

**MSc. Applications of Mathematics**

## Linear Algebra - Homework 4

(Due on 01/02/2017 at 10:30 a.m.)

## Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Let  $A$  be a non-singular  $n \times n$  matrix such that  $\det(\Delta_k) \neq 0$  for each  $1 \leq k \leq n$  ( $\Delta_k$  is the top-left  $k \times k$  submatrix of  $A$ ). Prove the uniqueness of the  $LU$  factorization of  $A$ .

2. Find the  $LU$  factorization of:

$$A = \begin{pmatrix} 2 & -1 & 4 & 0 \\ 4 & -1 & 5 & 1 \\ -2 & 2 & -2 & 3 \\ 0 & 3 & -9 & 4 \end{pmatrix}$$

3. Suppose  $A$  is a  $n \times n$  matrix.

- (i) What is the operation count for calculating the determinant of  $A$  using minors for  $n = 3, 4, 5$ .
- (ii) Guess (and if possible prove) a general formula for the operation count.

4. Given  $A \in \mathbb{C}^{m \times n}$  with  $m \geq n$ , show that  $A^*A$  is nonsingular if and only if  $A$  has full rank.