Particle Physics 2, Spring 2015 CMI

Problem set 6

Due at the beginning of lecture on Tuesday Jan 13, 2015

Reducible and adjoint representations of Lie algebras

1. $\langle \mathbf{6} \rangle$ Show that the operators $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, $\hat{x} = i\hbar\frac{\partial}{\partial p} + x$ acting on functions on phase space $\psi(x, p)$ furnish a representation of the Heisenberg algebra $[\hat{x}, \hat{p}] = i\hbar, [\hat{x}, \hat{x}] = 0 = [\hat{p}, \hat{p}]$. Is this representation reducible? Hint: Look for an invariant subspace on which this representation restricts to the standard Schrödinger representation.

2. $\langle \mathbf{6} \rangle$ Define three 3×3 matrices I_1, I_2, I_3 with matrix elements $(I_\alpha)_{ab} = -i\epsilon_{\alpha ab}$ where $\epsilon_{\alpha ab}$ is the totally anti-symmetric Levi-Civita symbol. Show (using only the total antisymmetry of $\epsilon_{\alpha\beta\gamma}$) that I_α furnish a 3d representation of the SU(2) Lie algebra $[I_\alpha, I_\beta] = i\epsilon_{\alpha\beta\gamma} I_\gamma$. After checking that the commutation relations are satisfied, write out the 3 matrices I_α and say whether any generator is diagonal in this representation. Note: This is called the adjoint representation of the Lie algebra and generalizes to any set of totally anti-symmetric structure constants $C_{\alpha\beta\gamma}$.