01/09/2013

MSc. Applications of Mathematics

Linear Algebra - Homework 5

(Due on 10/09/2013 at recitation)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Find the Cholesky factorization of

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 10 \\ 3 & 1 & 35 & 5 \\ 4 & 10 & 5 & 45 \end{pmatrix}$$

2. Let A be a symmetric, positive definite matrix.
   (a) Prove the uniqueness of Cholesky factorization $A = BB^T$ in the case when B is chosen to have positive diagonal elements.
   (b) How many distinct Cholesky factorizations of A can there be if it is not assumed that the diagonal elements of B are positive?

3. Show that LU factorization preserves the band structure of band matrices in the following sense. Let $L = (l_{ij})$, $U = (u_{ij})$ and $A = (a_{ij}) = LU$. Prove that:

$$a_{ij} = 0 \;\text{ for }\; |i-j| \geq p \implies \begin{cases} l_{ij} = 0 & \text{if } \; i - j \geq p \\ u_{ij} = 0 & \text{if } \; j - i \geq p \end{cases}$$

4. Suppose the $n \times n$ real matrix A admits a Cholesky factorization $A = BB^T$. Show that:

$$\kappa(B) = \kappa(A)^{\frac{1}{2}}.$$