Quantum Mechanics 1, Spring 2011 CMI

Problem set 5

Due by the beginning of class on Friday February 11, 2011

Bohr atom, de Broglie hypothesis, Double slit interference

1. A photon is absorbed by a hydrogen atom that was in its first excited Bohr energy level. The hydrogen atom is ionized and in the process, an electron is emitted, moving non-relativistically with a kinetic energy of 10eV. Find the frequency of the absorbed photon in Hertz. ⟨2⟩

2. What are the frequencies of photons (near the frequency of the above absorbed photon) that can be absorbed by a hydrogen atom in its first excited state? ⟨3⟩

3. Is the energy spectrum of a system consisting of a proton and an electron discrete or continuous? ⟨2⟩

4. Where does a smooth function of one real variable $ph(k)$ not change very much? ⟨1⟩

5. Suppose the phase function $ph(k) = i(kx - \omega(k)t)$ for some appropriate smooth dispersion relation $\omega = \omega(k)$. At what k is the phase function nearly constant? ⟨2⟩

6. Assume that the dominant contribution to the wave packet integral $\psi(x,t) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{i(kx-\omega(k)t)} \tilde{\psi}(k)$ comes from the neighborhood of the value of k where the phase is nearly constant. (You may assume there is only one such value of k). This means $\psi(x,t)$ is peaked near that x where the phase is nearly constant. Find the time evolution of the point $x_{peak}(t)$ where $\psi(x,t)$ is peaked. What is the speed at which the point $x_{peak}(t)$ moves? ⟨2⟩

7. As an example, take $\omega(k) = \frac{\hbar k^2}{2m}$. Find the value of k (for given x and t) at which the phase function is nearly constant. Use this to find the group speed of the corresponding wave packet. ⟨1⟩

8. A standard double slit interference experiment is conducted with electrons as described in the lecture & text books. Suppose the intensity of electrons detected on the screen with slit S_1 alone open is $I_1(x)$ and with S_2 alone open is $I_2(x)$. Here x is the coordinate along the screen. Find the intensity on the screen when both slits are open. ⟨2⟩

9. A non-relativistic particle of mass m with energy E is moving in a potential $V(x)$. Find an expression for the de Broglie wavelength of the associated matter wave. ⟨2⟩

10. Suppose $V(x) = kx^2$ with $k > 0$ and energy $E > 0$. Where along the classical trajectory is the de Broglie wavelength maximal and minimal? ⟨2⟩

11. Where along the classical trajectory of the above particle in the quadratic potential would you expect quantum effects to be most pronounced? ⟨1⟩