Nonlinear Dynamics, Spring 2020 CMI
Problem set 12
Due by 5pm on Friday April 10, 2020
Limit Cycles

1. $\langle \mathbf{6} \rangle$ Give a simple example of a vector field on the plane that displays a stable limit cycle. Mention the limit cycle. Hint: Use polar coordinates.

2. $\langle \mathbf{10} \rangle$ A non-trivial example of a limit cycle arises in the equation:

$$m\ddot{x} = -kx - \mu(x^2 - 1)\dot{x} \quad \text{for} \quad m, k, \mu > 0. \tag{1}$$

(a) $\langle \mathbf{3} \rangle$ Describe the qualitative physical character of the nonlinear term by comparing it with equations we have studied. (b) $\langle \mathbf{4} \rangle$ Write the equation as a 1st order pair and discuss the nature of its fixed points as a function of the parameters m, k and μ. (c) $\langle \mathbf{3} \rangle$ For *large* x and p approximate the equations by keeping only the leading terms and mention the qualitative behavior (do x and/or p grow/decay/remain bounded?).