COMBINATORIAL METHODS IN INDIAN MUSIC:

PRATYAYAS IN SAṄGĪTARATNĀKARA OF ŚĀRṄGADEVA[1]

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Six combinatorial tools (called *pratyayas*) have been in systematic use in India for the study of Sanskrit prosody and these go back in time at least to Piṅgala (c. 300 BC). Among these, three − *prastāra* (an enumeration rule for generating all the possible metrical patterns of a given class as a sequence of rows), *uddiṣṭa* (the process for finding, for any given metrical pattern, the corresponding row number in the enumeration) and *naṣṭa* (the converse of *uddiṣṭa*) are found in Bharata's *Nāṭyaśāstra*, in the chapter where prosody is discussed. The notion of *pratyayas* was perhaps discussed in other ancient texts of music also. However, the first extant text on music where the *pratyayas* are systematically dealt with, both in connection with patterns of musical phrases (*tānas*) and patterns of musical rhythms (*tālas*), is *Saṅgītaratnākara* of Śārṅgadeva (c.1225 AD). Nārāyaṇa Paṇḍita in his *Gaṇitakaumudī* (1356 AD) deals with some of these questions in a more general context, though his theory does not cover the kind of *tāla-prastāra* considered by Śārṅgadeva.

Our aim in this talk is to highlight the contributions of Śārṅgadeva and explain his work in a mathematical set up. We first discuss the sequential generation or enumeration of patterns of musical phrases, called *tāna-prastāra*. The method of generating these patterns, as discussed in the first chapter of *Saṅgītaratnākara*, is essentially a rule for generating sequentially the $n!$ permutations of n symbols. We note that the *naṣṭa* and *uddiṣṭa* processes are indeed encoded in a certain unique representation of any integer in terms of sums of factorials. We also explain how Śārṅgadeva employs a tabular figure, *khaṇḍa-meru*, to essentially go back and forth between any integer and its representation as a sum of factorials.

[1] The talk is based on the article, Raja Sridharan, R. Sridharan and M. D. Srinivas, Combinatorial Methods in Indian Music: *Pratyayas* in *Saṅgīta-ratnākara* of Śārṅgadeva, in C.S. Seshadri Ed., *Studies in the History of Indian Mathematics,* Hindustan Publishing Agency, Delhi, 2010, pp. 55-112.

We then discuss the *pratyayas* for patterns of musical rhythms, *tāla-prastāra*. This theory has been dealt with at length in the sixth chapter of *Saṅgītaratnākara*. It is in fact a generalisation of the theory of *pratyayas* for moric metres or *mātrā-vṛttas*, where the short syllable (*laghu*) is taken to be of one *mātrā* (metrical time unit) and the long syllable (*guru*) is taken to be of two mātrās. *Saṅgītaratnākara* considers musical rhythmic patterns (*tāla*) made up of *druta* (of one time unit) and *laghu, guru* and *pluta*, which are of 2, 4 and 6 durations respectively, of that unit of *druta*. *Saṅgītaratnākara* first presents a systematic method of enumerating all the *tālas* of a given time duration in a *prastāra*, and follows this up with a complete mathematical theory of *pratyayas* which is a generalisation of the corresponding theory for moric metres. An interesting feature of *tāla-prastāra* is that the total number of patterns (the *saṅkhyāṅka*), if laid out in a sequence, is generated by a generating function which involves a polynomial of the sixth-degree.

The discussion of *pratyayas* in prosody and music lead to the study of combinatorics related to three important ways of representing any non-negative integer, representations which are widely in use even today. While discussing the pratyayas for syllabic metres (*varṇa-vṛttas*), Piṅgala gave the procedure for finding the binary representation of integers. Much later, Nārāyaṇa Paṇḍita, in his *Gaṇitakaumudī* (c.1356 AD), generalised the method of Piṅgala for an arbitrary radix. The discussion of *pratyayas* for moric metres (*mātrā-vṛttas*) led to the discovery of the so called Fibonacci representation of integers which expresses any integer uniquely as a sum of Fibonacci numbers. In his discussion of *tāla-prastāra*, Śārṅgadeva introduced a generalisation of the Fibonacci representation, where the Fibonacci numbers are replaced by the *saṅkhyāṅkas* of *tāla-prastāra*. Another form of generalisation was considered later by Nārāyaṇa Paṇḍita. As we already noted, Śārṅgadeva, in his construction of *khaṇḍa-meru*, implicitly employed the so called factorial representation of integers in his discussion of the *pratyayas* for *tāna-prastāra*.