Reflection Groups - Assignment 1
Due on January 11 2016

1. Show that the only isometry of $\mathbb{R}^n$ fixing $\mathbf{0}$ and the standard basis is the identity. Further show that this need not be the case if the isometry does not fix $\mathbf{0}$.

2. Prove that the group $\mathrm{Iso}(\mathbb{R}^n)$ is isomorphic to the semidirect product $\mathbb{R}^n \rtimes \mathcal{O}_n$, where $\mathbb{R}^n$ is the subgroup of translations and $\mathcal{O}_n$ is the subgroup of orthogonal transformations.

3. Let $t_v \in \mathrm{Iso}(\mathbb{R}^n)$ denote the translation through the vector v and let $A \in \mathrm{Iso}(\mathbb{R}^n)$ be an orthogonal transformation. Then prove that
$$A t_v A^{-1} = t_{Av}.$$

4. Let A be an $n \times n$ real matrix. Show that $v \cdot w = 0 \Rightarrow Av \cdot Aw = 0$ if and only if A is a scalar multiple of an orthogonal matrix.

5. Prove that two isometries of $\mathbb{R}^n$ that are equal at $\mathbf{0}$ and at a basis of $\mathbb{R}^n$ are the same.