Reflection Groups - Assignment 3
Due on Thursday, February 18, 2016
(please submit in class)

1. Prove that the reflection group of type BC_2 is isomorphic to the dihedral group D_8.
2. Prove that the set of roots $\{\pm e_i \mid i = 1, 2, \ldots, n\}$ is a root system of type $A_1 \oplus A_1 \oplus \cdots \oplus A_1$ (n summands).
3. Let Φ be the root system of type A_n. Prove that the intersection Ψ of Φ with the hyperplane $x_1+x_2+\cdots+x_n = 0$ is a root system of type A_{n-1}.
4. Make a sketch of the roots systems $A_1 \oplus A_1$ in $\mathbb{R}^2$ and $A_1 \oplus A_1 \oplus A_1$ in $\mathbb{R}^3$.
5. For $\alpha \in \Delta$ and $w \in W$, let $\Pi(w)$ denote the set $\Pi \cap w^{-1}(-\Pi)$. Show that $\Pi(ws_\alpha)$ is the disjoint union of $s_\alpha\Pi(w)$ and $\{\alpha\}$. Use it to prove the following properties of the length function and the n function: :
   (a) $w\alpha > 0 \implies n(ws_\alpha) = n(w) + 1;$
   (b) $w\alpha < 0 \implies n(ws_\alpha) = n(w) - 1;$
   (c) $w^{-1}\alpha > 0 \implies n(s_\alpha w) = n(w) + 1;$
   (d) $w^{-1}\alpha < 0 \implies n(s_\alpha w) = n(w) - 1;$
6. For $w \in W$, prove that $\det(w) = (-1)^{n(w)}$.
7. If $w, w' \in W$, prove that $n(ww') \leq n(w) + n(w')$ and $n(ww') \equiv n(w) + n(w') \mod (2)$.