Nonlinear Dynamics, Spring 2020 CMI

Problem set 8

Due by 5pm on Friday Mar 27, 2020

Conservative systems: Double well potential

1. $\langle \mathbf{25} \rangle$ Consider the motion of a particle of mass m in a double-well potential $V(x) = (g/4)(x^2 - a^2)^2$ with $g, a > 0$.

   (a) $\langle \mathbf{2} \rangle$ Plot the potential and indicate the value of the height of the potential barrier between wells.

   (b) $\langle \mathbf{2} \rangle$ Write Newton's 2nd law as a pair of first order equations for x and $p = m\dot{x}$

   (c) $\langle \mathbf{2} \rangle$ Identify a conserved energy (Hamiltonian H) and verify that Hamilton's equations reduce to these first order equations.

   (d) $\langle \mathbf{2} \rangle$ Find the fixed points of the Hamiltonian vector field.

   (e) $\langle \mathbf{4} \rangle$ Find the linearization (Jacobian matrix) of the vector field around the fixed points. What does the linear theory predict about the fixed points.

   (f) $\langle \mathbf{4} \rangle$ Argue what the nonlinear nature of the fixed points must be.

   (g) $\langle \mathbf{9} \rangle$ Use the conservation of energy to sketch a phase portrait showing all fixed points and at least 5 qualitatively different types of trajectories with arrows indicating direction of motion. Mention the physical nature of the motion along each of the trajectories. Mention the composition of the *separatrix* (that separates motion that is confined to one or other potential well and that where the motion is not confined). Hint: Energy level curves must be (unions of) trajectories.