11/01/2017

MSc. Applications of Mathematics

Linear Algebra - Homework 2

(Due on Wednesday, 18 January at 10:30 a.m.)

1. Let A be an $m \times n$ matrix. Define

$$||A||_F = (\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2)^{\frac{1}{2}}.$$

   (i) Prove that this is indeed a matrix norm.

   (ii) Evaluate $||A||_1$, $||A||_2$, $||A||$ and $||A||_F$ for

$$A = \begin{pmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{pmatrix}.$$

2. Explain why $||I_{n\times n}|| = 1$ for every induced matrix norm. What is $||I_{n\times n}||_F$?

3. If $x \in \mathbb{C}^m$ and A is a $m \times n$ matrix, prove the following inequalities:

   (a) $||x||_\infty \leq ||x||_2$.

   (b) $||x||_2 \leq \sqrt{m}||x||_\infty$.

   (c) $||A||_\infty \leq \sqrt{n}||A||_2$.

   (d) $||A||_2 \leq \sqrt{m}||A||_\infty$.

4. Prove that if a matrix is both triangular and unitary, then it is diagonal. What are its diagonal entries?

5. Prove that for the induced 2-norm and the Frobenius norm on matrices are invariant under multiplication by unitary matrices.