Nonlinear Dynamics, Spring 2020 CMI
Problem set 13
Due by noon on Tuesday April 14, 2020
Periodic trajectories, Poincaré recurrence

1. $\langle \mathbf{7} \rangle$ Consider the system on the x-y plane:

$$\dot{x} = x - x(x^2 + y^2) \quad \text{and} \quad \dot{y} = y - y(x^2 + y^2). \tag{1}$$

   Show that it has no periodic trajectories by expressing it as a *gradient flow* for a suitable potential function $W(x, y)$. Plot a graph of W and describe its shape. What is the circle $x^2 + y^2 = 1$ in this system?

2. $\langle \mathbf{6} \rangle$ **Poincaré recurrence:** Let N_0 be a connected, simply connected open set in the phase space of a Hamiltonian dynamical system. There can be initial conditions lying in N_0 for which a trajectory that begins there returns to N_0 a finite number of times, and then stops doing so. Give an example where a trajectory intersects N_0 two times and never comes back after exiting for the second time. Hint: Try out the Hamiltonian systems we have studied. Use the freedom to pick N_0 as you wish.