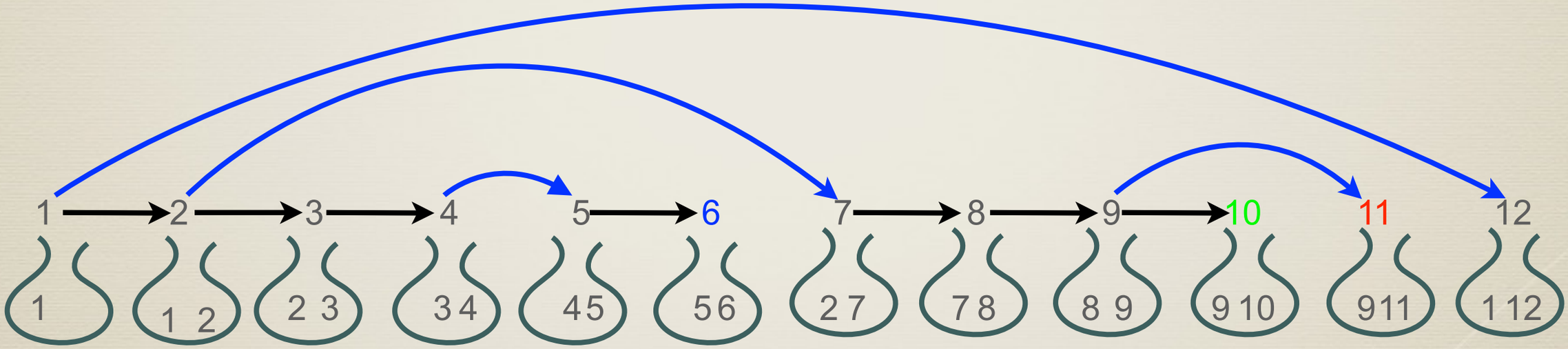

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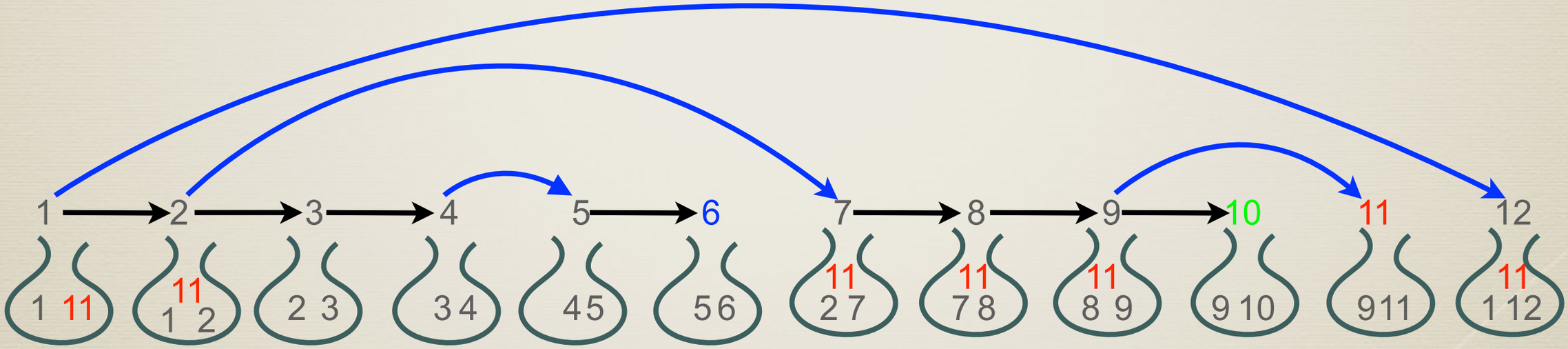

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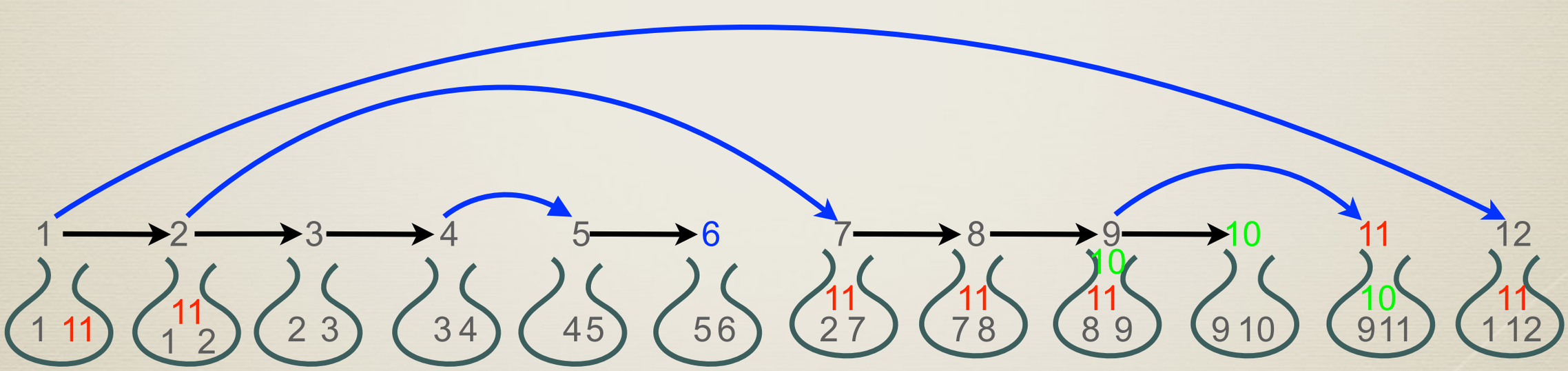

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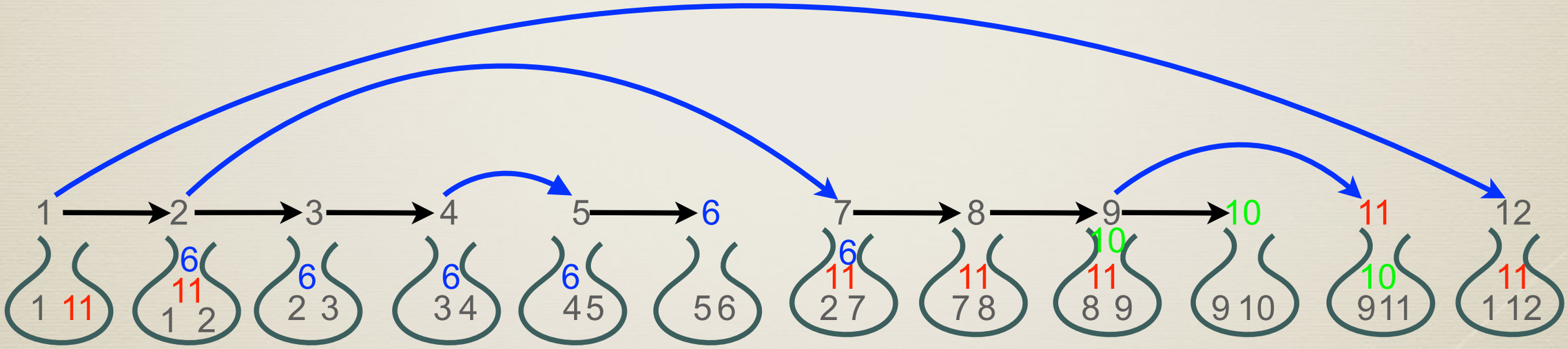

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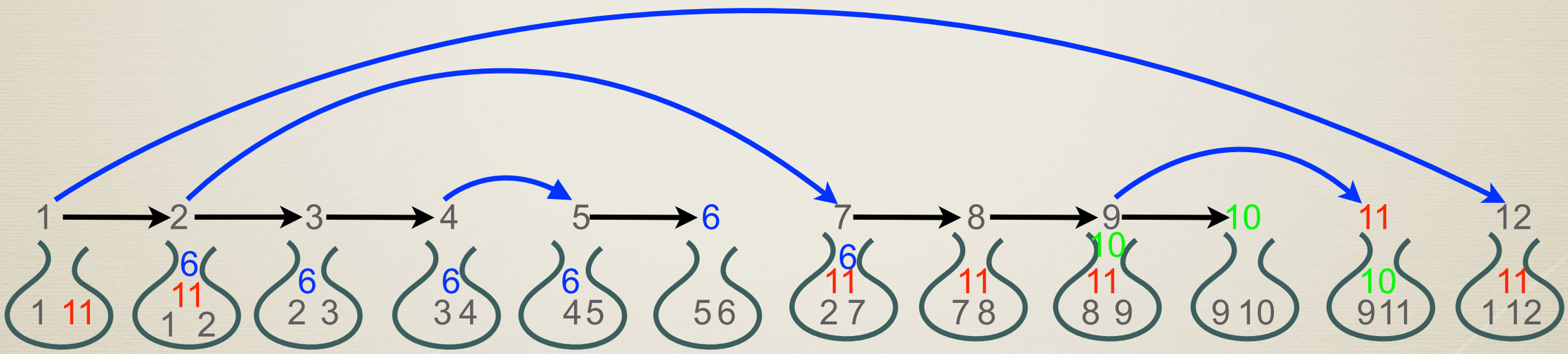
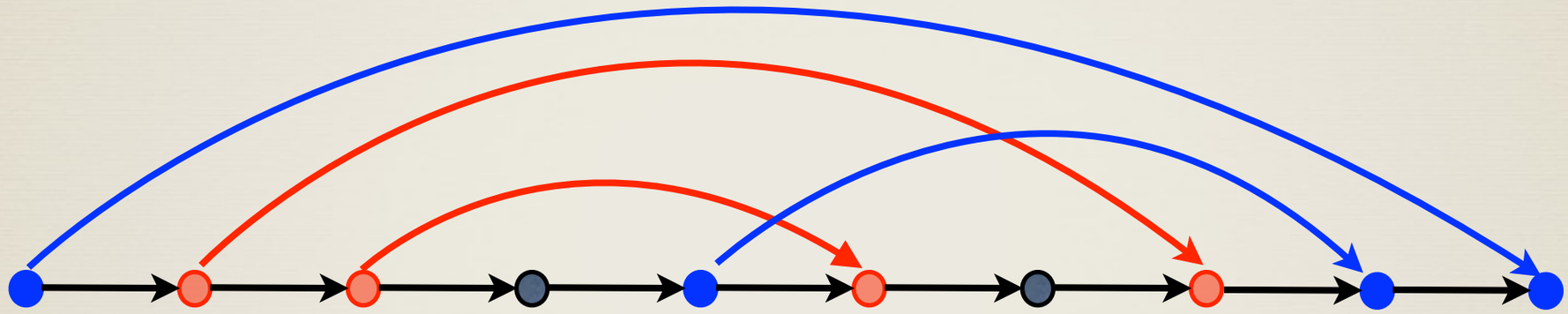


A bit more work gives a bound of 2

Example: Bounded Context Runs

* A CPDS with a single process and collection of Stacks
* A context is a segment where only one stack is accessed.



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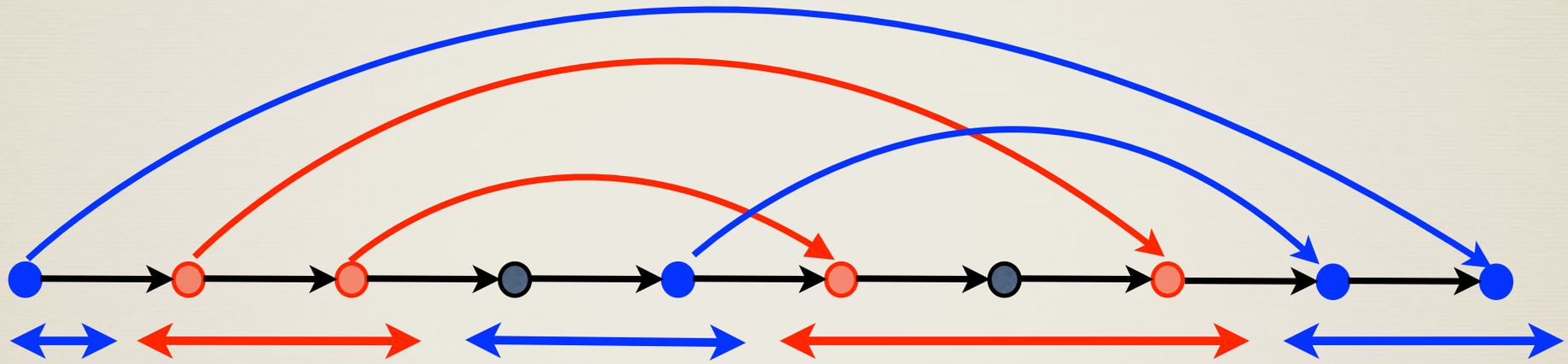

A behaviour with 5 contexts

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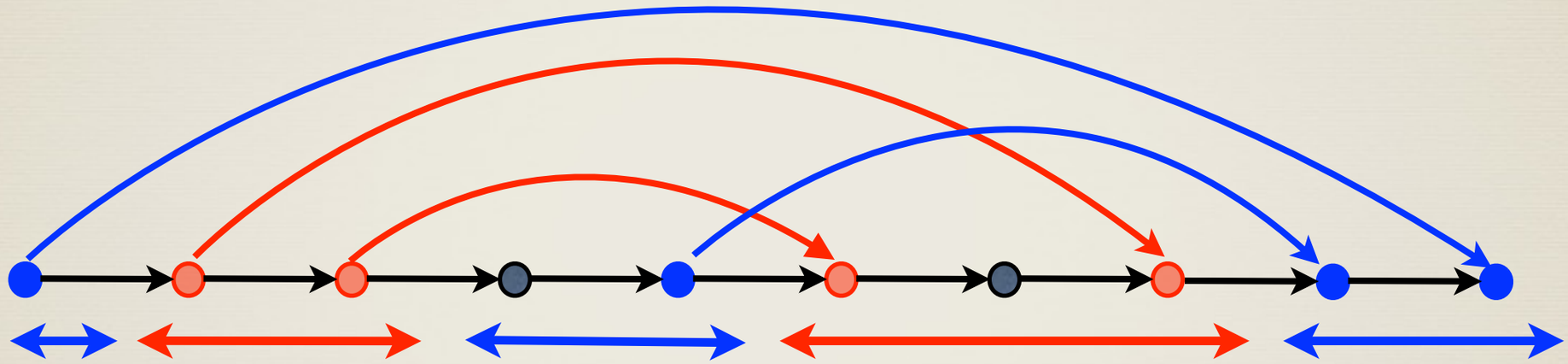
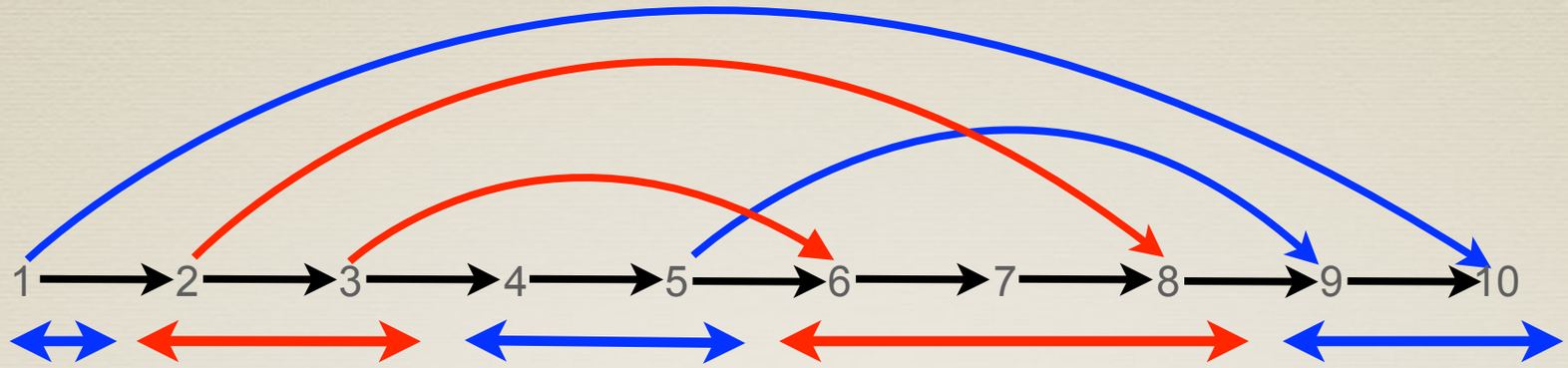


A behaviour with 5 contexts

What is the tree-width of the set of nested words with at most k contexts?

Tree-width of Bounded Context Runs



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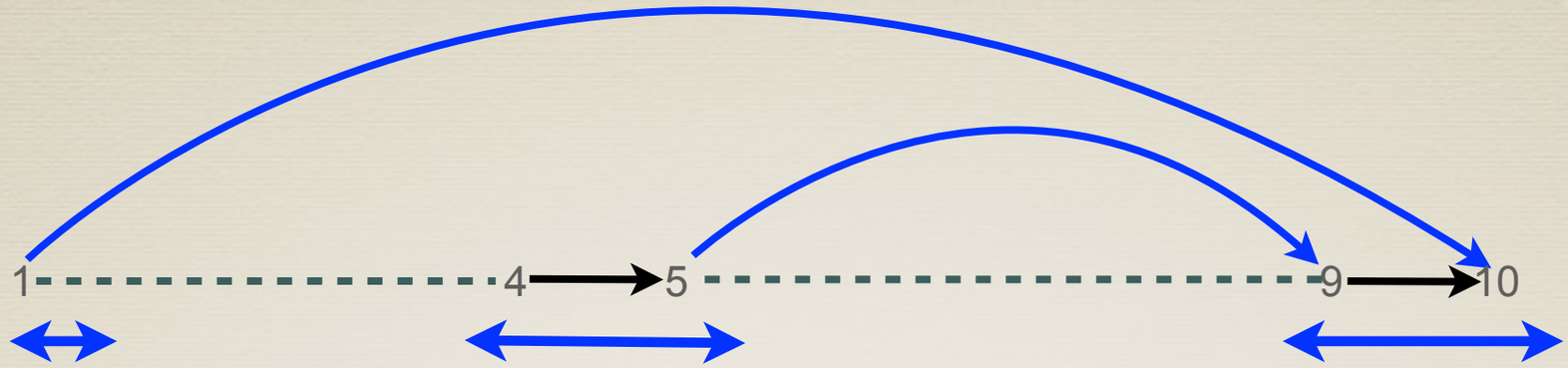
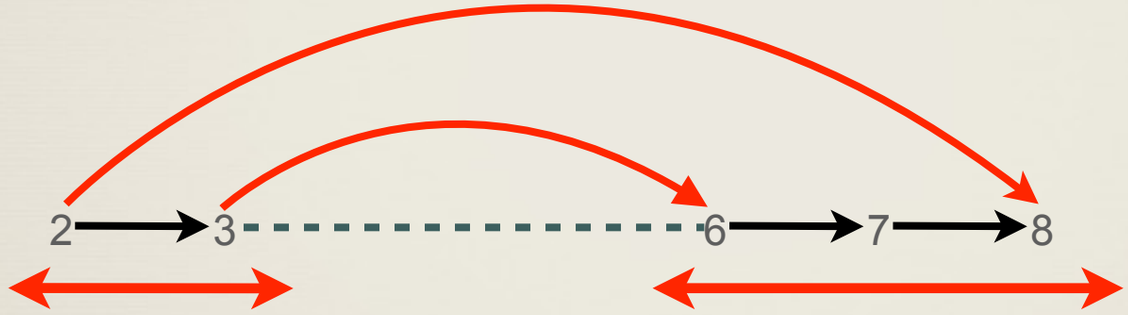
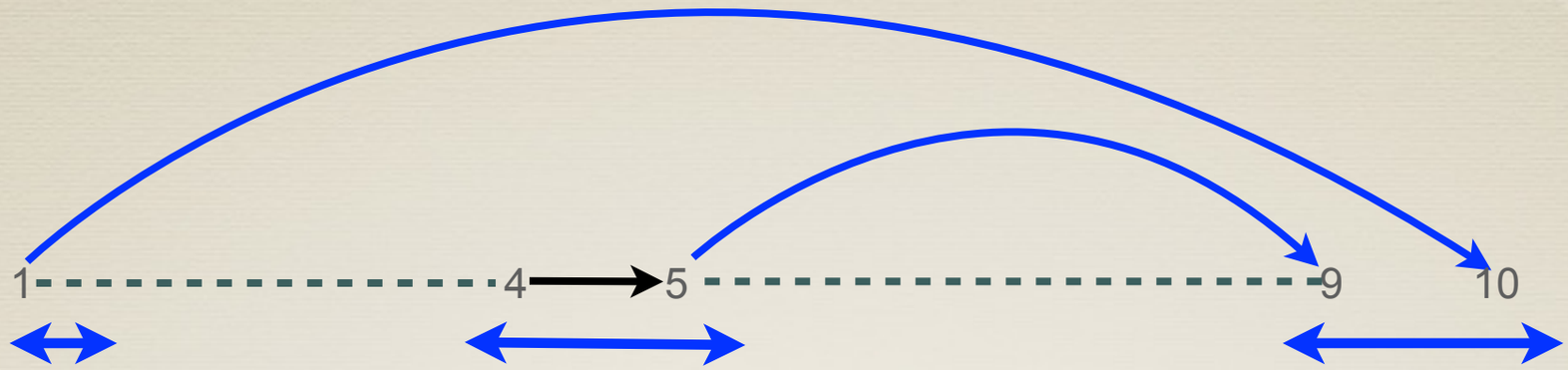
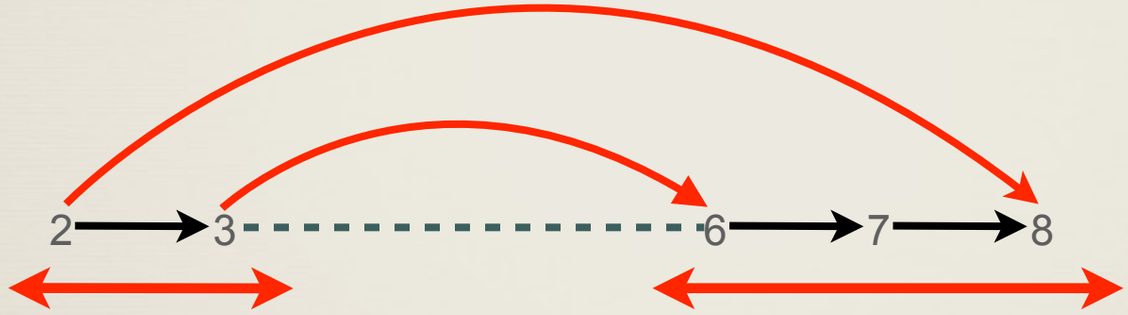


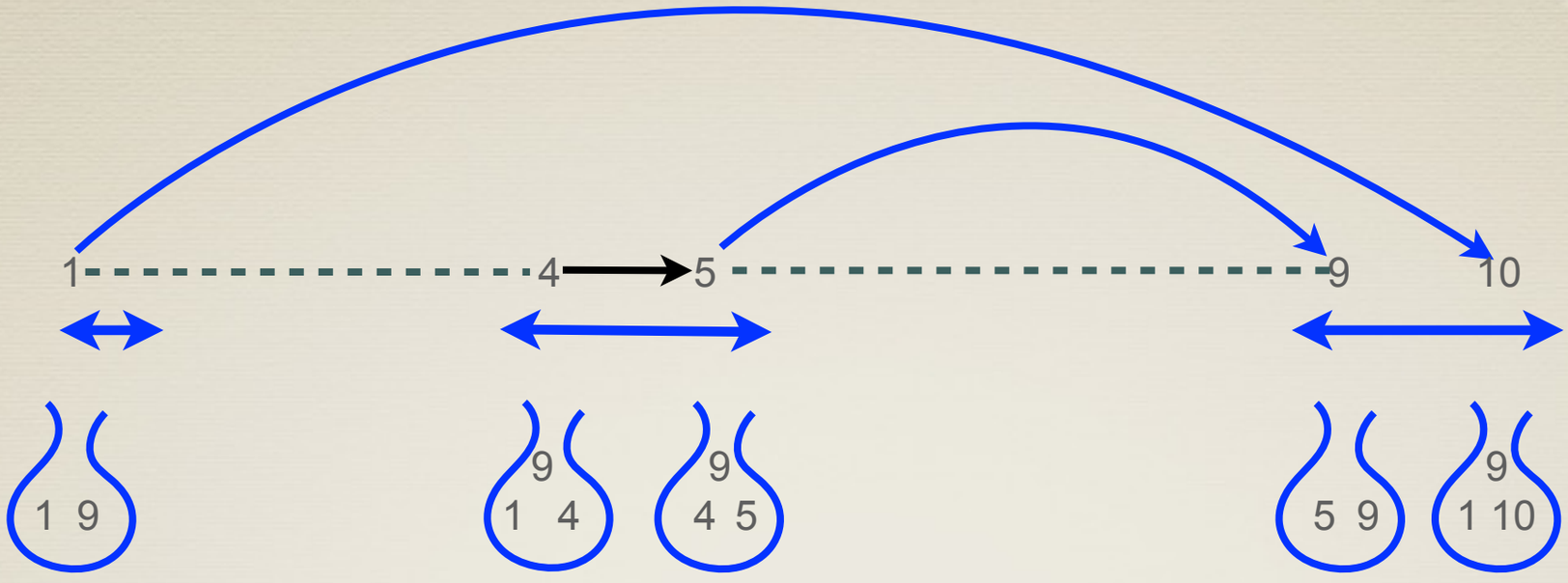


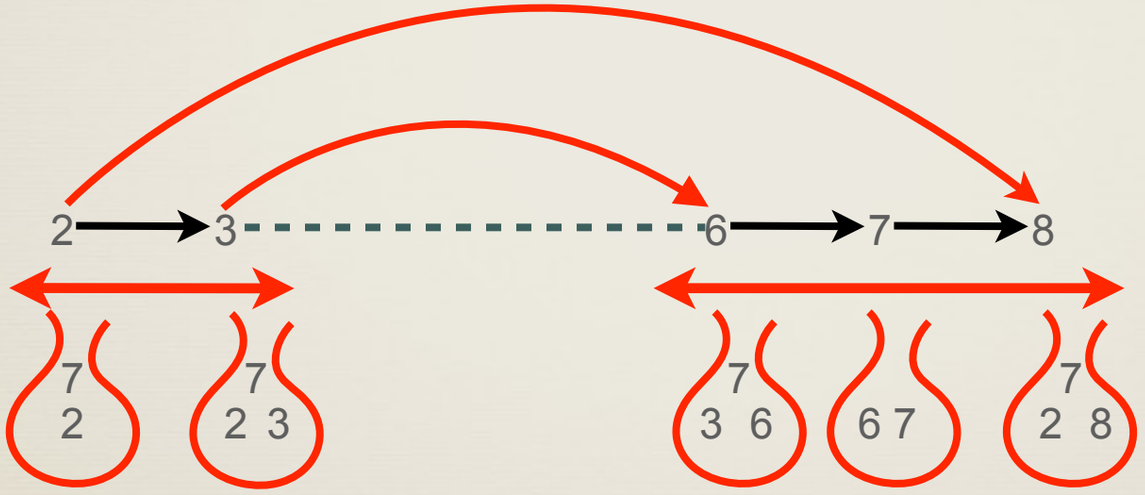
Tree-width of Bounded Context Runs





Tree-width of Bounded Context Runs





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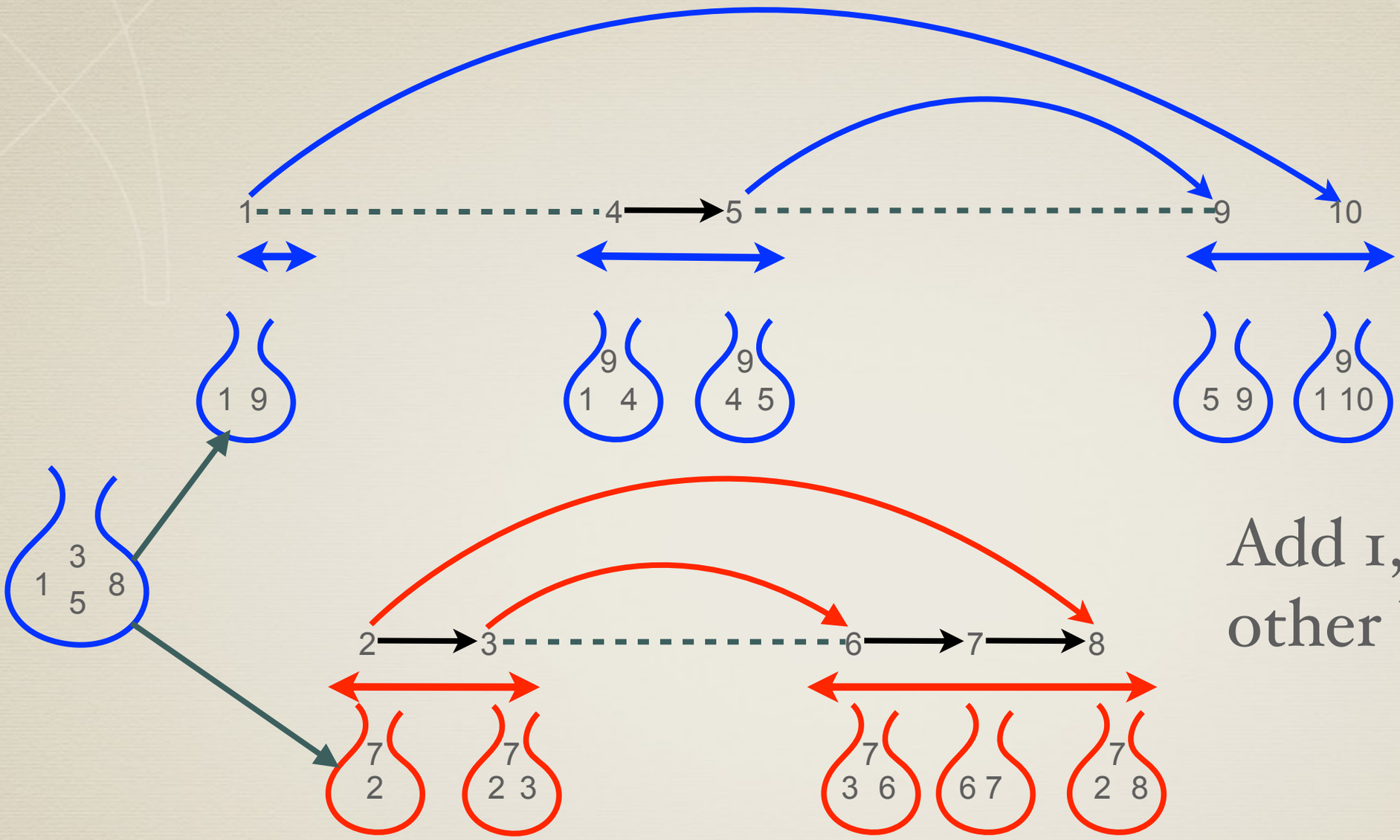

Add 1,3,5,8 to all other bags as well.

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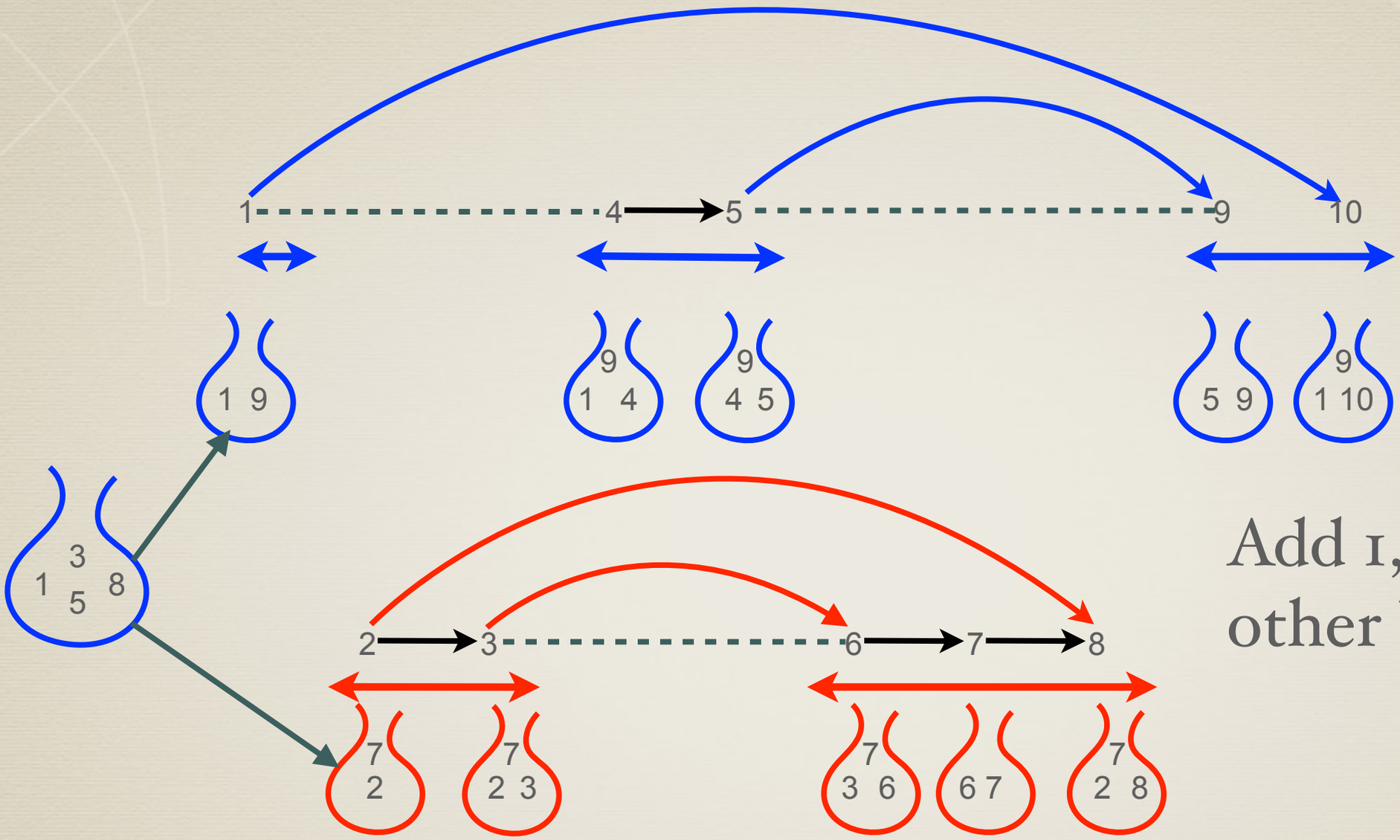

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The set of nested words with at most k contexts has tree-width <= 2+(k-1) = k+1

Tree-width bounds for other Under-approximations

- Multi-stack pushdown automata with bounded context-switching: This is the class of multi-stack automata where each computation of the automaton can be divided into k stages, where in each stage the automaton touches only one stack (proved decidable first in [14]). We show that they can be simulated by graph automata on graphs of tree-width $O(k)$.

- Multi-stack pushdown automata with bounded phases: These are automata that generalize the bounded-context-switching ones: the computations must be dividable into k phases, for a fixed k, where in each phase the automaton can push onto any stack, but can pop only from one stack (proved decidable recently in [11]). We show that graph automata on graphs of tree-width $O(2^k)$ can simulate them.

- Ordered multi-stack pushdown automata: The restriction here is that there a finite number of stacks that are ordered, and at any time, the automaton can push onto any stack, but pop only from the first non-empty stack. Note that the computation is not cut into phases, as in the above two restrictions. We show that automata on graphs of tree-width $O(n \cdot 2^n)$ (where n is the number of stacks) can simulate them.

- Distributed queue automata on polyforest architectures: Distributed queue automata is a model where finite-state processes at n sites work by communicating to each other using FIFO channels, modeled as queues. It was shown recently, that when the architecture is a polyforest (i.e. the underlying undirected graph is a forest), the emptiness problem is decidable (and for other architectures, it is undecidable) [12]. We

Tree-width bounds for other Under-approximations

MadhuParlato.pdf (page 2 of 28)

Search

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Non-trivial arguments are necessary

Why Tree-width?

Theorem: (D. Seese)

1. If the MSO(2) satisfiability problem for a class C of graphs is decidable then C has bounded tree-width.
2. For any k, checking MSO(2) satisfiability among the class of graphs with tree-width at most k is decidable.

Why Tree-width?

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2. For any k, checking MSO(2) satisfiability among the class of graphs with tree-width at most k is decidable.

Corollary:
If C is any MSO(2) definable family of graphs then, for any k, checking MSO(2) satisfiability among graphs in C with tree-width at most k is decidable.

Applying Seese's Theorem

There is a formula Φ_{CBM} in MSO over graphs which describes the class of CBMs.

* Satisfiability of Φ over CBMs is equivalent to satisfiability of $\Phi \wedge \Phi_{CBM}$ over graphs.

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The MSO theory of nested words is decidable.

Applying Seese's Theorem

* Identify an MSO definable under-approximation class C of behaviours which guarantees a bound on the tree-width.

  eg. all behaviours with at most k context-switches

* MSO satisfiability w.r.t. C is decidable via Seese's Theorem.

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* Model-checking restricted to the class C is decidable via Seese's Theorem.

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Emptiness, universality, containment is decidable w.r.t. k context-bounded behaviours.

Automata for better complexity

Theorem:(MadhusudanParlato 2011)
Emptiness problem for graph automata over any MSO definable class of graphs of tree-width <= k is decidable in time exponential in k.

* The formula for the class is fixed and so plays no role in the complexity.

* Convert the graph automaton running on CBMS into a tree automaton running on their tree-decompositions.

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Yields the same complexity as handcrafted algorithms in almost all cases.

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Graph Structure and Monadic Second-Order Logic

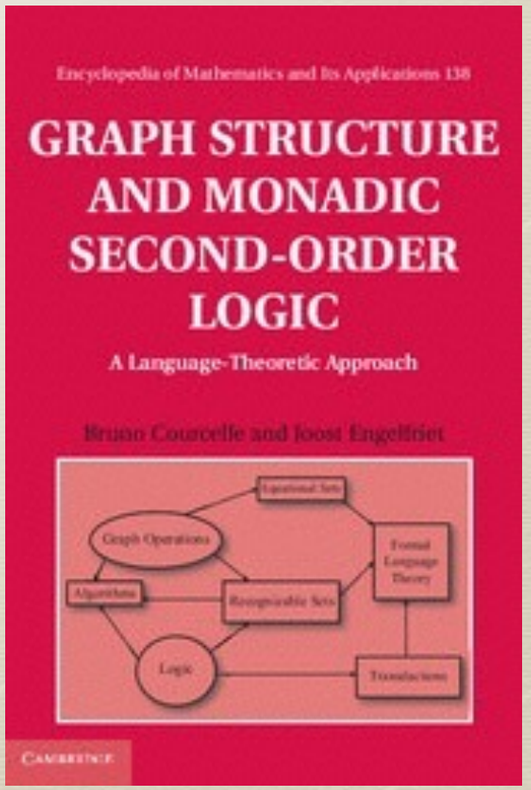
A Language-Theoretic Approach

BRUNO COURCELLE

Université de Bordeaux

JOOST ENGELFRIET

Universiteit Leiden



Other Measures: Desiderata

* MSO definable under-approximation classes of behaviours with bounded measure.

  eg. all behaviours with at most k context-switches

* Decidable MSO satisfiability for such bounded classes via ?? Theorem.

  eg. MSO satisfiability w.r.t k context-bounded CBMs

* Model-checking ...

With a translation to tree-automata to obtain efficient solutions.

Clique-width

* An algebraic measure for graphs.
  * An algebra to construct graphs, each expression has a size.
  * Size of a graph is the size of the smallest expression generating it.
* MSO decidability for graphs with bounded measure.

> Translation to tree-automata : the expression trees can be used to interpret the graphs

Co-graphs: An example

* Family of graphs generated by the following algebra:

$$G ::= a \in \Sigma \mid G \oplus G \mid G \otimes G$$

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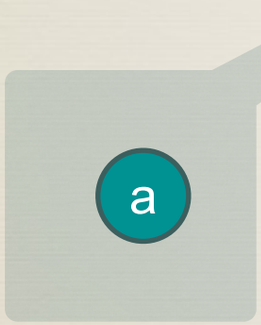

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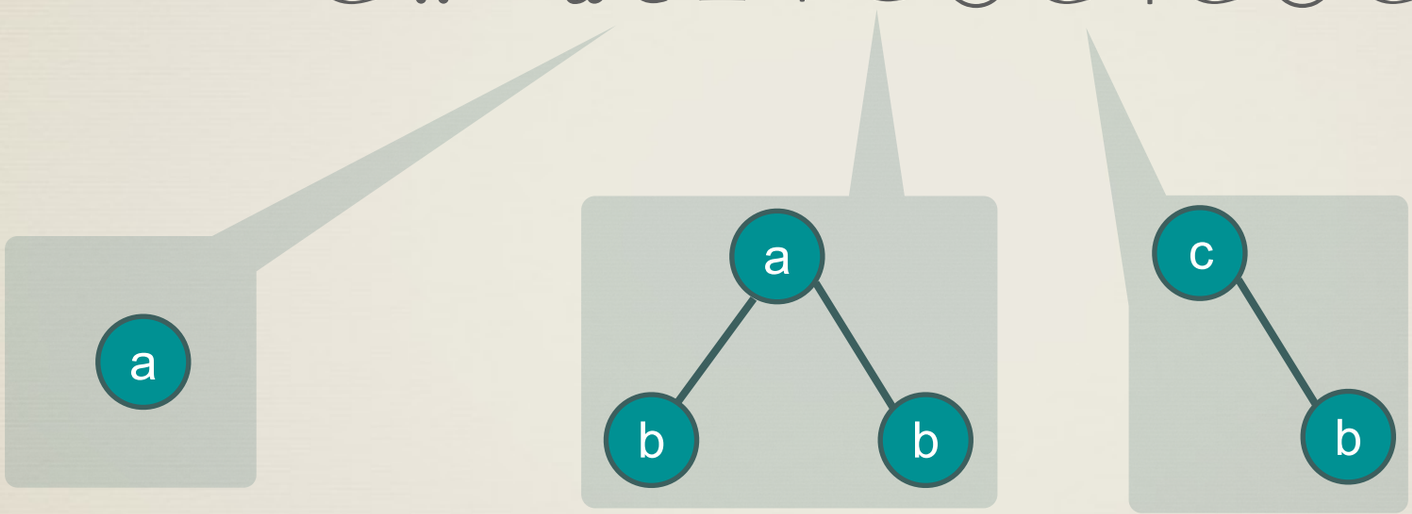

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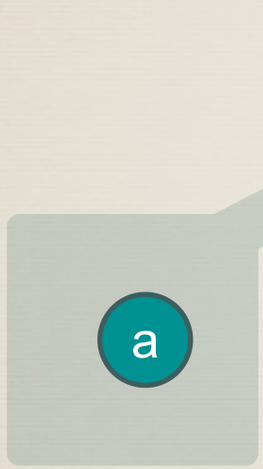

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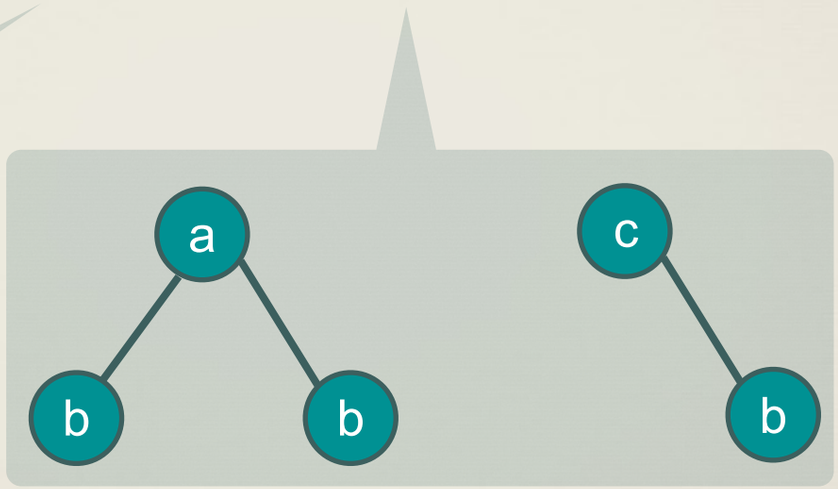

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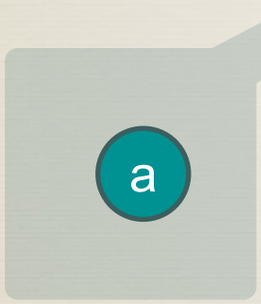

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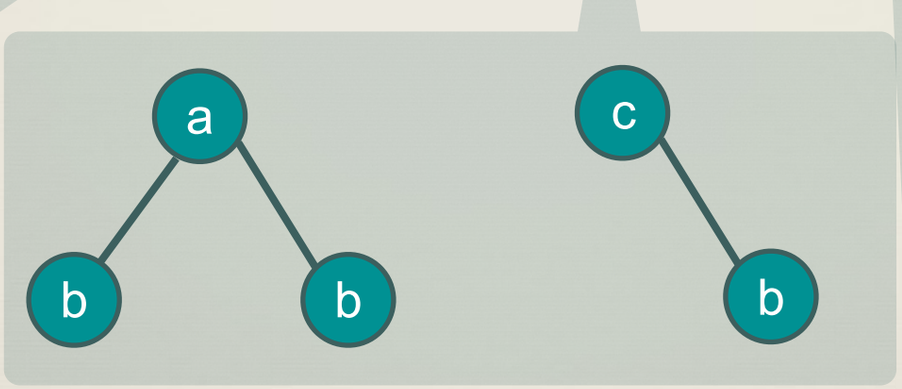

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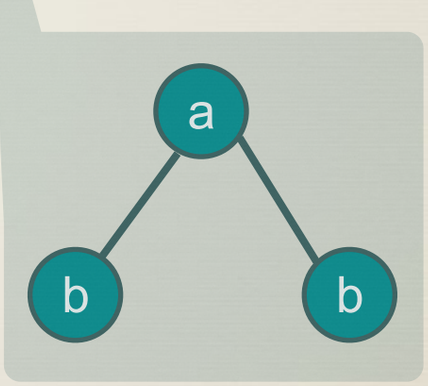

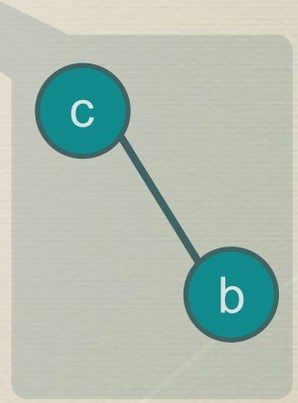

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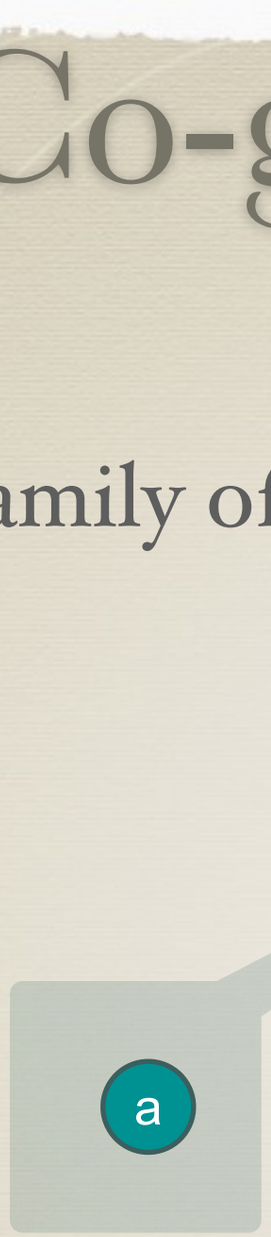

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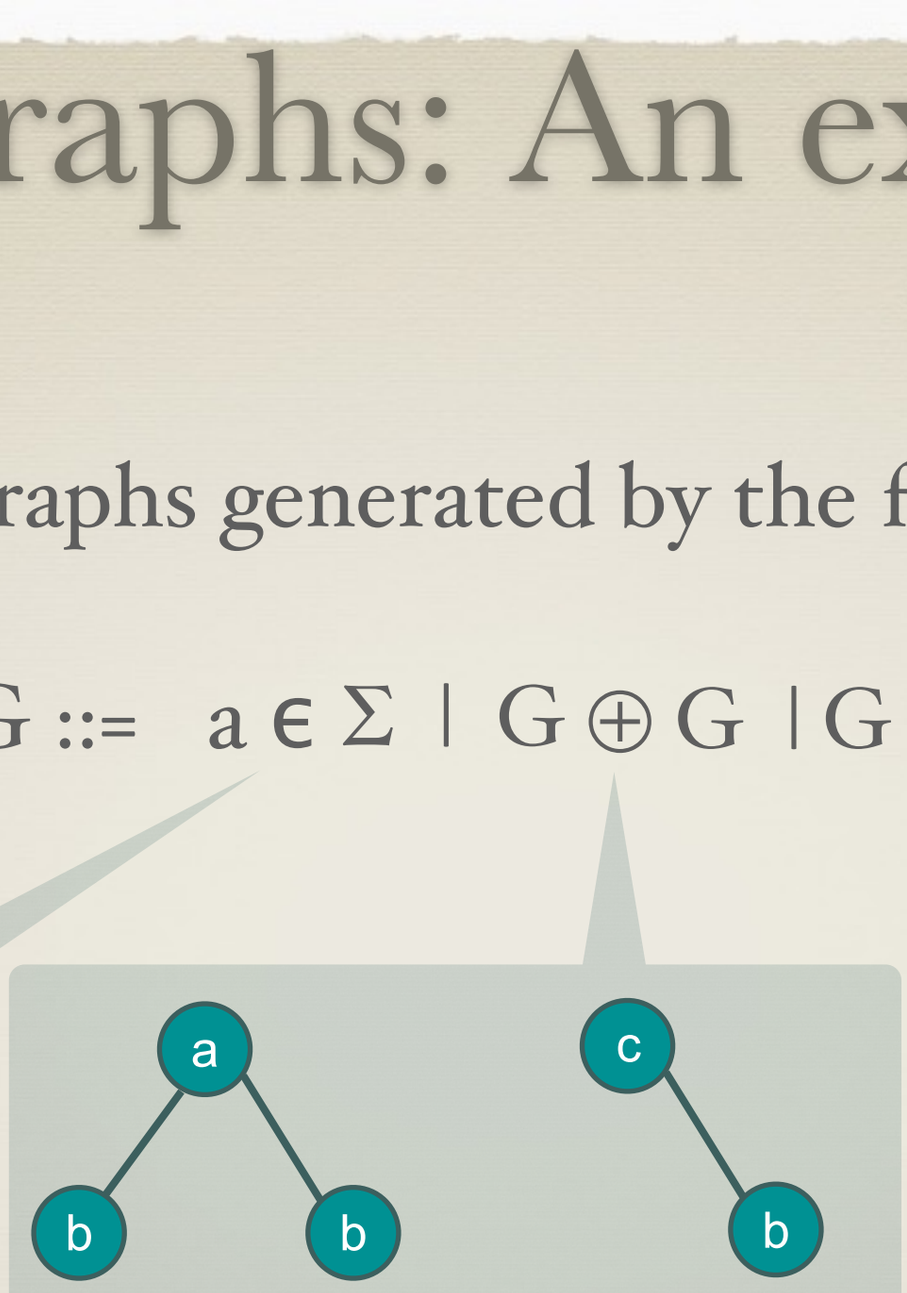

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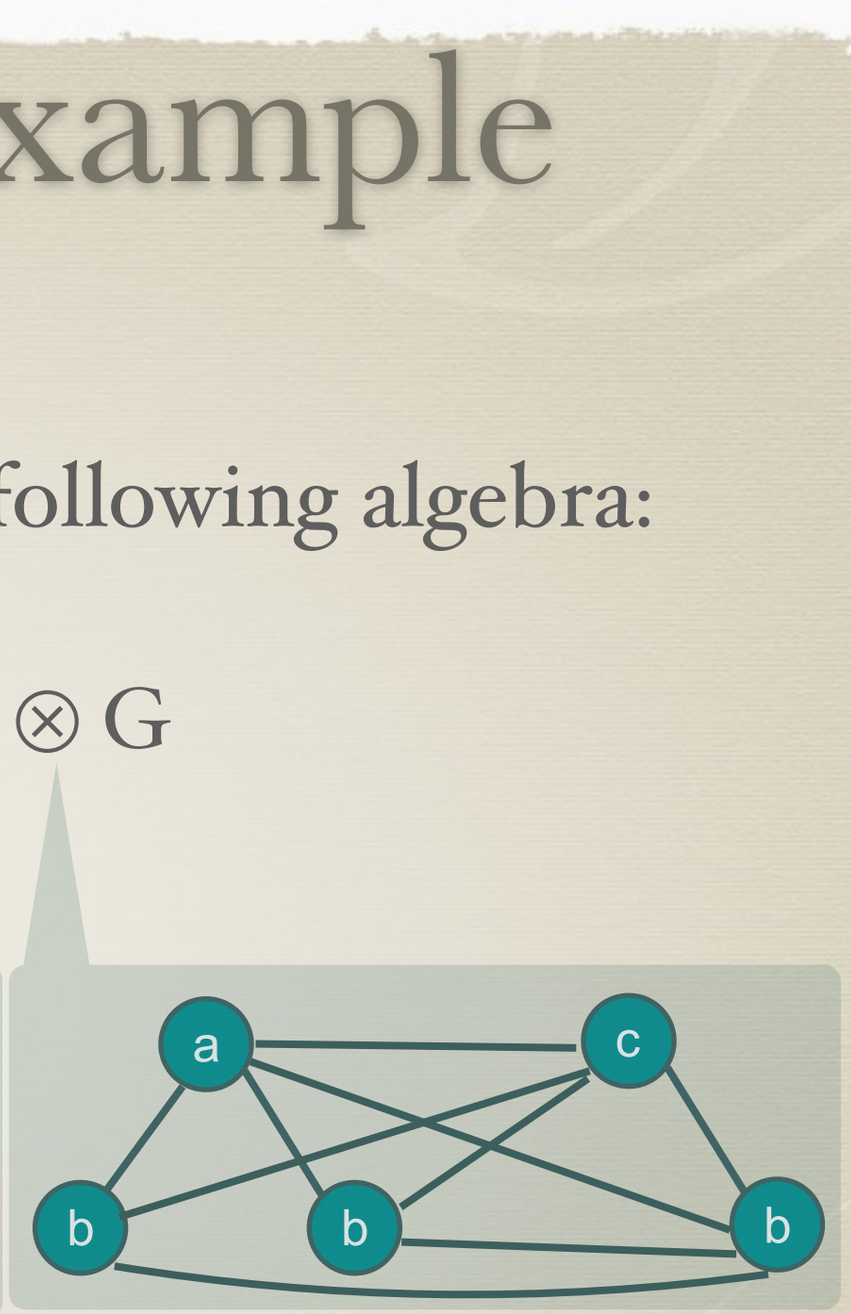
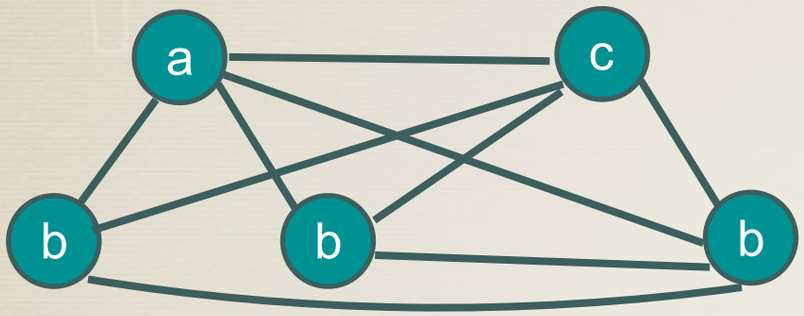


Disjoint union and connect all pairs

Co-graphs to Trees

Every co-graph has an expression generating it.



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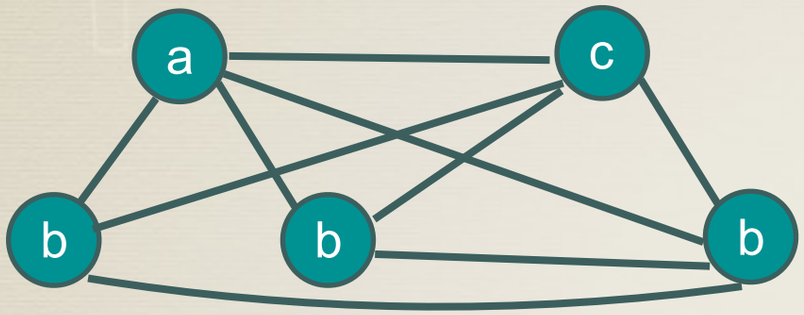

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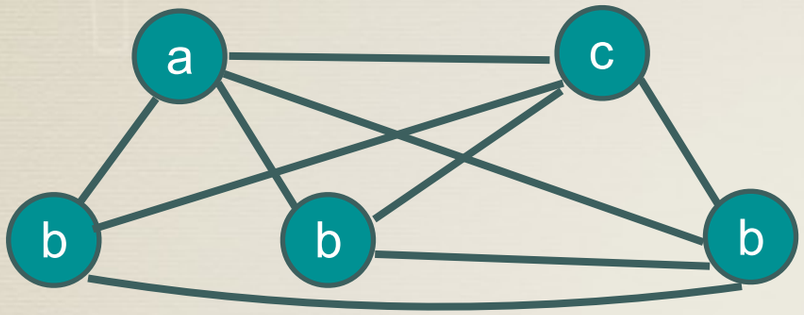

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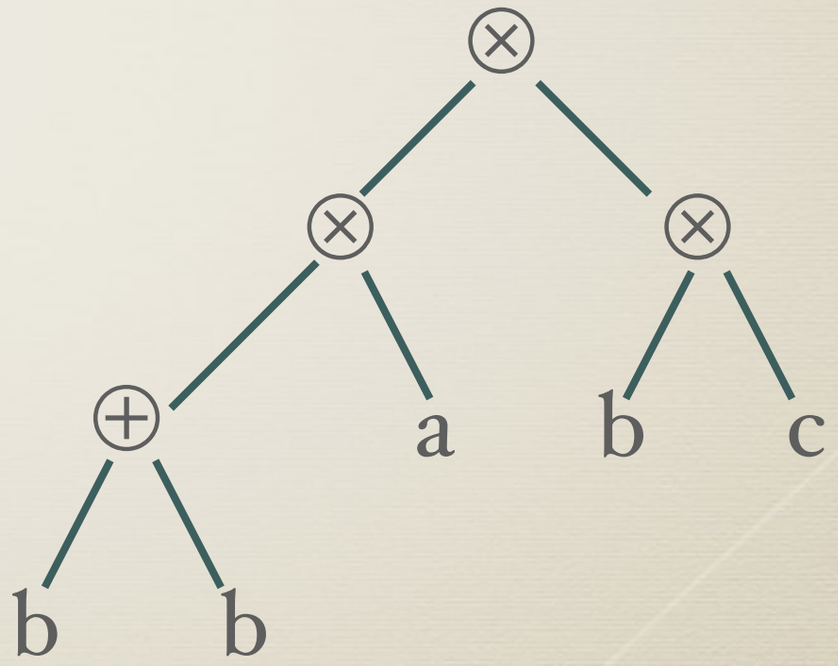

Vertices of the graph correspond to leaves of the tree.

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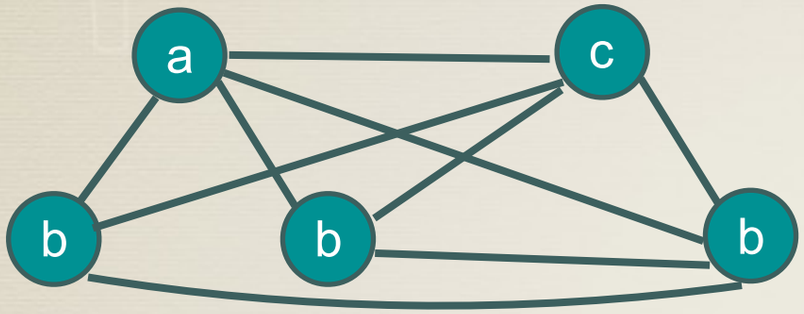

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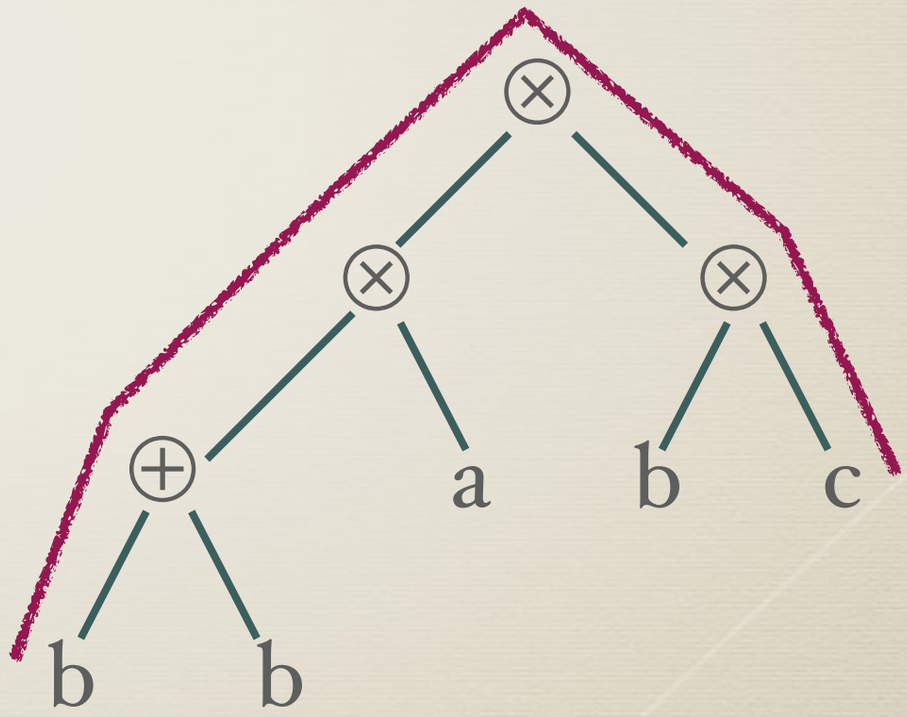

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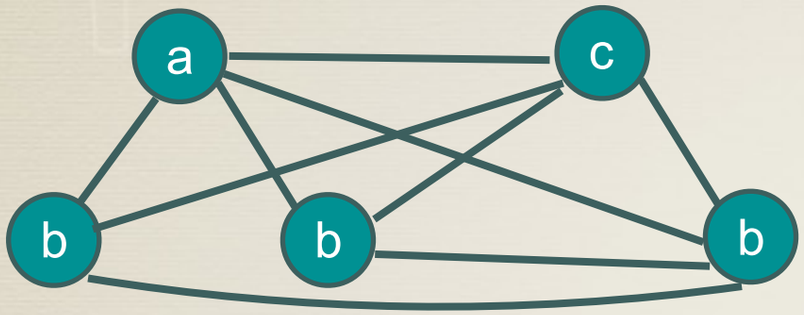

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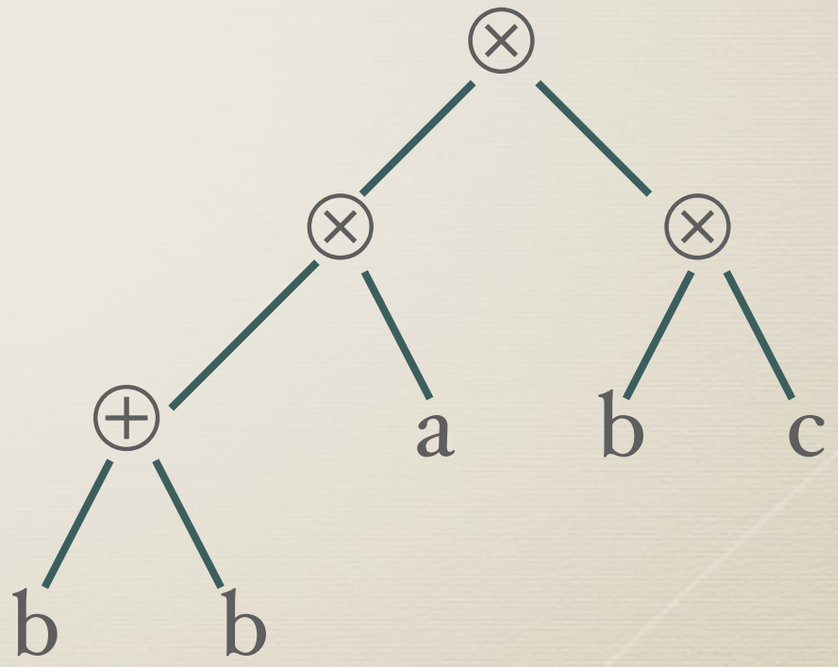

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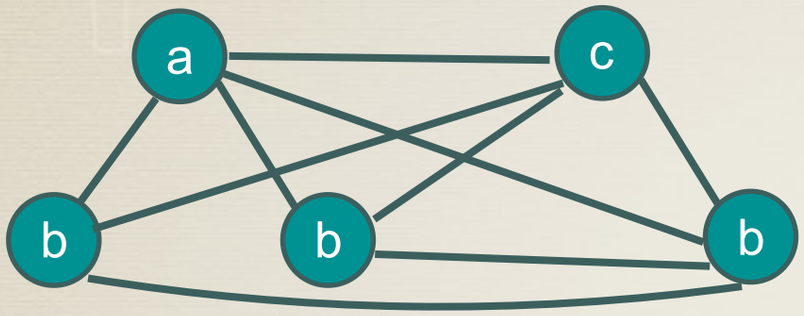

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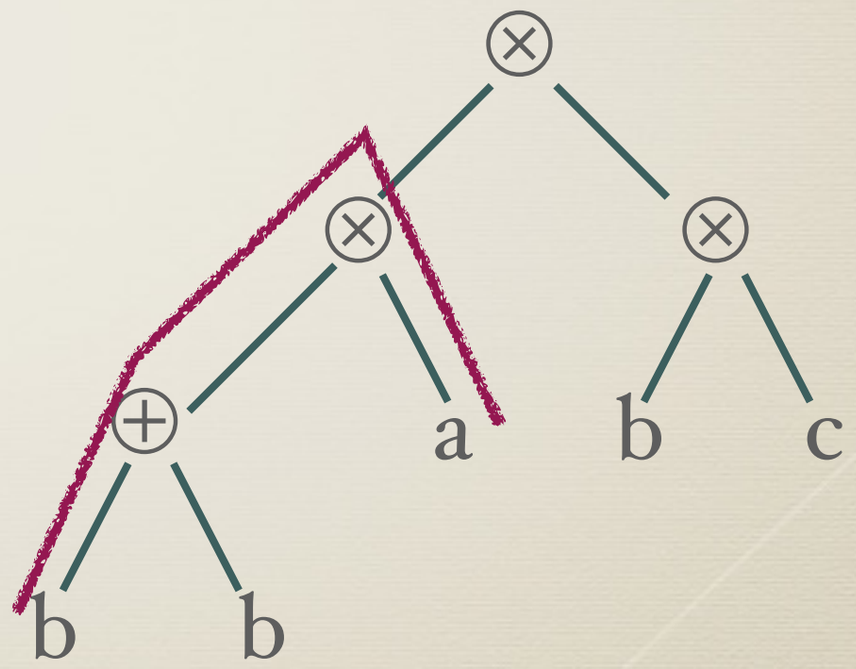

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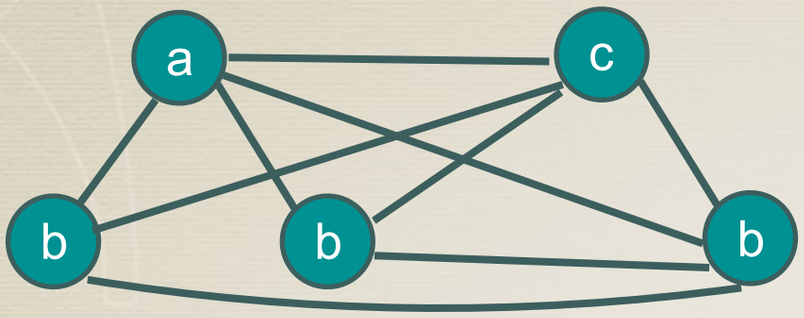
Interpretation on Trees



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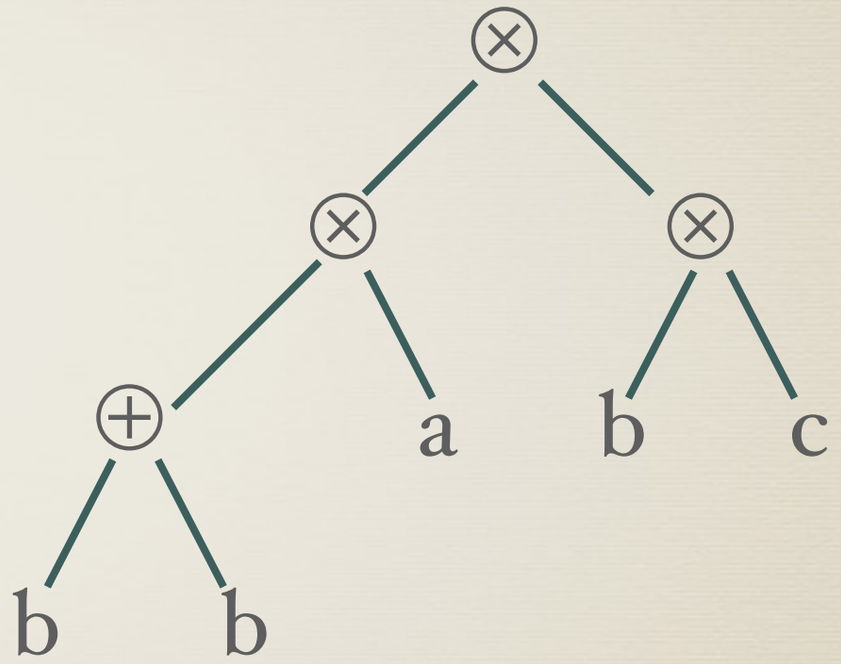

Graph	Tree
There is a vertex x	There is a leaf x
There is a set of vertices X	There is a set of leaves X
a(x)	a(x)
E(x,y)	There is path from x to y whose highest node is a ⊗

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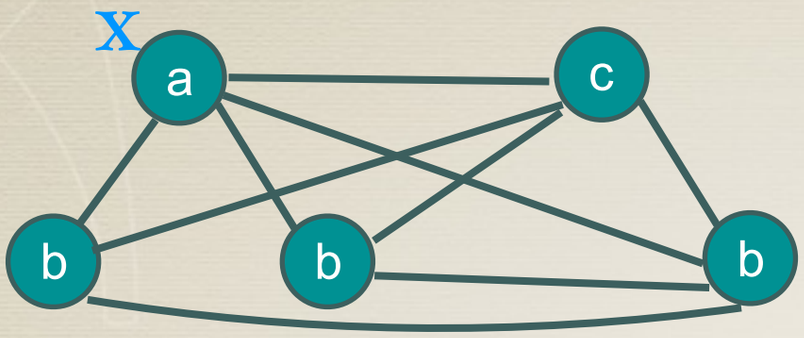

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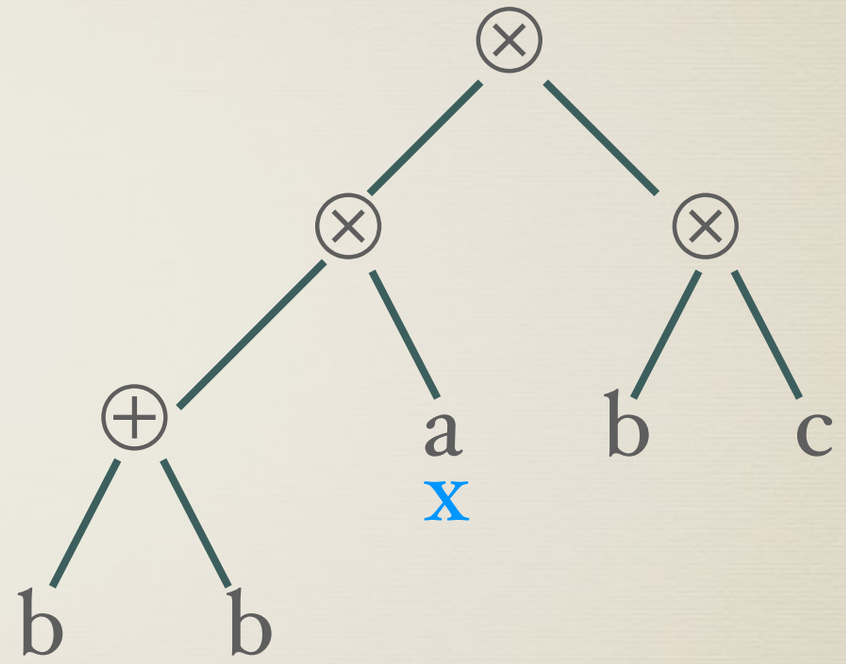

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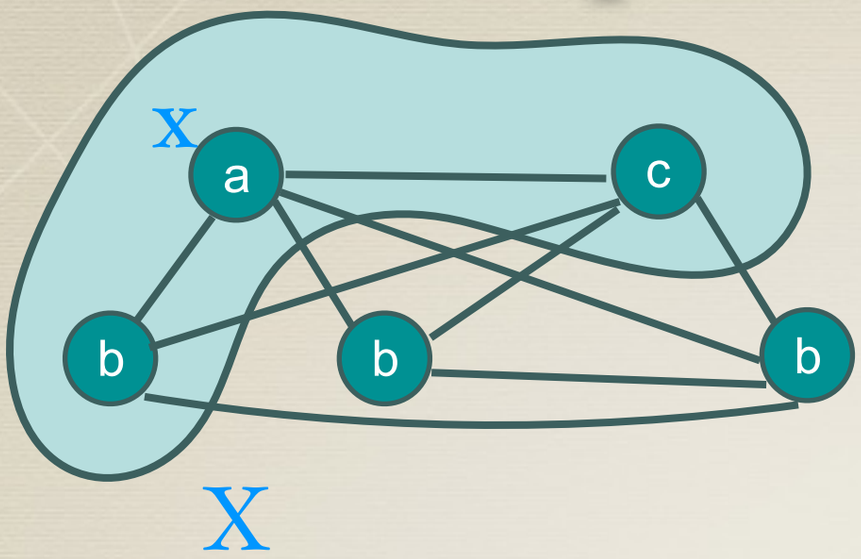

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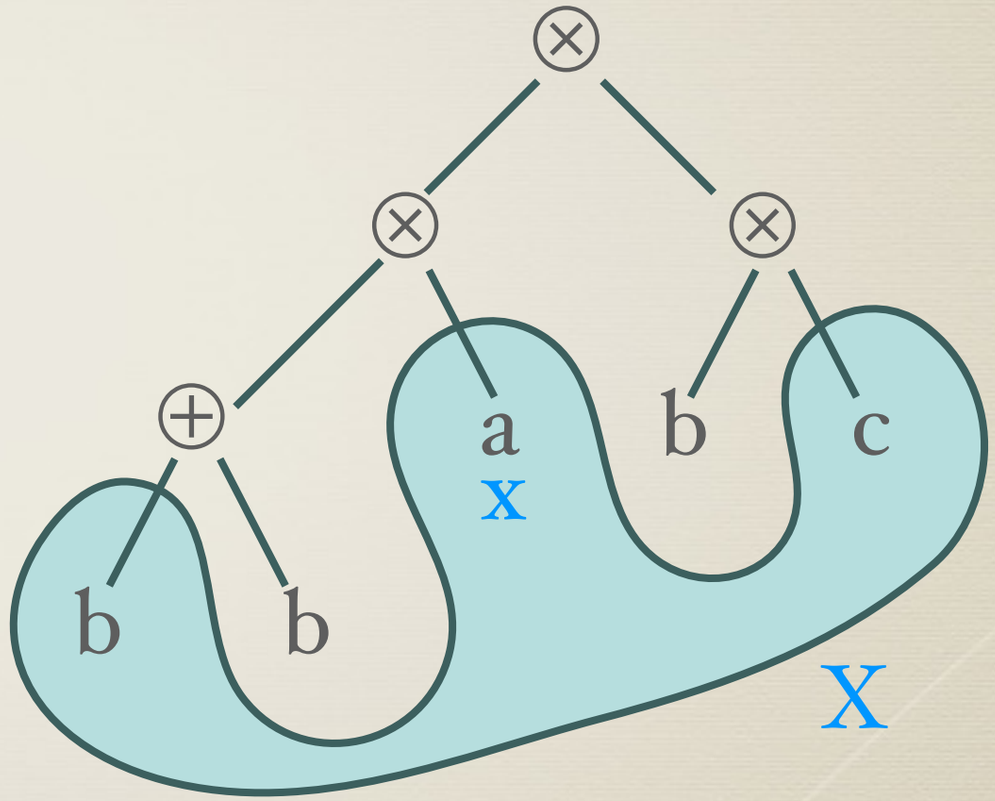

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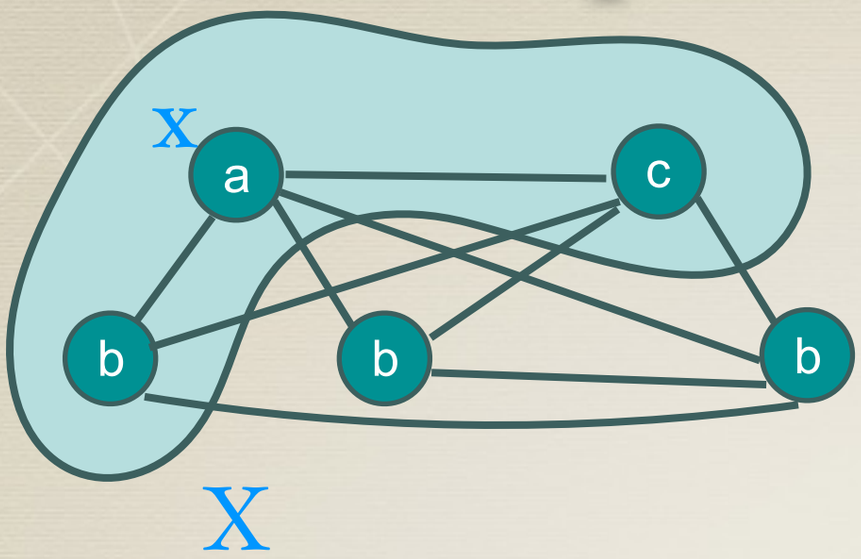

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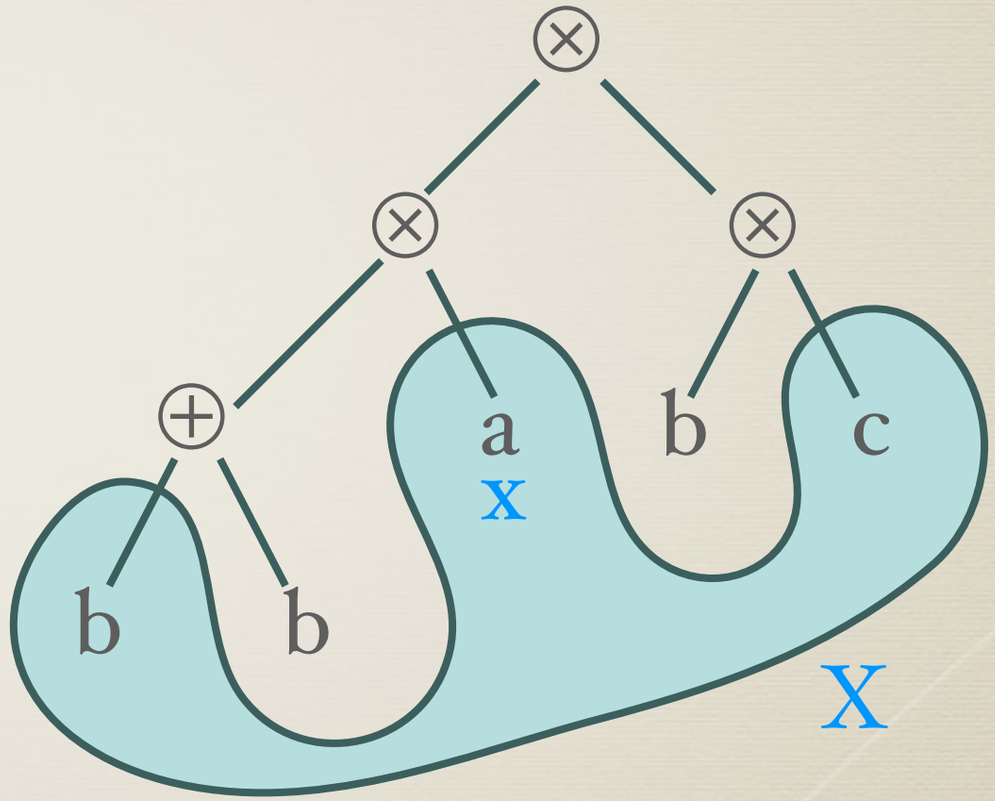

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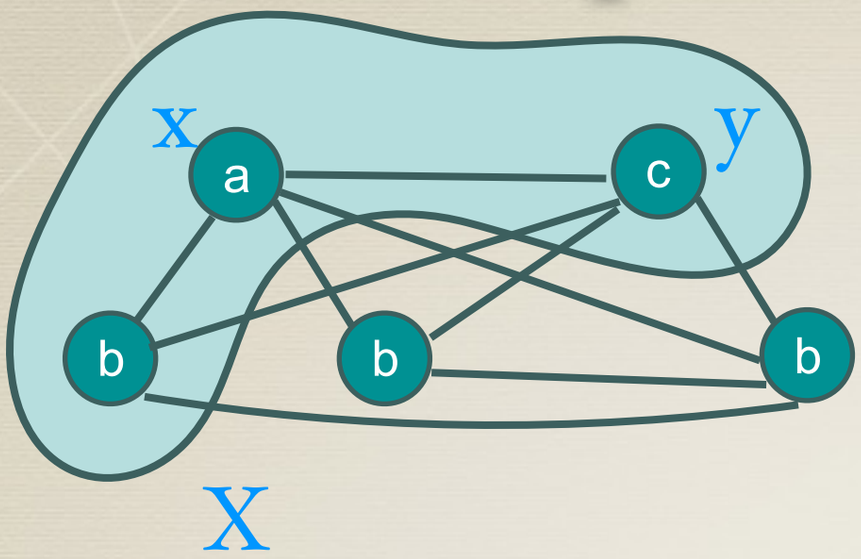

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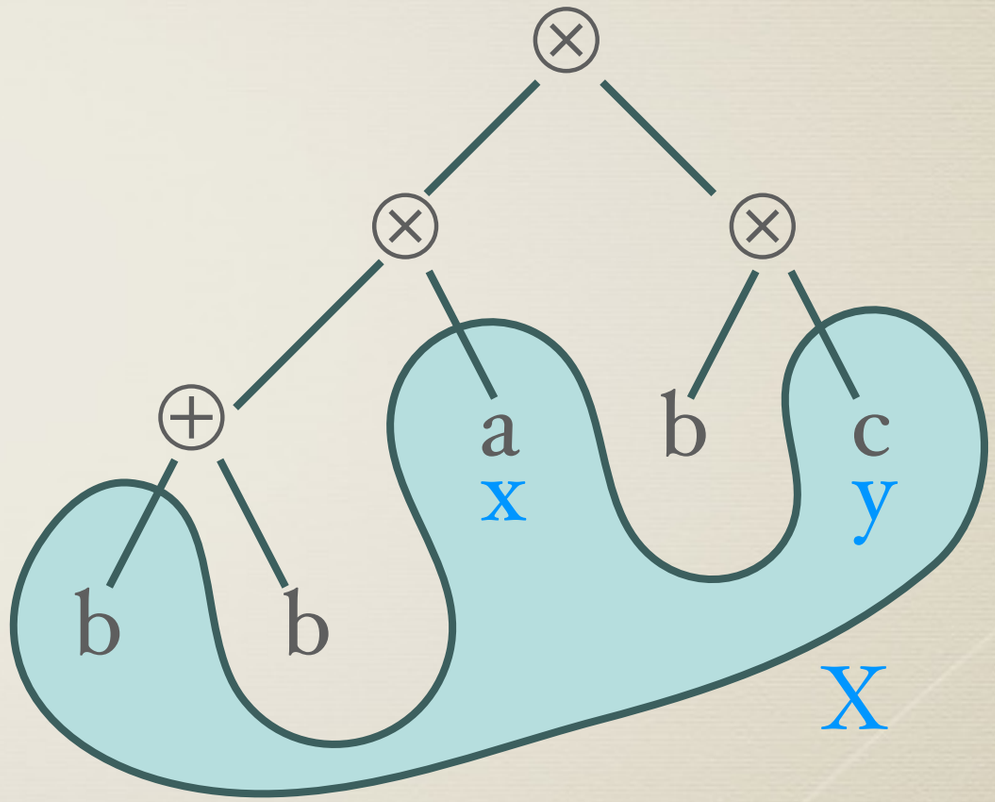

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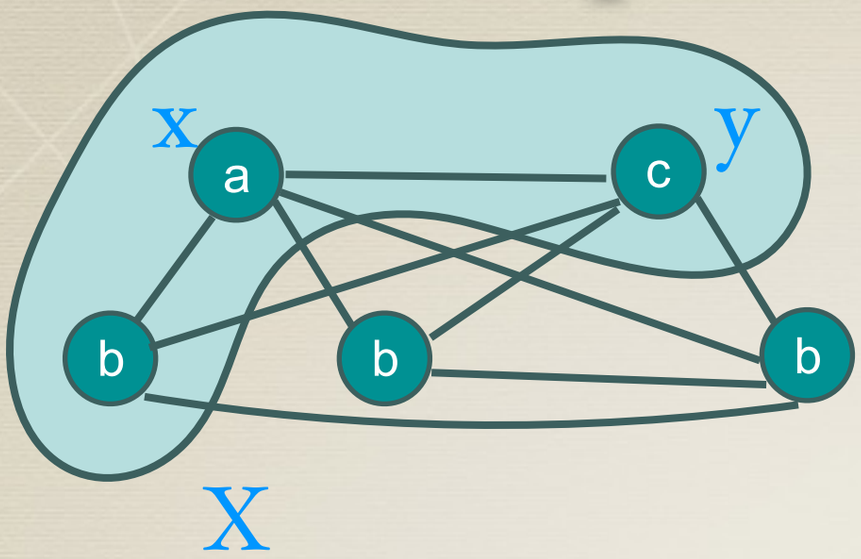

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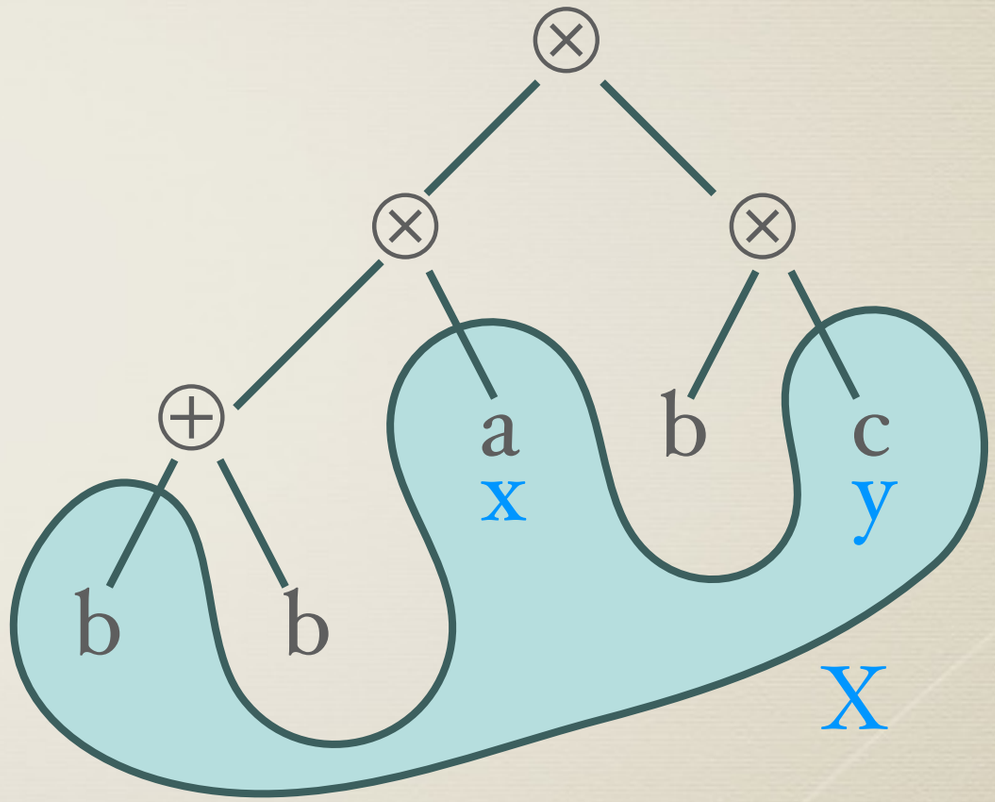
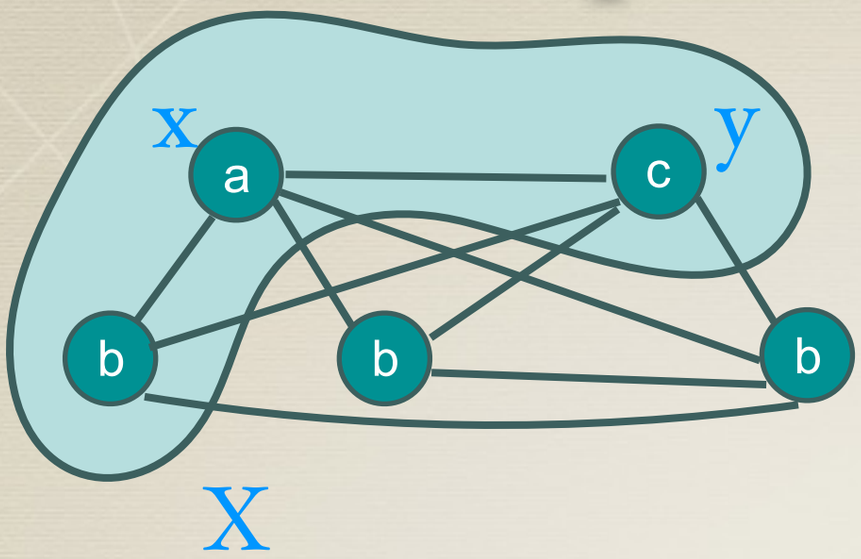


Graph	Tree
There is a vertex x	There is a leaf x
There is a set of vertices X	There is a set of leaves X
a(x)	a(x)
E(x,y)	There is path from x to y whose highest node is a ⊗

Interpretation on Trees



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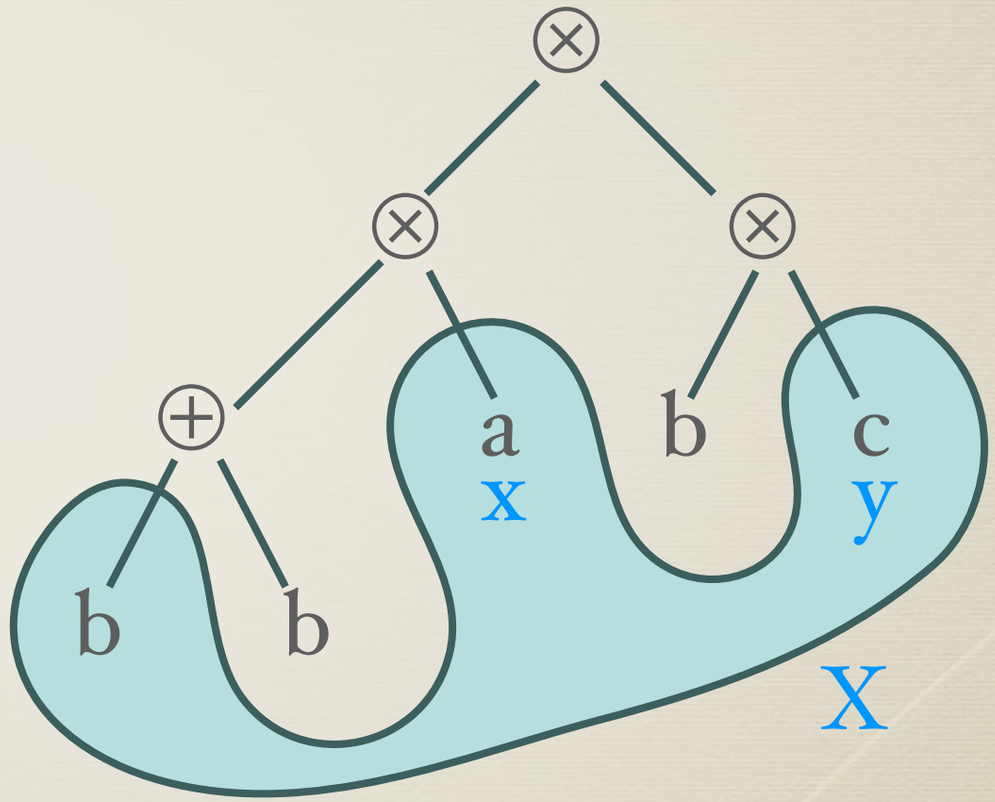

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We translate a formula Φ over graphs labelled with Σ to a formula Φ_{tree} over trees labelled with $\Sigma \cup \{\oplus, \otimes\}$.

MSO decidability for Co-graphs

* The collection of trees labelled by $\Sigma \cup \{\oplus, \otimes\}$ that constitute valid co-graph expressions is a regular tree language.

  Expressible in MSO over trees (φ_{co})

A formula Φ in MSO over graphs is satisfiable over co-graphs
iff
the formula $\Phi_{tree} \wedge \varphi_{co}$ is satisfiable over trees.

The MSO theory of co-graphs is decidable.