Nonlinear Dynamics, Spring 2020 CMI
Problem set 14
Due by noon on Wednesday April 22, 2020
Bifurcations in 2d

1. $\langle \mathbf{8} \rangle$ Consider the planar vector field

$$\dot{x} = x(a - x^2), \quad \text{and} \quad \dot{y} = -y. \tag{1}$$

Draw the phase portraits for two typical values of a ($a < 0$ and $a > 0$), indicating the qualitatively different trajectories and stable/unstable fixed points. What kind of bifurcation does it display at $a = 0$ (suggest a suitable name)?

2. $\langle \mathbf{15} \rangle$ Consider the vector field in plane polar coordinates

$$\dot{r} = ar + r^3 - r^5 \quad \text{and} \quad \dot{\theta} = \omega + br^2. \tag{2}$$

We will suppose that b and ω are fixed (and say positive for simplicity) and that a is varied. (i) $\langle \mathbf{3} \rangle$ Find the nature of the fixed point at the origin as a function of a mainly by analysing the radial equation. (ii) $\langle \mathbf{4} \rangle$ Working in cartesian coordinates, find the Jacobian at the origin and its eigenvalues and describe their behavior as a crosses zero. (iii) $\langle \mathbf{4} \rangle$ Find the limit cycles for $-1/4 < a < 0$ and $a > 0$. (iv) $\langle \mathbf{4} \rangle$ Sketch a phase portrait for two values of a ($-1/4 < a < 0$ and $a > 0$).