Quantum Mechanics 2, Autumn 2011 CMI

Problem set 1

Due by beginning of class on Wednesday August 17, 2011

Matrix representation of angular momentum

Consider the $l = 1$ subspace of the space of square-integrable functions on the sphere (i.e., functions of θ and ϕ in spherical coordinates). Choose as orthonormal basis for this vector space the spherical harmonics $Y_{11}, Y_{10}, Y_{1,-1}$ in that order.

1. What is $L_+ Y_{11}$? $\langle \mathbf{1} \rangle$

2. We will show in lecture that

$$L_+ Y_{10} = \sqrt{2}\hbar Y_{11} \quad \text{and} \quad L_+ Y_{1,-1} = \sqrt{2}\hbar Y_{10} \tag{1}$$

   Find the 3×3 matrix representation of L_+ in the above basis. $\langle \mathbf{2} \rangle$

3. Use the above result to write down the matrix representation of L_-. $\langle \mathbf{1} \rangle$

4. Find L_y in the above basis. $\langle \mathbf{2} \rangle$

5. What are the possible results if L_y is measured in a state with $l = 1$? $\langle \mathbf{2} \rangle$

6. For each possible measured value of L_y, find the state vector to which the system collapses after the measurement. $\langle \mathbf{9} \rangle$

7. Suppose the initial state of the system is Y_{10}. What are the probabilities of the various possible results of a measurement of L_y ? $\langle \mathbf{3} \rangle$