Thermal Physics, Autumn 2019 CMI

Problem set 6

Due by the beginning of lecture on Thu Nov 14, 2019

Thermodynamic potentials, van der Waals gas

1. ⟨**6**⟩ We used extensivity of internal energy to derive the Gibbs-Duhem relation (in the so-called energy representation)

$$d\mu = \frac{V}{N}dp - \frac{S}{N}dT. \tag{1}$$

Use extensivity to obtain another version of the Gibbs-Duhem relation (in the so-called entropy representation)

$$d\left(\frac{\mu}{T}\right) = \frac{V}{N}d\left(\frac{p}{T}\right) + \frac{U}{N}d\left(\frac{1}{T}\right) \tag{2}$$

2. ⟨**5**⟩ Show that the heat capacity at constant volume (C_V) for a van der Waals gas is independent of volume when regarded as a function of T, V.

3. ⟨**7**⟩ Consider n moles of a van der Waals gas satisfying the equation of state

$$(p + \frac{n^2 a}{V^2})(V - nb) = nRT. \tag{3}$$

Suppose we measure p, V, T in units of their critical values

$$p_c = \frac{a}{27b^2}, \quad V_c = 3nb \quad \text{and} \quad T_c = \frac{8a}{27Rb}, \tag{4}$$

by defining $\mathcal{P} = p/p_c$, $\mathcal{V} = V/V_c$ and $\mathcal{T} = T/T_c$. Show that the EOS takes a universal form, i.e. the same form irrespective of the values of the material parameters a and b. Find this universal EOS.