03/03/2014

Representations of symmetric groups

Homework 7

(Due on 10/03/2014 at 9:10 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must **write your own solutions**. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. List a basis for $S^{(3,2)}$.

2. Let $t = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2} 7 & 6 \\ \cline{1-2} 8 \\ \cline{1-1}\end{array}$.

   (a) Find the Garnir element corresponding to t.

   (b) Use the straightening law to undo the row inversion in t.

   (c) Express e_t as a linear combination of standard polytabloids.

3. Let H be a subgroup of G and let V denote the coset representation of G w.r.t H. Let C_x denote the conjugacy class of an element $x \in G$ and let $C_G(x) = \{y \in G \mid xy = yx\}$ be the centralizer of x in G. Prove that

$$\chi_V(x) = [G : H]\frac{|C_x \cap H|}{|C_x|}.$$

Deduce that for $\lambda, \mu \vdash n$, if χ_λ is the character of M^λ, then

$$\chi_\lambda(C_\mu) = \frac{n!\ |C_\mu \cap S_\lambda|}{(\lambda_1! \cdots \lambda_r!)\ |C_\mu|}.$$

(Here $\lambda = (\lambda_1, \ldots, \lambda_r)$.)