13/02/2015

Representations of the symmetric group

Homework 5

(Due on 20/02/2015 at 9:10 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions.

1. Suppose χ is a non-zero, non-trivial character of G and that $\chi(g)$ is a non-negative real number for all $g \in G$. Prove that χ is reducible.

2. Suppose χ is an irreducible character of G. Suppose $z \in Z(G)$ and that z has order m. Prove that there exists an mth root of unity $\lambda \in \mathbb{C}$ such that for all $g \in G$,
$$\chi(zg) = \lambda\chi(g).$$

3. Let $G = S_4$.
   (a) Calculate the character table of A_4 giving proper reasoning for each step.
   (b) Restrict each irreducible representation of S_4 to A_4 and decompose it as a direct sum of irreducible representations of A_4.
   (c) Induce each irreducible representation of A_4 to S_4 and decompose it as a direct sum of irreducible representations of S_4.

4. Show that induction is transitive as follows: suppose we have groups $K \subseteq H \subseteq G$ and a representation ρ of K, then
$$\rho\uparrow_K^G \;\cong (\rho\uparrow_K^H)\uparrow_H^G .$$