

Representations of the symmetric group

Homework 5

(Due on 10/02/2014 at 9:10 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Show that induction is transitive i.e. suppose we have groups $K \subseteq H \subseteq G$ and a representation X of K , then

$$X \uparrow_K^G \cong (X \uparrow_K^H) \uparrow_H^G.$$

2. Let $\rho : G \rightarrow \text{Gl}(V)$ be a representation of G . Its kernel is the set $N = \{g \in G \mid \rho(g) = I\}$. V is said to be a faithful representation if ρ is one-to-one.

- (a) Suppose the character of ρ is χ and its degree is d . Prove that $g \in N$ if and only if $\chi(g) = d$. (Hint: Show that $\chi(g)$ is a sum of roots of unity.)
- (b) Show that for the coset representation w.r.t $H \subseteq G$, $N = \cap g_i H g_i^{-1}$, where g_i are the transversal.
- (c) Find the conditions under which each of the following representations are faithful: trivial, regular, coset, sign (for $G = S_n$), defining (for $G = S_n$), degree 1 for C_n .

3. Let $\lambda = (3, 2)$. List all tabloids of shape λ .

4. Let λ be a partition of n . As seen in class, the action of S_n on the set of tableaux of shape λ induces an action on the set of λ -tabloids. Show that under this action, any 2 λ -tabloids lie in the same S_n -orbit.