10/03/2014

Representations of the symmetric group

Homework 8

(Due on 17/03/2014 at 9:10 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Let $\lambda = (3, 2, 1)$. Consider representation S^λ of S_6.
   (a) Express $S^\lambda \uparrow^{S_7}$ as a direct sum of S_7-irreducibles.
   (b) Express $S^\lambda \downarrow_{S_5}$ as a direct sum of S_5-irreducibles.

2. Consider the bijective map $\theta : M^\mu \to \mathbb{C}[T_{\lambda\mu}]$ defined in class. Let $\lambda = (3, 1, 1)$, $\mu = (3, 2)$ be two partitions of $n = 5$. Let the tableau t deciding the numbering of boxes of λ be $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array}$.
   (a) What is the image of $\{s'\} = \begin{array}{ccc}\hline 2 & 3 & 1 \\ \hline 5 & 4 & \\ \hline\end{array}$ under θ?
   (b) What is the pre-image of $T = \begin{array}{|c|c|c|}\hline 2 & 1 & 2 \\ \hline 1 \\ \cline{1-1} 1 \\ \cline{1-1}\end{array}$? Call it s.
   (c) What is the action of $\pi = (123)(45)$ on the generalized tableau T in part (b)? Check for this example that $\theta(\pi\{s\}) = \pi T$.

3. Let t be a tableau of shape λ and let A and B by subsets of the jth and $(j + 1)$st columns of t respectively. Recall that the standardized choice of π_i's for the Garnir element $g_{A,B}$ is: take those $\pi_i \in S_{A\cup B}$ such that the elements of $A \cup B$ are increasing down the columns of πt.

   Prove that this choice of π_i's is indeed a transversal for $S_A \times S_B$ in $S_{A\cup B}$.