Quantum Mechanics 2, Autumn 2011 CMI

Problem set 3

Due by beginning of class on Monday September 5, 2011

Angular momentum and spin

1. Recall that the angular momentum raising operator is $L_+ = \hbar e^{i\phi}(\partial_\theta + i\cot\theta\ \partial_\phi)$. Use this to find L_-.

2. Use the above formulae for $L_\pm$ to find the coordinate representation of the angular momentum basis states Y_{11}, Y_{10} and $Y_{1,-1}$ up to normalization.

3. Write out the 9 equations summarized in the formula for products of Pauli matrices

$$\sigma_i\sigma_j = \delta_{ij} + \mathrm{i}\epsilon_{ijk}\sigma_k \tag{1}$$

4. Check that these formulae hold for the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

5. The hamiltonian for an electron in a vertical magnetic field $B\hat{z}$ is

$$H = E\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad E = \frac{g|e|\hbar B}{4m}. \tag{3}$$

   Find the spin state $\psi(t)$ if the initial spin wavefunction is $\psi(t=0) = \frac{1}{\sqrt{2}}(\uparrow + \downarrow)$.

6. Compute $\langle S_z\rangle$, $\langle S_x\rangle$ and $\langle S_y\rangle$ at time t in the state $\psi(t)$

7. Physically interpret the obtained expectation values for S_x, S_y, S_z.