17/03/2014

Representations of the symmetric group

Homework 9

(Due on 24/03/2014 at 9:10 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Let $\mu = (2, 2, 2)$.
   (a) Find all $\lambda \vdash 6$ such that λ dominates μ.
   (b) For each λ, find $K_{\lambda\mu}$.
   (c) Find the decomposition of M^μ as a direct sum of S_6-irreducibles.
2. Recall that for a permutation $\pi = x_1\, x_2 \ldots x_n$, $P(\pi) := r_{x_n} \ldots r_{x_1}(\emptyset)$ and $Q(\pi)$ is the corresponding recording tableau.
   (a) Compute the pair $(P(\pi), Q(\pi))$ for $\pi = 3\ 7\ 2\ 4\ 1\ 6\ 5$.
   (b) Given $\tilde{P} = \begin{array}{|c|c|c|}\hline 1 & 3 & 7 \\ \hline 2 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} 8 \\ \cline{1-1}\end{array}$ and $\tilde{Q} = \begin{array}{|c|c|c|}\hline 1 & 2 & 6 \\ \hline 3 & 8 \\ \cline{1-2} 5 \\ \cline{1-1} 7 \\ \cline{1-1}\end{array}$, find the permutation σ such that $P(\sigma) = \tilde{P}$ and $Q(\sigma) = \tilde{Q}$. Also find the permutation τ such that $P(\tau) = \tilde{P}^t$ and $Q(\sigma) = \tilde{Q}^t$.
3. We saw in class that if P is a standard tableau then the P-tableau of π_P is P itself. Describe the Q-tableau of π_P.
4. An involution is a map π from a set to itself such that π^2 is the identity. Prove that a permutation π is an involution if and only if $P(\pi) = Q(\pi)$. Conclude that the number of involutions in S_n is given by $\sum_{\lambda \vdash n} f^\lambda$.