17/01/2017

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MSc. Applications of Mathematics
Linear Algebra - Homework 3
(Due on 25/01/2017 at 10:30 a.m.)

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Instructions:

- Solutions must be complete and legible in order to earn maximum points.

- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Prove that the rank of a matrix is equal to the number of its nonzero singular values.

2. Prove that $||A||_F = \sqrt{\sigma_1^2 + \cdots + \sigma_r^2}$, where σ_i are the singular values of A.

3. Calculate the full and reduced singular value decompositions for the matrix

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$$

4. Prove that for a $n \times n$ matrix A,

$$|\det A| = \prod_{i=1}^{n} \sigma_i$$

where σ_i are the singular values of A.

5. Discuss the differences between the eigenvalue decomposition and the singular value decomposition of a matrix. Include one or more examples to illustrate your points.