Inflation after Planck 2013 & 2015

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 $V\left(\phi
ight)\equiv M^{4}\left[1
ight]$

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 $_{3}V(\phi)=M^{4}$

(MPI)

 $\alpha + (\phi | M_{Pl})^2$



<u>Outline</u>

Inflation in brief

□ The Planck CMB data and their implications for inflation

Constraints on Vanilla models

□ Model comparison: what is the best model of inflation?

□ First constraints on the reheating epoch



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Inflation in brief



- Inflation is a phase of <u>accelerated expansion</u> taking place in the very <u>early Universe</u>.





Inflation does not replace the Hot Big Bang model. It is a new ingredient which completes the standard model. It takes place before the Hot Big Bang phase



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$$\frac{\ddot{a}}{a}=-\frac{1}{6M_{_{\mathrm{Pl}}}^2}\left(\rho+3p\right)$$

In GR, any form of energy weighs including pressure



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Inflation in brief





If the scalar field moves slowly (the potential is flat), then pressure is negative



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- In HEP, matter is described by field theory; we take a scalar field, the inflaton, since compatible with the cosmological principle
- Inflation is also a phase of exponential expansion during which the

Hubble radius is almost constant



$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\downarrow$$

$$p \simeq -\rho$$

$$\downarrow$$

$$\frac{d\rho}{dt} = -3H(\rho + p) \simeq 0$$

$$\downarrow$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_{\rm Pl}^2} \simeq \text{Constant}$$

$$\downarrow$$

$$a(t) \sim e^{Ht}$$





The Hubble radius 1/H is almost constant during inflation





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The field oscillates, decays and the decay products thermalize ... Then the radiation dominated era starts ...



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Quantum fluctuations as seeds of CMB anisotropy and large scale structures



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- The properties of the fluctuations can be characterized by the correlation function: 2-point functions (power spectrum), 3-point function (bispectrum) etc ...

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- The reheating phase depends on all the couplings between the inflaton and the other fields (scalar, fermions, gauge bosons)

- It can be parametrized by the the reheating temperature and the mean equation of state during reheating.

$$T_{\rm reh} = \left(g_* \frac{30}{\pi^2} \rho_{\rm reh}\right)^{1/4}$$
$$\bar{w}_{\rm reh} = \frac{1}{\Delta N} \int_{N_{\rm T}}^{N_{\rm reh}} w_{\rm reh}(n) dn$$

In fact, the CMB only depends on a specific combination, the <u>Reheating parameter</u>

$$\ln R_{\rm rad} = \frac{1 - 3\bar{w}_{\rm reh}}{12 + 12\bar{w}_{\rm reh}} \ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right)$$



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➤ The relevant quantities are $\epsilon_{1*} = \epsilon_1(\phi_*)$ and $d_* = \epsilon_2(\phi_*)$ and the reheating dependence enters here sin ϕ_{Θ} depends on the reheating parameter





nflationary predictions: the two-point correlation function



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$$\mathcal{P}_{\zeta} = \frac{H^2}{\pi \epsilon_{1*} m_{\mathrm{Pl}}^2} \left[1 - 2\left(C + 1\right) \epsilon_{1*} - C \epsilon_{2*} - \left(2\epsilon_{1*} + \epsilon_{2*}\right) \ln\left(\frac{k}{k_{\mathrm{P}}}\right) \right]$$

$$\mathcal{P}_h = \frac{16H^2}{\pi m_{\mathrm{Pl}}^2} \left[1 - 2\left(C + 1\right) \epsilon_{1*} - 2\epsilon_{1*} \ln\left(\frac{k}{k_{\mathrm{P}}}\right) \right]$$
- The amplitude is controlled by H
- For the scalar modes, the amplitude also depends on ϵ_1
 $C \approx -0.7$
The power spectra are scale-invariant plus logarithmic corrections the amplitude of which depend on the sr parameters, ie on

the microphysics of inflation

Consistency relation:

$$r = \frac{T}{S} \equiv \frac{\mathcal{P}_h}{\mathcal{P}_{\zeta}} = 16\epsilon_{1*} = -8n_{\scriptscriptstyle \mathrm{T}}$$

Gravitational waves are subdominant

The spectral indices are given by

$$n_{\rm s} - 1 \equiv \frac{\mathrm{d}\ln\mathcal{P}_{\zeta}}{\mathrm{d}\ln k} , \ n_{\rm T} \equiv \frac{\mathrm{d}\ln\mathcal{P}_{h}}{\mathrm{d}\ln k}$$
$$n_{\rm s} = -2\epsilon_{1*} - \epsilon_{2*} \ n_{\rm T} = -2\epsilon_{1*}$$

The running, i.e. the scale dependence of the spectral indices, of dp and gw are $\alpha_{\rm s} \equiv \frac{\mathrm{d}^2 \ln \mathcal{P}_{\zeta}}{\mathrm{d} \left(\ln k\right)^2} \qquad \alpha_{\rm T} \equiv \frac{\mathrm{d}^2 \ln \mathcal{P}_h}{\mathrm{d} \left(\ln k\right)^2} \qquad \alpha_{\rm T} = -2\epsilon_{1*}\epsilon_{2*} - \epsilon_{2*}\epsilon_{3*}$ $\alpha_{\rm T} = -2\epsilon_{1*}\epsilon_{2*}$



















$$\langle \zeta_{\mathbf{k}_{1}} \zeta_{\mathbf{k}_{2}} \zeta_{\mathbf{k}_{3}} \rangle = -\frac{3}{10} f_{\mathrm{NL}} \left(\mathbf{k}_{1}, \mathbf{k}_{2}, \mathbf{k}_{3} \right) \frac{(2\pi)^{5/2}}{k_{1}^{3} k_{2}^{3} k_{3}^{3}} \delta \left(\mathbf{k}_{1} + \mathbf{k}_{2} + \mathbf{k}_{3} \right) \left[k_{1}^{3} \mathcal{P}_{\zeta} \left(k_{2} \right) \mathcal{P}_{\zeta} \left(k_{3} \right) + 2 \text{ permutations} \right]$$



 $f_{\rm NL}$ is of the order of the slow-roll parameters and hence unobservable with the current technology ...



There are literally hundreds of different models of inflation








Multiple field inflation





Multiple field inflation



Inflation with non-minimal kinetic term





Multiple field inflation







Inflation with features





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- Even if one considers single field slow-roll models with minimal kinetic only it remains at least two hundreds models ...



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- ➢ All these models were recently compared to Planck data in "Encyclopedia Inflationaris" (JM, C. Ringeval & V. Vennin, arXiv:1303.3787)





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But different models make different predictions so we can distinguish among them. Non-vanilla models typically predict non-adiabatic perturbation or non-Gaussianities.



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Planck results



Planck 2013 results in brief:

$$\Omega_{\kappa} = -0.058^{+0.046}_{-0.026}$$
$$\alpha_{\mathcal{RR}}^{(2,2500)} \in [0.98, 1.07]$$

$$\Omega_{\kappa} = -0.040^{+0.038}_{-0.041}$$

$$\alpha_{RR}^{(2,2500)} \in [0.985, 0.999]$$

 $f_{_{\rm NL}}^{
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$$f_{\rm NL}^{\rm eq} = -42 \pm 75$$
 $f_{\rm NL}^{\rm eq} = -4 \pm 43$

- $f_{\rm \scriptscriptstyle NL}^{\rm ortho} = -25 \pm 39 \qquad \qquad f_{\rm \scriptscriptstyle NL}^{\rm ortho} = -26 \pm 21$
- Spatially flat universe with adiabatic and Gaussian fluctuations

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- Spatially flat universe with adiabatic and Gaussian fluctuations
- Single field slow-roll inflation with minimal kinetic term is preferred
- We can focus on single field models, not because they are the simplest ones but because they are favored by Planck (Giannantonio & Komatsu 20)



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Planck 2015 constraints on the inflaton potential

$$\begin{split} M_{\rm Pl} \frac{|V_{\phi}|}{V} &< 0.14 \\ M_{\rm Pl}^2 \frac{V_{\phi\phi}}{V} &= -0.01^{+0.005}_{-0.009} \end{split}$$







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 $H_*^2 \sim \mathcal{P}_{\zeta} \epsilon_{1*}$

 $0.010 \begin{bmatrix} 0.010 \\ 0.010 \\ 0.001 \\ 0.00$

Planck 2015 constraints on the energy scale of inflation

$$\begin{array}{l} & \rho_*^{1/4} < 2.2 \times 10^{16} \, {\rm GeV} \\ \\ & H_* < 1.2 \times 10^{14} \, {\rm GeV} \end{array} \end{array}$$



- PLANCK+BICEP2 / $log(\epsilon_1)$ -prior
- – PLANCK / $log(\epsilon_1)$ -prior
- -- PLANCK+BICEP2 / ϵ_1 -prior
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One can derive constraints on power-law parameters

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 Planck 2013 n_s = 0.9603 ± 0.0073
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No detection of scalar running

 $\frac{\mathrm{d}n_{\rm s}}{\mathrm{d}\ln k} = -0.0134 \pm 0.009$





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But what is the best one?

To answer this question one can calculate the Bayesian evidence of each r (the integral of the likelihood over the prior space). Using Bayes theorem, thi leads to the probability of a model

$$p\left(\mathcal{M}_{i}|\mathcal{D}\right) = \mathcal{E}\left(\mathcal{D}|\mathcal{M}_{i}\right)\pi\left(\mathcal{M}_{i}\right)$$



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This is a highly non-trivial computing problem ... needs to set a pipeline of numerical codes: CAMB, CosmoMC, Multinest ...

Planck: and the winners are ...





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posterior-to-prior ratio of inflationary models



Plateau inflation are the winners!





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-mflation in the evidence-Number of unconstrained parameter plane





Displayed Models: 193/193

Displayed Models: 151/193





Displayed Models: 66/193



Displayed Models: 180/193

75





NB: Here, the reference is the best model!!

Statistics





Summary 26 % inconclusive zone 21 % weak zone 18 % moderate zone 34 % strong zone

15 different potentials in the inconclusive zone



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Planck can constrain the reheating epoch





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Technically, this means putting contraints on the reheating parameter introduced before Planck can constrain the reheating epoch

 \succ Technically, this means putting contraints on the reheating parameter introduced before

 \succ Reheating is contrained if the posterior has a width smaller than that of the prior



Constrain on reheating = width of prior/width of posterior>



Planck 2013 constraints on reheating



Displayed Models: 170/193

Planck2013 constraints on reheating







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Recap



Planck 2013: single field inflation with a plateau-like potential. More complicated models (multiple field scenarios, non-minimal kinetic term scenario etc ...) should all have a "bad" Bayesian evidence ...

□ This could change if non standard features are found (NG etc ...)

□ Planck2013: 1/3 of the models are now ruled out

□ KMIII, ESI, Starobinsky model, ... are the winners

Reheating is now constrained, average reduction of the prior to posterior width of about 40%

□ Future CMB experiments such as COrE+: can ruled out 3/4 of the models and provide very good constraints on the reheatin epoch.



