09/04/2014

Representations of the symmetric group

Homework 11

(Due on 20/04/2014)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. For each of the following pairs (λ, μ), express the product $s_\lambda s_\mu$ of Schur polynomials as a sum of other Schur polynomials. Explain your reasoning.
   (a) $\lambda = (2,1,1),\ \mu = (5)$,
   (b) $\lambda = (2,1,1),\ \mu = (1^5)$,
   (c) $\lambda = (3,1),\ \mu = (2,2)$,
   (d) $\lambda = (2,2,1),\ \mu = (3,3)$.
2. Let $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 7 & 1 & 6 & 8 & 3 & 5 & 2 \end{pmatrix}$. Construct the pair (P, Q) that corresponds to π under the R-S correspondence using Viennot's construction. Draw the shadow diagram and construct the sequences $P_0 = \emptyset, P_1, \ldots, P_k = P$ and $Q_0 = \emptyset, Q_1, \ldots, Q_k = Q$. Explain the reasoning behind obtaining each row of P and Q.
3. For $\lambda = (2,2,1)$, express p_λ as a sum of monomial symmetric polynomials,
$$p_\lambda = \sum_\mu \phi_\lambda^\mu m_\mu.$$
For $\mu = (3,2)$, verify that your answer is correct as follows: express M^μ as a direct sum of irreducible S_5-representations and calculate the character ϕ_λ^μ using this direct sum decompostion.
4. Express $s_{(3,2,1)}$ as a sum of monomial symmetric polynomials.
5. Construct the character table of S_6. (Suggestion: do this as a class activity, each person calculating one or two rows.)