Mathematical Physics 1 - Linear Algebra

Chennai Mathematical Institute, B.Sc. Physics 1st Year
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Syllabus (Rough list of topics covered)

1. Physical examples of Vectors and Matrices
2. Dot product
3. Systems of Linear equations
4. Linear combinations, linear independence
5. Matrix multiplication
6. Gaussian elimination, augmented matrix
7. Elementary row operations and echelon form, $A = LU$
8. Pivots, Rank of a matrix
9. Permutation matrices
10. Inverse of a matrix
11. Transpose, Symmetric matrix
12. Vector spaces: real and complex,
13. span of vectors, subspace, basis, dimension, intersection, sum
14. Examples of vector spaces
15. Linear transformations, isomorphism
16. Matrix of a linear map
17. Gauss-Jordan elimination for A^{-1}
18. Vector-spaces associated to a matrix (Column space, Null space (kernel), Row space)
19. Dimension of kernel and rank-nullity theorem
20. Compatibility of $Ax = b$ and adjoint equations
21. General solution of $Ax = b$: homogeneous and particular solution
22. Inner product, norm, right triangles.
23. Orthogonality, orthogonal complement
24. Triangle and Cauchy-Schwarz inequality
25. Orthonormal basis

26. Orthogonal projection to a subspace
27. Projection matrices, rank-1 matrices
28. Trace of a matrix
29. Best possible solution of over-determined systems $Ax = b$
30. Normal equation $A^T A\hat{x} = A^T b$.
31. Application to curve fitting of experimental data
32. Positive matrix
33. Orthogonal and unitary matrices
34. Matrix exponential
35. Gram-Schmidt orthogonalization
36. Transformation of vectors and matrices under change of basis
37. Similarity or general linear transofrmations
38. Invariance of matrix equations under orthogonal and general linear changes of basis
39. Determinant: Invertibility and volume of parallelopiped
40. Properties and formulas for determinant
41. Cofactor formula for inverse and Cramer's rule
42. Jacobian determinant and change of integration variables
43. Eigenvalues
44. Characteristic polynomial and Cayley-Hamilton theorem
45. Eigenvectors
46. Diagonalizaion of a matrix with distinct eigenvalues
47. Principle axis transformation, quadratic forms
48. Reality of spectrum of hermitian matrices, orthogonality of eigenvectors
49. Spectrum of real symmetric, hermitian, orthogonal and unitary matrices
50. Powers of a matrix and matrix exponential via diagonalization
51. Passage to normal modes for coupled oscillations of bodies connected by springs
52. Hilbert spaces: complete orthonormal basis, Dirac bra-ket notation, completeness relation.