Nonlinear Dynamics, Spring 2020 CMI

Problem set 9

Due by 12 noon on Tuesday Mar 31, 2020

Hamiltonian systems and recurrence

1. ⟨**13**⟩ Consider a Hamiltonian system with planar x-p phase space and real Hamiltonian $H(x,p)$. Suppose (x_*, p_*) is a fixed point of the Hamiltonian vector field.

   (a) ⟨**2**⟩ Find the Jacobian matrix J of the linearization of the Hamiltonian vector field around the fixed point.

   (b) ⟨**3**⟩ What is the trace of J? What are the possible forms of the eigenvalues of J?

   (c) ⟨**3**⟩ Assuming the eigenvalues are not both zero, what sort of (linear) fixed points do the above possibilities correspond to?

   (d) ⟨**5**⟩ For the free particle ($H = p^2/2m$), what are the fixed points of the Hamiltonian vector field? What is J at these fixed points and how many linearly independent eigenvectors does it have? Draw the phase portrait showing at least three qualitatively different trajectories.

2. ⟨**5**⟩ Does the motion of a free particle on a line display Poincaré recurrence? What about the dynamics of the Kepler problem? Why or why not? Hint: Think about the various types of motion that these systems display.