14/01/2015

Representations of symmetric groups

Homework

(Due on 20/01/2015 at 2:00 p.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must **write your own solutions**. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Let $k = \mathbb{Z}/p\mathbb{Z}$. Consider the two dimensional representation of the cyclic group $G = \mathbb{Z}/p\mathbb{Z}$ over k defined as
$$a \mapsto \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$
where a is a generator of G. Find a subspace to show that Maschke's theorem does not hold.
2. Write out the matrices $\rho(g)$ for every $g \in G$ for the representation of the symmetric group S_3 on $\mathbb{C}^3$ given by permuting coordinates.
3. Suppose G is a finite group such that every irreducible $\mathbb{C}[G]$-module is 1-dimensional. Prove that G is abelian.