Particle Physics, Autumn 2014 CMI
Problem set 9
Due at the beginning of lecture on Tuesday Feb 3, 2015
Isospin, alpha decay and null vectors

1. $\langle \mathbf{4} \rangle$ We saw that baryons and mesons do not have isospin I more than $3/2$ and 1 respectively. But nuclei can have higher isospin. Consider a nucleus with baryon number $B = A$ and atomic number Z. What is its I_3 value? Give all the isospin multiplets (i.e. values of I) of which the nucleus could be a member of. Hint: There is a lowest and a highest possible I.

2. $\langle \mathbf{5} \rangle$ The integral $I = \int_\epsilon^1 \sqrt{\frac{1}{r} - 1} dr$ appears in estimating the tunneling probability of alpha particles from a nucleus. Show that for $1 > \epsilon \geq 0$,

$$I = \frac{\pi}{2} - 2\sqrt{\epsilon} + \cdots \qquad (1)$$

Hint: Express the integral as the difference between an integral over $[0, 1]$ and over $[0, \epsilon]$. Evaluate the first by trigonometric substitution and the second by expanding in powers of small r.

3. $\langle \mathbf{6} \rangle$ Suppose w^μ and p^μ are two *non-zero light-like* 4-vectors that are orthogonal with respect to the Minkowski inner product $p^\mu w_\mu = 0$. Show that $w^\mu = \lambda p^\mu$ for some constant λ. Hint: Before considering the general case where $p^\mu = (p^0, \vec{p})$ and $w^\mu = (w^0, \vec{w})$, try the special case where $p^\mu = (E, 0, 0, E)$. [For future reference: Appropriately interpreted, λ is the helicity and $|\lambda|$ the spin of the massless particle with momentum p and 'Pauli-Lubanski vector' w.]