25/08/2013

MSc. Applications of Mathematics

Linear Algebra - Homework 4

(Due on 02/09/2013 at 10:30 a.m.)

Instructions:

- Solutions must be complete and legible in order to earn maximum points.
- You may discuss and work together if necessary but you must write your own solutions. Copied solutions (from each other or books or the internet) are easy to identify and easier to grade as they can only earn a zero.

1. Let A be a non-singular $n \times n$ matrix such that $\det(\Delta_k) \neq 0$ for each $1 \leq k \leq n$ (Δ_k is the top-left $k \times k$ submatrix of A). Prove the uniqueness of the LU factorization of A.

2. Consider a $n \times n$ triband matrix A:

$$\begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \ddots & \ddots & \ddots & \\ & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & a_n & b_n \end{pmatrix}$$

In class we wrote a formula for the LU factorization of A in terms of δ_k's.

(i) Verify the recursive formula:

$$\delta_0 = 1, \quad \delta_1 = b_1, \quad \delta_k = b_k\delta_{k-1} - a_k c_{k-1}\delta_{k-2} \quad (2 \leq k \leq n).$$

(ii) Prove by induction that for $1 \leq k \leq n$

$$\delta_k = \det(\Delta_k)$$

where Δ_k is the top-left $k \times k$ submatrix of A.

3. Find the LU factorization of:

$$A = \begin{pmatrix} 2 & -1 & 4 & 0 \\ 4 & -1 & 5 & 1 \\ -2 & 2 & -2 & 3 \\ 0 & 3 & -9 & 4 \end{pmatrix}$$

4. Suppose A is a $n \times n$ matrix.

(i) What is the operation count for calculating the determinant of A using minors for $n = 3, 4, 5$.

(ii) Guess (and if possible prove) a general formula for the operation count.