**Quantum Mechanics 2, Autumn 2011 CMI**

Problem set 4

Due by beginning of class on Monday September 12, 2011

Addition of angular momenta

1. In the problem of addition of two angular momenta, find $[L^2, L_1^2]$ where $\vec{L} = \vec{L}_1 + \vec{L}_2$. Can $L^2$, $L_1^2$ and $L_2^2$ be simultaneously specified? ⟨3⟩

2. In the problem of combining two spin half particles, calculate $S^2\,(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ and check that you get the expected (what is expected?) answer. ⟨4⟩

3. Consider an electron in a $p$-wave state of a hydrogen atom. What are the possible results if the observable $\vec{J}^2 = (\vec{L} + \vec{S})^2$ is measured? ⟨4⟩

4. Suppose we form a composite system from three spin half particles (a proton is one example). Let $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$. ⟨15⟩

   (a) What is the dimension of the combined Hilbert space? ⟨1⟩

   (b) Give an 'uncoupled' basis for this tensor product Hilbert space consisting of simultaneous eigenvectors of $S_{1z}, S_{2z}$ and $S_{3z}$. Group the basis vectors by the eigenvalue of $S_z$. ⟨2⟩

   (c) What are the possible spins $s$ of the combined system and the multiplicities with which they occur? ⟨4⟩

   (d) Verify that the dimension of the resulting space of spin states in problem 4c is as found in problem 4a. ⟨2⟩

   (e) The state $\uparrow\uparrow\uparrow$ is annihilated by $S_+$. Find $u = S_-\,\uparrow\uparrow\uparrow$. ⟨2⟩

   (f) Now find a maximal set of orthonormal eigenstates of $S_z$ which are orthogonal to $u$ and annihilated by $S_+$. What is the use of these states? ⟨4⟩